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Fuzzy Relation Properties and Fuzzy Composition Deep Explain with RTAs

Sophiya A[1], Subramanian S[2]
[1]Research Scholar, Department of Mathematics, PRIST University, Thanjavur, Tamilnadu, India
E-mail:sophiyajohn1229@gmail.com
[2]Professor, Department of Mathematics, PRIST University, Thanjavur, Tamilnadu, India
E-mail:mathsspmanian@gmail.com

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Abstract:
As the number of fatalities on the roads rises, analysts forced to look for models that might foretell a driver's tendency for traffic accidents (RTAs). This study intends to investigate the relationship between drivers' abilities to judge speed and available space in relation to the occurrence of RTAs. The Circulant Composition of Fuzzy Relations Inference System employed as the foundation for the approach used for this goal (CCFRIS). The first CCFRIS's inputs related to drivers' capacity for speed assessment. 200 novice drivers participated in the trial, which tested these skills at test speeds of 35, 55, and 75 km/h. In the driving simulator, the participants evaluated the aforementioned speed numbers from four distinct vantage points. On the other side, space evaluation capabilities of drivers make up the second CCFRIS's inputs. The identical set of drivers underwent both 2D and 3D space evaluation exams. The quantity of RTAs a motorist has experienced is the third CCFRIS structure taken into account. The research found that the space assessment qualities explained participation in RTAs better than the drivers' speed assessment abilities after testing three proposed CCFRIS on empirical data. This study also looks into binary fuzzy relations and fuzzy relation composition characteristics.

Keywords: Fuzzy relation properties, Relational join, Binary relation, fuzzy composition.

1. Introduction

The number of road traffic accidents (RTAs) and fatalities on roadways is one of the largest issues facing the world today. RTAs result in 1.24 million fatalities and 20 to 50 million injuries annually [1,2]. Road safety is a social and economic problem that causes serious physical and mental harm as well as enormous property losses [3]. In emerging and poor nations, the expenses of RTAs range from 1.5% to 2.5% of gross domestic product [2,4]. By 2030, RTAs may overtake all other causes of death, according to the present trends, unless immediate action is taken. Additionally, 92% of these fatalities take place in developing and poor nations [2,3,5].

Car affordability rises as a result of technological advancement, which causes a sharp rise in vehicle ownership. On the one hand, this increases the risk of RTAs happening, but on the other side, it generates a lot more RTA data, providing more opportunities to learn new things about traffic safety. Many universities, research institutes, and safety organisations examine RTA data in order to suggest sufficient safety measures due to the ongoing improvement of research methodologies, computer performance, information processing, etc. [6,7].

The abundance of RTA data unavoidably results in more explanatory variables, which could lead to more precise models for modelling the occurrence of RTAs. But it's well recognised that "more is not always better," particularly when it comes to RTA prediction, because too many variables can lead to model overfitting. Additionally, this may have an effect on related tasks including lengthy execution times and incorrect forecast outcomes.

New kinds of cars are created as a result of the advancement of technology and software. Through car tracking and smartphone applications that identify unsafe driving behaviours, such as speeding or illegal lane switching, safer driving can be identified in the current market for new vehicles. Additionally, the introduction of partially or fully autonomous vehicles can dramatically improve traffic safety by lowering the number of RTAs. Even for the most progressive markets, this is still a far cry from reality.

RTAs are caused by a variety of random circumstances, but they are most frequently categorised by the features of the road, the vehicle, and the driver, such as vehicle speed, aggressiveness, skills, etc. A driver's attention will be on the traffic issue if they have mastered spatial abilities while driving; otherwise, it will be diverted and focused on spatial orientation. On the other hand, according to some reports, nearly one-third of all fatal RTAs are caused by speeding.

Given everything mentioned above, one reason for conducting the research reported in this work is to provide a model whose application would help to explain the causes of RTAs.

The goal of the experimental investigation was to determine how vehicle speed and drivers' abilities to judge their surrounding environment affect the frequency of RTAs.

Speed is linked to both the likelihood of being involved in an RTA and the severity of a crash. Speeding, driving too fast for the road conditions, inappropriate lane changes, and improper passing were the most common types of unsafe driving in the late 20th century. Numerous empirical research have typically shown a correlation between the occurrence of RTAs and traffic speed. However, several research have shown that increased speeds do not always translate into more accidents. It is recommended that the speed-RTA relationship be taken into consideration in the appropriate context, taking into account any potential extra complicating factors including traffic exposure and road conditions, but also taking into account a vehicle's perceived speed. An increased chance of getting in a collision is linked to both speeding and erroneous vehicle speed.

The survey carried out by the state authorities in the period from 2017 to 2019 illustrates the current situation in the subject field in the Republic of Serbia, the nation where our research was done. According to this survey, roughly half of drivers in metropolitan areas drive faster than the posted limit (2017: 49.4%; 2018: 48.4%; 2019: 51%). The statistics for this time period show that the majority of the worst RTAs, as well as the majority of fatalities, fall into the category of unadjusted speed (53.14% in 2017, 52.14% in 2018, and 51.41% in 2019 of the cases where unadjusted speed is the cause of RTAs with fatalities). Based on participant replies, the Road Traffic Safety Agency in Serbia estimates that every third passenger car driver in metropolitan areas exceeds the speed limit by an average of more than 10 km/h. The findings reveal that nearly two-fifths of participants drive faster than the speed limit when it comes to the speed of cars outside of urban areas. The majority of drivers (41.7%) do not think that speeding in crowded areas by 10 km/h increases the probability of being involved in an RTA, according to a survey on drivers' views on speeding in the Republic of Serbia. The psychological features of the driver, driving abilities and restrictions, road and environmental variables, vehicle characteristics, as well as the driver's spatial perception skills, all have an impact on the degree of vehicle speed that is chosen.

Processes including the sense of time, speed, and most importantly space are all part of driving. The importance of these abilities for driving safely is extensively discussed in the literature. Offering a model whose implementation would add to the explanation of the causes of RTAs is one of the motivations for conducting the research discussed in this paper. The goal of the experimental investigation was to determine how vehicle speed and drivers' abilities to judge their surrounding environment affect the frequency of RTAs.

We thought that implementing fuzzy inference systems would be a useful tool for data processing because participant responses linked to the assessment of speed and space entail a certain level of imprecision and fuzziness. Driver behaviour is frequently explained using fuzzy reasoning. A connection is essential in all engineering disciplines. Relations can also be used to illustrate similarity. Relationships are a part of logic, classification, pattern recognition, and control. Some relationships exist between components of the same universe. One measurement is greater than another, one circumstance occurred before another, one object resembles another, and so on. The measurement is enormous and its rate of change is positive, for instance, or the x-coordinate is vast and the y-coordinate is little, to name a few relationships that incorporate elements from many universes. These are interactions between two things, but relationships between any number of things are theoretically possible. This idea can be broadened to include different levels of connection or interaction between the parts. Furthermore, fuzzy relations are crucial since they aid in describing how different variables interact.

2. Preliminary

An $n \times n$ CCFRIS has the form

$$\begin{bmatrix} a_1 & a_2 & a_3 & \cdots & \cdots & a_{n-1} & a_n \\ a_n & a_1 & a_2 & \cdots & \cdots & a_{n-2} & a_{n-1} \\ \vdots & \vdots & \ddots & & & \vdots & \vdots \\ \vdots & \vdots & & \ddots & & \vdots & \vdots \\ \vdots & \vdots & & & \ddots & \vdots & \vdots \\ a_3 & a_4 & a_5 & \cdots & \cdots & a_1 & a_2 \\ a_2 & a_3 & a_4 & \cdots & \cdots & a_n & a_1 \end{bmatrix}$$

Thus, a CCFRIS matrix is resolute by its first row.

Let F_{mn} signify the set of all $m \times n$ matrices over F, if $m = n,$, in short, we write F_n elements of F_{mn} are called as grade value matrices, binary fuzzy relation matrices (or) in short, fuzzy matrices.

A Fuzzy matrix $\tilde{A} = [a_{ij}^{\tilde{A}}]$ is said to be CCFRIS fuzzy matrix if all the elements of $\tilde{A}$ can be resolute completely by its first row. Supposing the first row of $\tilde{A}$ is $[a_1, a_2, ..., n]$. Then any element a_{ij} of $\tilde{A}$ can be resolute [during the element of the first row] as $a_{ij} = a_{1(n-i+j+1)}$ with $a_{1(n+k)} = a_{1k}$.

A CCFRIS fuzzy matrix is the form of

$$\begin{bmatrix} a_1 & a_2 & a_3 & \cdots & \cdots & a_{n-1} & a_n \\ a_n & a_1 & a_2 & \cdots & \cdots & a_{n-2} & a_{n-1} \\ \vdots & \vdots & \ddots & & & \vdots & \vdots \\ \vdots & \vdots & & \ddots & & \vdots & \vdots \\ \vdots & \vdots & & & \ddots & \vdots & \vdots \\ a_3 & a_4 & a_5 & \cdots & \cdots & a_1 & a_2 \\ a_2 & a_3 & a_4 & \cdots & \cdots & a_n & a_1 \end{bmatrix}$$

With entries in [0 , 1].

Let $\tilde{A} = [a_{ij}^{\tilde{A}}] \in$ circulant $FM_{m \times n}$, according to the description in the illustration of the complement of the Fuzzy Matrix $\tilde{A}$ which is denoted by $\tilde{A}^{\circ}$ and then $\tilde{A}^{\circ}$ is so-called CCFRIS fuzzy complement matrix if $\tilde{A}^{\circ} = [(1 - a_{ij}^{\tilde{A}})]m \times n$ for all $a_{ij}^{\tilde{A}}$ [0,1]. Then the matrix obtained from so so-called grade value would be the subsequent $\tilde{A}^{\circ} = a_{ij}^{\tilde{A}} = [(1 - a_{ij}^{\tilde{A}})]$ for all i and j.

Let $\tilde{A} = [a_{ij}^{\tilde{A}}] \in$ circulant FM $m \times n$, represent the fuzzy CCFRIS function of N_i. Also let $\tilde{B} = [b_{jk}^{\tilde{B}}]_{n \times p}$ characterize the fuzzy grade function N_i. We now define $\tilde{A} \circ \tilde{B}$ the product of $\tilde{A}$ and $\tilde{B}$ as $\tilde{A} \circ \tilde{B} = [d_{jk}^{\tilde{A}\tilde{B}}]_{m \times p} = \max\min(\mu_{ij}^{\tilde{A}}, \mu_{jk}^{\tilde{B}})_{m \times p,}$ $1 \le i \le m,\ 1 \le k \le p$ for $j = 1,2,3,...,n.$

Fuzzy composition

Fuzzy relations

A fuzzy relation is a mapping from $\tilde{A} \to \tilde{B}$ that contains fuzzy subsets $\tilde{A} \times \tilde{B}$. Fuzzy relationships are frequently used and are very important. Only binary relationships, or relationships between two sets, have been taken into consideration. Relation can be unary, binary, ternary, etc.

Let $\tilde{A}, \tilde{B} \subseteq \mathbf{R}$ be general set, then

$\tilde{R} = \{((\tilde{a}, \tilde{b}), \mu_{\tilde{R}}(\tilde{a}, \tilde{b})) \,|\, (\tilde{a}, \tilde{b}) \in \tilde{A} \times \tilde{B}\}$ is so-called a fuzzy relation from $\tilde{A}$ to $\tilde{B}$.

Binary fuzzy relation:

They have a particular connotation among n-dimensional relations because they are, in a certain sense, binary fuzzy relations.

The generalized mathematics function is different from $A \to B$ the function from. Each element of A may be assigned two or more items of B in a binary connection. Binary relations can also be employed with several fundamental operations, such as inverse and compositions. A fuzzy set on a domain R whose membership functions are defined by is given a fuzzy relation.

$$dom\, R(a) = \max_{b \in B} R(a,b)$$

for every $a \in A$i.e. Each component of set A belongs to the domain of R to the extent that its strongest possible relation to any member of set B exists.

whose membership function is defined by, the range is a fuzzy relation on the B range R.

$$ran\, R(a) = \max_{a \in A} R(a,b)$$

The strength of the strongest relationship that each element of a can have to another element of a determines how much of that element belongs in the range of R for each instance of $b \in B$ i.e.

The fuzzy relation's member's height is determined by

$$h(R) = \max_{b \in B} \max_{a \in A} R(a,b)$$

i.e., $h(R)$ is the largest grade reached by the pair (a,b) in R.

A expedient representation of binary relation $R(a,b)$ are grades matrices $R[r_{ab}]$ where

$$r_{ab} = R_{(a,b)}$$

Another useful way to depict binary relations is via a "sagittal diagram," in which each of the sets (a,b) is represented by a set of nodes that represents the other set. For elements having non-zero membership grades, lines linking the pertinent nodes are labelled with the values of the membership grade.

The solution on defined by is the inverse of the fuzzy relation represented by $B \times A$

$$R^{-1}(b,a) = R(a,b)\ \forall a \in A, b \in B$$

A grade matrix $R^{-1}[r_{ba^{-1}}]$ representing $R^{-1}(B,A)$ is the rearrange of the matrix R for $R\ (A,B)$ which means that the rows of R^{-1} is equal to columns of R and the column of R^{-1} equal to the row of R. clearly $(R^{-1})^{-1} = R$ for any binary relation.

Contemplate now two binary relations $P(A,B)$ *and* $Q(B,A)$ with a common set B.

This relation's ordinary composition, which is denoted by

$$P(A,B) \circ Q(B,A)$$

Yields a binary relation $R(A,C)$ *on* $A \times C$ well-defined by

$$R(a,c) = [P \circ Q](a,z)$$
$$= \max_{b \in B} \min[P(a,b),Q(b,c)] \ \forall a \in A, c \in C$$

Because it is based on accepted t-norms and t-conorms, this composition is also known as the max-min composition.

$$[P(a,b) \circ Q(b,c)]^{-1} = [Q^{-1}(c,b) \circ P^{-1}(b,a)]$$
$$[P(a,b) \circ Q(b,c)] \circ R[a,d] = P[a,b] \circ \{[Q(b,c)] \circ R[c,d]\}$$

i.e., Associative composition is the norm, and the composition of the inverse relation is the composition of the norm. The membership matrices of a binary fuzzy relation make it simple to compose the relation.
Let

$$P = [P_{ik}], Q = [q_{kj}], R = [r_{ij}]$$

Be grade matrices of binary relation. Such that $R = P \circ Q$. We can write using matrix representation

$$[r_{ij}] = [P_{ik}] \circ [q_{kj}]$$
$$= \max_{k} \min[P_{ik}, q_{kj}]$$ Is called relational join.

The minimal and maximum operations are used in place of the product and sum operations when computing "r," but the same "p" and "q" elements are used as they are when multiplying matrices.

3. Methodology

Application of CCFRIS Matrix in RTAs

The research was done in Belgrade at the Faculty of Transport and Traffic Engineering's Laboratory of Traffic Psychology. 200 new drivers were recruited to take part in the experiment (120 males and 80 females). The mean age of our participants was 23.05 years; the standard deviation (SD) was 2.42. All participants who took part in both trials and filled out questionnaires on their demographic information and engagement in RTAs followed the identical experimental protocol in terms of instructions, testing methods, and data collection. The participants, attesting to their willingness to participate in the study, also completed a written voluntary informed agreement. For taking part in the survey, the participants were not compensated in any way.

Algorithm

Step 1

Input the CCFRIS value over the set of Speed *S* over Assessment capabilities of drivers *D* and write the input value over the set of Speed *S* over *D* denoted by the knowledge matrix $\tilde{R}_1$ and $\tilde{R}_2$ respectively.

Step 2

Input the CCFRIS over the set *P* of drivers over *S* and write its relation *Q*.

Step 3

Compute the relation matrices under the composition (°).

(i) $\tilde{T}_1 = Q \circ \tilde{R}_1$

(ii) $\tilde{T}_2 = Q \circ R_2$

(iii) $T_3 = Q \circ (J - \tilde{R}_1)$

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Where J is the matrix with all its entries 1, which is the greatest element of F.

(iv) $\tilde{T}_4 = \tilde{Q} \circ (J - \tilde{R}_2)$

Step 4

Compute the diagnosis score $\tilde{SR}_1$ and $\tilde{SR}_2$.

$$ST_1 = \max\{T_1(p_i, d_j), T_3(p_i, d_j)\} \text{ for } i = 1,2,3,\ j = 1,2,3$$

$$ST_2 = \max\{T_2(p_i, d_j), T_4(p_i, d_j)\} \text{ for } i = 1,2,3,\ j = 1,2,3$$

Step 5

Find $S_k = \max[ST_1(p_i, d_j) - ST_2(p_i, d_j)]$ then we conclude the drivers p_i is affect from the assessment capabilities of drivers d_k.

Step 6

If S_k has more than one value then go to step 1 and repeat the process by reassessing the Speed for the driver.

Case Study

Let us consider 3 drivers Ravi, Mohan and Muthu are denoted by the set P = {Ravi, Mohan, Muthu} and the set of Speed S = {35Km/h, 55Km/h, 75Km/h}. Let the set of assessment capabilities of drivers be D = {Self Discipline assessment capabilities of drivers, Alertness assessment capabilities of drivers, Mechanical Skills assessment capabilities of drivers}.

Step 1

$$\tilde{R}_1 = \begin{pmatrix} 1 & 0.6 & 0.9 \\ 0.9 & 1 & 0.6 \\ 0.6 & 0.9 & 1 \end{pmatrix} \qquad \tilde{R}_2 = \begin{pmatrix} 0.6 & 1 & 0.5 \\ 0.5 & 0.6 & 1 \\ 1 & 0.5 & 0.6 \end{pmatrix}$$

Step 2

$$Q = \begin{pmatrix} 1 & 0.6 & 0.7 \\ 0.7 & 1 & 0.6 \\ 0.6 & 0.7 & 1 \end{pmatrix}$$

Step 3

$$\tilde{T}_1 = \begin{pmatrix} 1 & 0.7 & 0.9 \\ 0.9 & 1 & 0.7 \\ 0.7 & 0.9 & 1 \end{pmatrix} \qquad \tilde{T}_2 = \begin{pmatrix} 0.7 & 1 & 0.6 \\ 0.6 & 0.7 & 1 \\ 1 & 0.6 & 0.7 \end{pmatrix}$$

$$\tilde{T}_3 = \begin{pmatrix} 0.4 & 0.4 & 0.4 \\ 0.4 & 0.4 & 0.4 \\ 0.4 & 0.4 & 0.4 \end{pmatrix} \qquad \tilde{T}_4 = \begin{pmatrix} 0.5 & 0.6 & 0.6 \\ 0.6 & 0.5 & 0.6 \\ 0.6 & 0.6 & 0.5 \end{pmatrix}$$

Step 4

$$\tilde{ST}_1 = \max(\tilde{T}_1, \tilde{T}_3)$$

$$\tilde{ST}_2 = \max(\tilde{T}_2, \tilde{T}_4)$$

$$\tilde{ST}_1 = \begin{pmatrix} 1 & 0.7 & 0.9 \\ 0.9 & 1 & 0.7 \\ 0.7 & 0.9 & 1 \end{pmatrix} \qquad \tilde{ST}_2 = \begin{pmatrix} 0.7 & 1 & 0.6 \\ 0.6 & 0.7 & 1 \\ 1 & 0.6 & 0.7 \end{pmatrix}$$

Step 5

Now we have the difference for and against the assessment capabilities of drivers

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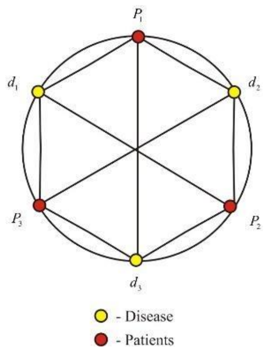
$\tilde{S}_{T_1} - \tilde{S}_{T_2}$	d_1	d_2	d_3
P_1	0.3	-0.3	0.3
P_2	0.3	0.3	-0.3
P_3	-0.3	0.3	0.3

This can be represented in the form of a graph namely network as follows:

CCFRIS graph between drivers and assessment capabilities of drivers



The graph above makes it clear that, if the RTAs is on board, Ravi affected from the alertness evaluation of drivers' abilities, Mohan is affected by the mechanical skills assessment of drivers' abilities, and Muthu is affected by the self-discipline assessment of drivers' abilities.

4. Results

In this attempt, 35 km/h, 55 km/h, and 75 km/h are taken as significant parameters for assessing the self-discipline, alertness, and mechanical skills of drivers based on a decision-making challenge with the operation of the CCFRIS matrix. This technique is used to accurately determine a driver's ability to judge their own self-discipline, alertness, and mechanical skills.

5. Conclusion

One area where the viability of fuzzy set theory was early recognized was RTAs. Clarify the CCFRIS theory matrices in the area of human evaluation capabilities of driver diagnostics in this study. This work strengthens a few novel ideas, such as the complement of the maximum CCFRIS matrix. The Department of Motor Vehicles typically learns information about a motorist via their prior behavior, the results of laboratory tests, and other investigative techniques. There are various levels of uncertainty associated with the information offered by each of these sources. As a result, the greatest and most helpful descriptions of the assessment skills of driving entities frequently employ ambiguous language. Fuzzy set structure has thus been used in this study in a variety of ways to simulate the assessment, diagnosis, and decision-making processes. In conclusion, it is clear from the study above that Ravi affected from the alertness evaluation of drivers' abilities, Mohan is affected by the mechanical skills assessment of drivers' abilities, and Muthu is affected by the self-discipline assessment of drivers' abilities. It would be necessary to do additional research in this area to see whether the theories advanced in this study provide positive outcomes.

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