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Generalization of ZZ Transform and Its Applications

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Abstract: ZZ integral transformation was presented via Zain UI A. Zafar in year (2016), [1,6], In this research paper, a generalized form of ZZ integral transform has been proposed, the general transform is introduced as a new integral transform called the " general ZZ integral Transform". The general ZZ Transform has been studied and proven for some fundamental functions It's applicability and ability to evaluate the exact solution have been proven via utilizing the transformation in solving n^{th} order ordinary differential equations.

Keywords: Integral transformation; ZZ Transform; Inverse of ZZ Transform, Ordinary differential equations.

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1. Introduction

In engineering and science ordinary differential operator equations (OD OE'S) play substantial role [2,5]. In order to find the exact Solution of the certain differential operator equations, the integral operator equations were extensively applied. The more importance of an integral transformations technique is that they provide powerful operational techniques to evaluate the solution of the initial value problems (I.V.P's) [3,4]. Mostly we apply Laplace integral transformation method to evaluate the exact solution of differential operator equations. The general ZZ integral transform technique is the new general integral transform technique and is very useful to find the exact solution of differential equations. In this paper, we investigate a general integral transform which is based on the previous integral transformations but in another domain. This integral transformation takes another form in process of transforming an equation and gets the solution of the equation where in it follows the set of functions C, an exponential order defined by:

$$C = \{\kappa(t) \colon \exists\, N, S_1, S_2 > 0, |\kappa(t)| \; > N e^{S_j |t|},$$
$$t \in (-1)^{j-1} \times (-\infty, 0] \;\; j = 1\,,2 \tag{1}$$

For the set-in equation (1), the arbitrary Constant N must be a finite and S_1 and S_2 can be infinite or finite constants. Introducing a general ZZ integral transform which is defined in equation (1). Let $\kappa(t)$ be function defined $\forall\ t \geq 0$. The general ZZ transform of $\kappa(t)$ is defined as:

$$H_g\{\kappa(t)\} = q(s,v)\int_{t=0}^{\infty} e^{-p(s,v)t}\,\kappa(t)dt\,. \tag{2}$$

Where q and p are functions of a parameters s and v.

2. Existence of General ZZ Transform

(i). If h(t) is a piecewise continuous function in every finite interval $0 \le t \le k$ and of exponential order $\lambda \quad \forall\, t \ge k$, then its kernel of general ZZ transform technique $H_g\{\kappa(t)\}$ exists $\forall\, s > \lambda$ and $v > \lambda^2$

(ii). General ZZ Transform of some Required Function

Properties:

1. If κ (t)=k, where k is a constant, then
   $H_g\{\kappa(t)\} = \frac{k\, q(s,v)}{p(s,v)}$, $p(s,v) \neq 0$.
   Proof: By definition,
   $H_g\{k\} = q(s,v)\int_{t=0}^{\infty} e^{-p(s,v)t}\,k\,dt = \frac{-k\,q(s,v)}{p(s,v)}\left[\,e^{-p(s,v)t}\right|_{t\,=\,0}^{\infty} = \frac{k\,q(s,v)}{p(s,v)}$, $p(s,v) \neq 0$
2. If κ (t)= e^{at} , where a is a constant, then
   $$H_g\{e^{at}\} = \frac{q(s,v)}{p(s,v)-a} \qquad , \; p(s,v) > a$$
   Proof: By definition,
   $$H_g\{e^{at}\} = q(s,v)\int_{t=0}^{\infty} e^{-p(s,v)t}\,e^{at}\,dt\,, = q(s,v)\int_{t=0}^{\infty} e^{-[p(s,v)-a]t}\;dt\,,$$
   $$= \frac{-\,q(s,v)}{p(s,v)-a}\left[\,e^{-\,[p(s,v)-a]t}\right|_{0}^{\infty} = \frac{q(s,v)}{p(s,v)-a}\;.$$
3. If κ (t)= cos (at) , where a is a constant, then
   $$H_g\{\cos\,(at)\} = \frac{p(s,v).\,q(s,v)}{[p(s,v)]^2 + a^2}\;.$$
   Proof: By definition,
   $$H_g\;\{\cos\,(at)\} = q(s,v)\int_{0}^{\infty} e^{-p(s,v)t}\cos(at)\,dt,$$
   $$= \frac{q(s,v)}{2}\int_{0}^{\infty} e^{-p(s,v)t}(e^{ait} + e^{-ait})\;dt\,,$$
   $$= \frac{q(s,v)}{2}\left[\int_{t=0}^{\infty} e^{-\,(p(s,v)+ai)t}\,dt + \int_{0}^{\infty} e^{-\,(p(s,v)-ai)t}\;dt\right],$$
   $$= \frac{q(s,v)\,.}{2}\Big[\frac{1}{-(p(s,v)+ai)}\left[\,e^{-\,[p(s,v)+ai]t}\right|_{t\,=\,0}^{\infty}$$
   $$+\frac{1}{-\,(p(s,v)-ai)}\left[\,e^{-\,[p(s,v)-ai]t}\right|_{0}^{\infty},$$
   $$= \frac{q(s,v)}{2}\Big[\frac{1}{(p(s,v)+ai)} + \frac{1}{(p(s,v)-ai)}\Big]\,,$$

$$= \frac{q(s,v)}{2} . \left[\frac{p(s,v) - ai + p(s,v) + ai}{(p(s,v))^2 + a^2} \right],$$

$$= \frac{p(s,v)q(s,v)}{(p(s,v))^2 + a^2} \ .$$

4. If κ (t)= sin (at) , then

   $H_g\{sin\ (at)\} = \frac{aq(s,v)}{(p(s,v))^2 + a^2}$

   Proof: By definition,

$$H_g\{\sin\ (at)\} = q(s,v) \int_{t=0}^{\infty} e^{-p(s,v)t} \sin(at)\, dt,$$

$$= \frac{q(s,v)}{2i} \int_{t=0}^{\infty} e^{-p(s,v)t} (e^{ait} - e^{-ait})\ dt\ ,$$

$$= \frac{q(s,v)}{2i} \left[\int_{t=0}^{\infty} e^{-(p(s,v)-ai)t}\, dt - \int_{t=0}^{\infty} e^{-(p(s,v)+ai)t}\ dt \right].$$

   After simple computation, we get:

$$H_g\{\sin\ (at)\} = \frac{a\, q(s,v)}{(p(s,v))^2 + a^2} \ .$$

5. If κ (t)= sinh (at) , where a is a constant, then

   $H_g\{\sinh\ (at)\} = \frac{aq(s,v)}{(p(s,v))^2 - a^2}$.

   Proof: By definition,

$$H_g\{\sinh\ (at)\} = q(s,v) \int_{t=0}^{\infty} e^{-p(s,v)t} \sinh(at)\, dt,$$

$$= \frac{q(s,v)}{2} \int_{t=0}^{\infty} e^{-p(s,v)t} (e^{at} - e^{-at})\ dt\ ,$$

$$= \frac{q(s,v)}{2} \left[\int_{t=0}^{\infty} e^{-(p(s,v)-a)t}\, dt - \int_{t=0}^{\infty} e^{-(p(s,v)+a)t}\ dt \right].$$

$$= \frac{q(s,v)}{2} \left[\frac{1}{(\ p(s,v) -\ a)} - \frac{1}{(\ p(s,v) +\ a)} \right],$$

$$= \frac{a\, .\, q(s,v)}{[p(s,v)]^2 - a^2} \ , \quad p\ (s,v) > a\, .$$

6. If κ (t)= cosh (at) where a is a constant, then

$$H_g\{\cosh\ (at)\} = \frac{p(s,v).\, q(s,v)}{[p(s,v)]^2 - a^2} \ , \ p(s,v) > a\, .$$

   Proof: By definition,

$$H_g\{\cosh\ (at)\} = \frac{q(s,v)}{2} \int_{t=0}^{\infty} e^{-p(s,v)t} (e^{at} + e^{-at})\ dt\, .$$

   After simple computations, we have:

$$H_g\{\cosh\ (at)\} = \frac{p(s,v).\, q(s,v)}{[p(s,v)]^2 - a^2} \ ,, \quad p\ (s,v) > a\, .$$

7. If κ (t)=t , then $H_g\{t\} = \frac{q(s,v)}{(p(s,v))^2}$, $p(s,v) \neq 0$.

Proof: By definition,

$$H_g\{t\} = q(s,\nu)\int_{t=0}^{\infty} e^{-p(s,\nu)t}\, t\, dt\,,$$

Let $u = t$, $du = dt$, $dv = e^{-p(s,\nu)t}$, $v = \frac{-1}{p(s,\nu)}.\, e^{-p(s,\nu)t}$.

$$H_g\{t\} = q(s,\nu)[\frac{-t}{p(s,\nu)}\left[e^{-[p(s,\nu)]t}\Big|_{t\,=\,0}^{\infty} + \frac{1}{p(s,\nu)}\int_{t=0}^{\infty} e^{-\,p(s,\nu)t}\, dt\right].$$

$$H_g\{t\} = \frac{q(s,\nu)}{[p(s,\nu))]^2}\,.$$

8. If κ (t)=t^2 ,then $H_g\{t\} = \frac{2q(s,\nu)}{(p(s,\nu))^3}$, $p(s,\nu) \neq 0$.

   Proof: By definition,

$$H_g\{t^2\} = q(s,\nu)\int_{t=0}^{\infty} e^{-\,p(s,\nu)t}\, t^2\, dt\,.$$

   Integration by parts we get

$$H_g\{t^2\} = \frac{2\, q(s,\nu)}{(p(s,\nu))^3}\,.$$

9. Similarly, if κ (t)=t^3 then $H_g\{t^3\} = \frac{3!q(s,\nu)}{(p(s,\nu))^4}$, $p(s,\nu) \neq 0$.

   In general, $H_g\{t^n\} = \frac{n!q(s,\nu)}{(p(s,\nu))^{n+1}}$, $p(s,\nu) \neq 0 \; and \; n \in z^{+}$.

3. General ZZ Integral Transform for Derivatives

In this part, we provide the derivative of general ZZ integral transform.

Theorem (3.1). Let $H_g\ \{\kappa(t)\} = z_g\ (s,\nu)$ then:

$$H_g\{\kappa'(t)\} = -\kappa(0).\, q(s,\nu) + p(s,\nu).\, z_g(s,\nu).$$

Proof: From definition $H_g\{\ \kappa'(t)\} = q(s,\nu)\int_{t=0}^{\infty} e^{-\,p(s,\nu)t}\,\kappa'(t)\, dt$

Integration by parts:

$$u = e^{-p(s,\nu)t}\,, du = -p(s,\nu)e^{-p(s,\nu)t}dt\,,$$

$dv = \kappa'(t)dt\,, \quad v = \kappa(t)$.

$$H_g\{\kappa'(t)\} = q(s,\nu)\left[e^{-\,p(s,\nu)t}.\,\kappa(t)\Big|_{o}^{\infty} +\ p(s,\nu)\int_{t=0}^{\infty} e^{-p(s,\nu)t}.\,\kappa(t)dt\right],$$

$H_g\ \{\kappa'(t)\} = -\kappa(0).\, q(s,\nu) + p(s,\nu).\, z_g(s,\nu)$

Theorem (3.2). Let $H_g\{\kappa(t)\} = z_g(s,\nu)$ then:

$H_g\{\kappa''(t)\} = -\,\kappa'(0).\, q(s,\nu) - \kappa(0).\, q(s,\nu).\, p(s,\nu) + \big(p(s,\nu)\big)^2.\, z_g(s,\nu)$.

Proof: From definition, $H_g\{\kappa''(t)\} = q(s,\nu)\ \int_{t=0}^{\infty} e^{-\,p(s,\nu)t}.\,\kappa''(t)dt,$

Let $u = e^{-p(s,\nu)t}\,, du = -p(s,\nu)e^{-\,p(s,\nu)t}dt$,

$dv = \kappa''(t)\, dt\,, \quad v = \kappa'(t)$.

$$H_g\{\kappa''(t)\} = q(s,v)\left[e^{-p(s,v)t}.\kappa'(t)\Big|_{o}^{\infty} + p(s,v)\int_{t=0}^{\infty} e^{-p(s,v)t}.\kappa'(t)\,dt\right],$$

$$= -\kappa'(0).q(s,v) + p(s,v).H_g\{\kappa'(t)\},$$

$$= -\kappa'(0).q(s,v) + p(s,v)\big[-\kappa(0).q(s,v) + p(s,v).z_g(s,v)\big],$$

$$= -\kappa'(0)q(s,v) - \kappa(0).p(s,v).q(s,v) + \big(p(s,v)\big)^2.z_g(s,v).$$

Theorem (3.3). If $H_g\{\kappa(t)\} = z_g(s,v)$, then:

$$H_g\{\kappa'''(t)\} = -\kappa''(0).q(s,v) - \kappa'(0).q(s,v).p(s,v) - \kappa(0).q(s,v).\big(p(s,v)\big)^2 + \big(p(s,v)\big)^3.z_g(s,v)$$

Proof: From definition, $H_g\{\kappa'''(t)\} = q(s,v)\int_{t=0}^{\infty} e^{-\,p(s,v)t}.\kappa'''(t)dt,$

$$u = e^{-\,p(s,v)t}\,, du = -p(s,v).e^{-\,p(s,v)t}dt\,,$$

$dv = \kappa'''(t)\,dt\,,\quad v = \kappa''(t).$

$$H_g\{\kappa'''(t)\} = q(s,v)\left[e^{-\,p(s,v)t}.\kappa''(t)\Big|_{o}^{\infty} + p(s,v)\int_{t=0}^{\infty} e^{-\,p(s,v)t}.\kappa''(t)\,dt\right],$$

$$= -\kappa''(0).q(s,v) + p(s,v).H_g\{\kappa''(t)\},$$

$$= -\kappa''(0)q(s,v) + p(s,v)[-\kappa'(0)q(s,v) - \kappa(0)p(s,v)q(s,v) + \big(p(s,v)\big)^2 z_g(s,v)],$$

$$= -\kappa''(0).q(s,v) - \kappa'(0).q(s,v).p(s,v) - \kappa(0).q(s,v).\big(p(s,v)\big)^2 + \big(p(s,v)\big).^3\,z_g(s,v).$$

Theorem (3.4). If $H_g\ \{\kappa(t)\} = z_g(s,v)$ then:

$$H_g\{\,\kappa^{(n)}(t)\} = \ q(s,v)\sum_{m=1}^{n} -\kappa^{(n-m)}.(p(s,v))^{m-1} + (p(s,v))^n.z_g(s,v)$$

Where n is a positive integer number.

Proof. By Mathematical induction

4. Application of General ZZ Integral Transform in a Problem on Newton's Law of Cooling of Analysis

Newton's Law of Cooling cases that the average of alteration of a body temperature is proportional to the difference of the body temperature and that of its surrounding medium,[2]. Suppose that φ is the body temperature at any time t, φ_0 the temperature of its surrounding medium. Then via Newton's Law of Cooling analysis, we get:

$$\frac{d\varphi}{dt}\alpha.[\,\varphi(t) - \ \varphi_0(t)]\,, then\ \frac{d\varphi}{dt} = -\lambda.[\varphi(t) - \varphi_0(t)]\,. \qquad (4.1)$$

Where λ is a constant proportionality (λ is a positive number).

4.1 Problem. A body temperature is come down from $80^o c\ \ to\ \ 60^o c$ in 20 minutes, when it was placed at which the surrounding air temperature is at 40° C.

Then:

(1) What will be the body temperature after 40 minutes?

(2) When will be the temperature is 55° C?

Solution: Given that, the surrounding of the air temperature is $\varphi_0 = 40°$ C , initial temperature that is for time t=0, $\varphi(0)$=80°C and after t=20 minutes, $\varphi(0)$=60° C.

From Cooling Newton's Law (eq. (4.1)), we obtain:

$$\frac{d\varphi}{dt}\alpha.[\varphi(t)-\varphi_0(t)],\ \ then\ \frac{d\varphi}{dt}=-\lambda.[\varphi(t)-\varphi_0(t)],$$

Where λ is a constant, then:

$$\frac{d\varphi}{dt}=-\lambda.[\varphi(t)+\varphi_0(t)]\,.$$

Then:

$$\frac{d\varphi}{dt}+\lambda.\varphi(t)=\lambda.\varphi_0(t)\,. \tag{4.2}$$

Taking " general ZZ transform " on both sides' eq. (4.2), we get:

$H_g\{\varphi'(t)\}+\lambda H_g\{\varphi(t)\}=40\,\lambda H_g\{1\}$.

Now, by Appling property " general ZZ Transform "of derivative,

$$-\varphi(0).\,q(s,\nu)+p(s,\nu).H_g\{\varphi(t)\}+\lambda.H_g\{\varphi(t)\}=40.\lambda\frac{q(s,\nu)}{p(s,\nu)}\,. \tag{4.3}$$

Since at the time t=0 give $\varphi(0)$=80°C, substituting this in equation (4.3), we get:

$-80\,.\,q(s,\nu)+p(s,\nu).H_g\{\varphi(t)\}+\lambda.H_g\{\varphi(t)\}=40.\lambda\frac{q(s,\nu)}{p(s,\nu)}$.

$$[p(s,\nu)+\lambda].H_g\{\varphi(t)\}\ =40.\lambda\frac{q(s,\nu)}{p(s,\nu)}+80.\,q(s,\nu)\,,$$

$$H_g\{\varphi(t)\}=z_g(s,\nu)=\frac{40.\lambda\,q(s,\nu)}{p(s,\nu).[p(s,\nu)+\lambda]}+\frac{80.\,q(s,\nu)}{[p(s,\nu)+\lambda]},$$

$$=q(s,\nu)\ [\,\frac{40\,.\lambda+80.p(s,\nu)}{p(s,\nu).[p(s,\nu)+\lambda]},$$

$$=q(s,\nu)\left[\frac{A}{[p(s,\nu)+\lambda]}+\frac{B}{p(s,\nu)}\right]=\frac{A.p(s,\nu)+B.p(s,\nu)+B.\lambda}{p(s,\nu).[p(s,\nu)+\lambda]},$$

$$=\frac{(A+B).p(s,\nu)+B.\lambda}{p(s,\nu).[p(s,\nu)+\lambda]},$$

$40\lambda=B\lambda$ then B=40, and A+B=80 → A=40, then:

$$H_g\{\varphi(t)\}=z_g(s,\nu)=q(s,\nu).\left[\frac{40}{[p(s,\nu)+\lambda]}\right]+q(s,\nu).\frac{40}{p(s,\nu)}\,. \tag{4.5}$$

Applying inverse of " general ZZ transform " on both sides' eq. (4.5) then we obtain:

$$H^{-1}{}_g\{H_g\{\varphi(t)\}\} = H^{-1}{}_g[\frac{40q(s,v)}{[p(s,v)+\lambda]}] + H^{-1}{}_g\{\frac{40q(s,v)}{p(s,v)}\}$$

$$\varphi(t) = 40.e^{-\lambda t} + 40 = 40.\left(e^{-\lambda t}+1\right). \tag{4.6}$$

Using the condition at the time t=20 minute $\varphi(20)$=60°C, we get:

$60 = 40 + 40e^{-20\lambda}$ then, $20 = 40e^{-20\lambda}$

Then:

$$\tfrac{1}{2} = e^{-20\lambda} \to \left(e^{-\lambda}\right)^{20} = \tfrac{1}{2} \to e^{-\lambda} = \left(\tfrac{1}{2}\right)^{\frac{1}{20}}. \tag{4.7}$$

To evaluate the temperature after time t=40 minutes, from equation (4.7) we get $\varphi(t) = 40 + 40e^{-40\lambda} = 40 + 40(\frac{1}{2})^{\frac{40}{20}}$

$$\varphi(t) = 40 + 40(\frac{1}{2})^{20} \to \varphi(t) = 50℃.$$

Therefore, after 40 minutes of the time temperature of the body is $50℃$.

(2) To evaluate the time required to become the body temperature is equal to $55℃$, we have:

$$55 = 40 + 40e^{-\lambda t} \to 15 = 40(e^{-\lambda})^t \to \tfrac{15}{40} = (e^{-\lambda})^t. \tag{4.8}$$

Substituting equation (4.7) in equation (4.8) we have:

$$\frac{15}{40} = (\frac{1}{2})^{\frac{t}{20}}$$

Taking natural logarithms on both sides of the above equation, we have:

$$\ln\left(\frac{15}{40}\right) = \frac{t}{20}\ln\left(\frac{1}{2}\right) \to t = \frac{20\ln\left(\frac{15}{40}\right)}{\ln\left(\frac{1}{2}\right)}$$

t=28,3 minutes.

Therefore, after time (28.3) minutes, the body temperature is $equal\ to\ 55℃$.

5. Application to Beam

A beam which its hinged at its ends, $x = 0\ and\ X = L$ Carries a uniform Loud w_0 per unit Length. Evaluate the deflection at any point P,[3]

Solution:

The ordinary differential equation and boundary conditions are:

$$x^{(4)} = \frac{w_0}{EI} \quad or \quad \frac{d^4x}{dt^4} = \frac{w_0}{EI} \quad , \quad 0 < t < L. \tag{5.1}$$

$$x(0) = x_0 = 0, x''(0) = x_0'' = 0, x(L) = 0, x''(L) = x_L'' = 0. \tag{5.2}$$

Where E is a young's modulus, I be the moment of inertia of the cross section about on axis normal to the plane of bending and EI is called flexural rigidity of the beam. Some physical quantities associated with problem are $x'(t)$,

$$M(\mathrm{t}) = EIx''(\mathrm{t}) and\ s(\mathrm{t}) = M'(\mathrm{t}) = EIx'''(\mathrm{t})\,,$$

which respectively represent the slope, bending moment and shear at a point.

Taking " general ZZ transform" of both sides of above equation (5.1), we get:

$H_g\{\frac{d^4x}{dt^4}\} = \frac{w_0}{EI}H_g\{1\},$

$$-x'''(0)\mathrm{q}(s,v) - x''(0)\mathrm{q}(s,v)p(s,v) - x'(0)\mathrm{q}(s,v)(p(s,v))^2$$

$$-x(0)\mathrm{q}(s,v)\big(p(s,v)\big)^3 + \big(p(s,v)\big)^4 H_g\{x(\mathrm{t})\} = \frac{w_0}{EI}H_g\{1\}\,. \qquad (5.3)$$

Appling the 1^{st} and 2^{nd}conditions in eq. (5.2) and the unknow conditions $x'(0) = c_1\,, x'''(0) = c_2$ we find:

$$-c_2.\,\mathrm{q}(s,v) - c_1.\,\mathrm{q}(s,v).\,\big(p(s,v)\big)^2 + \big(p(s,v)\big)^4.\,H_g\{x(t)\} = \frac{w_0}{EI}.\,H_g\{1\},$$

$$\big(p(s,v)\big)^4.\,H_g\{x(t)\} = c_2.\,\mathrm{q}(s,v) + c_1.\,\mathrm{q}(s,v).\,\big(p(s,v)\big)^2 + \frac{w_0}{EI}.\frac{.\,\mathrm{q}(s,v)}{p(s,v)},$$

$$H_g\{x(t)\} = Z_g(s,v) \ = \frac{c_2.\,\mathrm{q}(s,v)}{\big(p(s,v)\big)^4} + \frac{c_1.\,\mathrm{q}(s,v)}{\big(p(s,v)\big)^2} + \frac{w_0}{EI.}\frac{.\,\mathrm{q}(s,v)}{\big(p(s,v)\big)^5}\,.$$

Inverting to find:

$$x(t) = \frac{w_0}{EI\ 24}\ t^4 + c_2\frac{t^3}{6} + c_1 t.$$

From the last two conditions in equation (5.2), we find:

$c_1 = \frac{w_0 t^3}{EI\ 24} \quad , c_2 = \frac{w_0 L}{2\ EI}\,.$

Thus, the required deflection is,

$$x(t) = \frac{w_0}{EI\ 24}\ t(L-t)(L^2 - Lt - t^2).$$

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