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Residual Power Series Method for Solving Linear Complex Differential Equations

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Abstract: This work uses the residual power series method (RPSM), primarily based on the general Taylor series formula with a residual error function, to offer solutions to instances of linear complex differential equations (LCDEs) as test problems. After finding approximate solutions to those problems, the current approach's findings have been compared with the exact solutions as shown in the tables and figures, which show the suggested method's reliability, precision, and rapid convergence. So, the importance of this research lies in demonstrating the method's ability to find an approximate solution close to the exact solution for complex differential equations.

Keywords: Residual power series, complex differential equations.

1. Introduction

An effective and simple method for creating power series solutions of differential equations without linearization, perturbation, or discretization is the residual power series method (RPS), which was put forth by the Jordanian mathematician Omar Abu Arqub. One benefit of RPS is that it doesn't suffer from computational round-off errors and doesn't need a lot of computer memory or time. This approach has attracted the interest of several researchers: Singular initial-value Lane-Emden-type issues were used to apply the process, and the suggested method obtains a Taylor expansion of the result and reproduces the solution exactly [1]. RPSM was effectively utilized to get the fractional Sharma-Tasso-Olever equation's solution [2].in [3] the method used for finding the solution to systems of initial value problems was introduced. The fractional differential equations are satisfactorily examined in [4]. The technique has been employed to successfully solve the time-fractional Gardner and Cahn-Hilliard equations[5].in [6], they created approximate analytical series solutions for differential equations with variable coefficients, including nonhomogeneous parabolic equations, fractional heat equations in 2D, and fractional wave equations in 3D. RPSM was used together with a fractional complex transform to provide the numerical solution for the time-fractional Korteweg-de Vries equation (KdV)[7], and[8] presented the method to solve the time fractional nonlinear gas dynamics equations. (RPSM) used to examine the approximate analytical solution for the fractional

Swift-Hohenberg problem[9]. In [10] RPS algorithm suggested to find the approximate solution of the factional Bagley-Torvik equation arising in fluid mechanics. Nonlinear Kaup-Boussinesq system approximate solution was described [11]. In [12], the system of fractional order BVPs was determined using the fractional RPS method. Residual power series approach was utilized to solve nonlocal pseudo hyperbolic partial differential equations [13]. an approximate solution to fuzzy Volterra integrodifferential equation was presented under strongly generalized differentiability by employed the Residual Power Series (RPS) method [14]. The Klein-Gordon-Schrödinger (KGS) equation approximate solutions were discovered using the RPSM[15]. In [16] the technique was applied to attain an analytical solution to the Hirota-Satsuma coupled KdV equation. For stiff differential equations treat with the (RPSM) to solve Different linear and nonlinear stiff systems [17]. Time-fractional partial differential equations (PDEs) were solved using (RPSM) together with the Aboodh transform in [18], also ARA transform was recommended to solve time-fractional partial differential equations [19], fractional Kawahara partial differential equation resolved by merging the Laplace transform with the (RPSM) [20]. Fractional integro differential equations (FIDEs) are solved by the (RPSM). Trigonometric Transform Method (TTM) and the Optimal Homotopy Asymptotic Method (OHAM) were used to compare the final result [21]. This paper aims to investigate and assess the solutions of complex differential equations using the residual power series methodology.

2. Procedures for applying a method

In order to achieve our goal, anticipate that this solution will take the following form:

$$w'(z) = f\big(z, w(z)\big) \tag{1}$$

$$z(0) = z_0 \tag{2}$$

where $z_{m(w)=c_m W^m}$ are approximate terms.

It follows that when $m = 0$, $z_0(w)$ satisfies the initial conditions.

This leads to that $c_0 = z_0(0) = z(0)$,

when we choose $z_0(0) = z(0)$ as an initial approximate estimation of $z(w)$ the $z_m(w)$ for $m = 1,2,...$

approximate solution by k^{th} -abbreviated series

$$w(z) = \sum_{m=0}^{k} c_m z^m \tag{3}$$

Now Eq.(1) and Eq.(2) can written in the form:

$$w'(z) - f\big(z, w(z)\big) = 0 \tag{4}$$

Then, by substituting the k^{th} -abbreviated series $z^k(w)$ into Eq.(4) , the k^{th} residual function can be defined as :

$$Res^k(z) = \sum_{m}^{k} m c_m z^{m-1} - f(z, \sum_{m=0}^{k} c_m z^m) \ , \tag{5}$$

knowing that ∞^{th} residual function is :

$$Res^{\infty}(z) = \lim_{k\to\infty} Res^{k}(z) \tag{6}$$

For each $t\epsilon[0, a]$ its clear that

$$Res^{\infty}(w) = 0 \text{ ,and } \frac{d^s}{dw^s} Res^{\infty}(0) = \frac{d^s}{dw^s} Res^{k}(0) = 0 \text{ for } s = 1,2, \dots , k \tag{7}$$

We put $k = 1$,to find the first approximate solutions , substituting $w = 0$ in Eq.(5) and concluding $c_1 = f(0, c_0) = f\big(0, z(0)\big)$ by utilizing the fact $Res^{\infty}(0) = Res^{1}(0) = 0$, so the first approximate solution written as:

$$w'(z) = w(0) + f\big(0, w(0)\big)z \tag{8}$$

To find the second approximation, suppose that k=2 therefor

$$w^{(2)}(w) = \sum_{m=0}^{2} c_m z^m \tag{9}$$

By differentiating Eq.(5) with respect to w we get :

$$\frac{d}{dz} Res^{(2)} = 2c_2 - \frac{\partial}{\partial z} f(z, c_0) - c_1 \frac{\partial}{\partial z^2} f(z, c_0) \tag{10}$$

Substituting $z = 0$ and using the fact in (7), Eq. (10) is written as:

$$c_2 = \frac{1}{2}\left[\frac{\partial}{\partial z} f(0, z(0)) + c_1 \frac{\partial}{\partial z^2} f(0, z(0))\ \right] \tag{11}$$

We can write the second approximation for (1) and (2) as:

$$w^{(2)}(z) = z(0) + \ f(0, z_1(0))z + \frac{1}{2}\left[\frac{\partial}{\partial z} f(0, z(0)) + f(0, z(0)) \frac{\partial}{\partial z^2} f(0, z(0))\ \right] z^2 \tag{12}$$

Repeat this process until the arbitrary-order coefficients of the RPS solution for (1) and (2) are achieved. Higher accuracy can be attained by examining more components of the answer and taking large k in the truncation series (3).

M. H. Al-Smadi present theorem with corollary to show the convergence of RPS method [22].

3. Application

This part presents three selected examples highlights the linear complex differential equations, from various sources in order to contrast the suggested method for solving in this paper with other approaches and to demonstrate its accuracy and efficacy in achieving satisfactory solutions quickly, readily, and accurately.

3.1. Example 1:

As the first example ,[23], consider the LCDE:

$$w''(z) + \ w(z) = 0 \tag{13}$$

The initial conditions are as follows:

$w(0)$=0 , $w'(0) = 1$, $-1 < w > 1$,$w \in C$,with the exact solution $w(z) = \sin{(z)}$

Let $w(z) = \sum_{m=0}^{k} c_m z^m$ (14)

$w'(z) = \sum_{m=0}^{k} m c_m z^{m-1}$ (15)

$w''(z) = \sum_{m=0}^{k} m(m-1) c_m z^{m-2}$ (16)

Form $k = 2$ to 5 the following equations are obtain respectively:

$w^{(2)}(z) = \sum_{m=0}^{2} c_m z^m$

$w''(z) = \sum_{m=0}^{2} m(m-1) c_m z^{m-2}$

$Re(z) = \sum_{m=0}^{2} m(m-1) c_m z^{m-2} + \sum_{m=0}^{2} c_m z^m$

$Re(0) = 2c_2 + c_0 = 0$, implies that $c_2 = 0$

$Re = 2c_2 + 6c_3 z + \sum_{m=0}^{3} c_m z^m$ for $k = 3$

$Re'(0) = 6c_3 + c_1 \rightarrow c_3 = -\frac{1}{3!}$

$Re(z) = 2c_2 + 6c_3 z + 12c_4 z^2 + \sum_{m=0}^{4} c_m z^m$, when $k = 4$

$Re''(0) = 24c_4 + 2c_2 \rightarrow c_4 = 0$

Also if $K = 5$

$Re'''(0) = 120c_5 + 6c_3$

$c_5 = \frac{1}{5!}$

$w(z) = z - \frac{z^3}{3!} + \frac{z^5}{5!} - \cdots$

$w(z) \cong \sum_{n=0}^{N} \frac{(-1)^n z^{2n+1}}{(2n+1)!}$, n=0,1,2,…,N

For $N = 4$, Table 1 shows the exact and approximate solutions together with the real (z) value's absolute error. while the results of the exact, approximate, and absolute errors for the imaginary (z) value are shown in Table 2.

Table 1. Displays the exact and approximate solutions (where $N = 4$) as well as the absolute error. For the real (z) value.

z	Exact solution	Approximate solution	$\lvert w(z)exact - w(z)approximate \rvert$
0+0i	0	0	0
0.03+0.03i	0.030008999189965	0.030008999189965	3.469446951953614e-18
0.06+0.06i	0.060071974075557	0.060071974075557	4.510281037539698e-16
0.09+0.09i	0.090242803094097	0.090242803094080	1.708355679141960e-14
0.12+0.12i	0.120575169991469	0.120575169991241	2.275818422603493e-13
0.15+0.15i	0.151122466039642	0.151122466037946	1.695726892236848e-12
0.18+0.18i	0.181937691730973	0.181937691722222	8.751138702578487e-12
0.21+0.21i	0.213073357776342	0.213073357741293	3.504921353147950e-11
⋮	⋮	⋮	⋮

.	.	.	.
0.87+0.87i	1.072300913875244	1.072288152175273	1.276169997055376e-05
0.90+0.90i	1.122575129542809	1.122557798571429	1.733097138023609e-05
0.93+0.93i	1.173997609557486	1.173974306943004	2.330261448268978e-05
0.96+0.96i	1.226571176871459	1.226540136305225	3.104056623359774e-05
0.99+0.99i	1.280294850063048	1.280253862863798	4.098719924994221e-05

Table 2. shows the exact and approximate results (where $N = 4$) with the absolute error for the imaginary (z) value.

z	Exact solution	Approximate solution	$\lvert w(z)exact - w(z)approximate\rvert$
0+0i	0	0	0
0.03+0.03i	0.029990999190035	0.029990999190035	3.469446951953614e-18
0.06+0.06i	0.059927974084444	0.059927974084443	4.440892098500626e-16
0.09+0.09i	0.089756803245937	0.089756803245920	1.706967900361178e-14
0.12+0.12i	0.119423171128986	0.119423171128759	2.274569421700789e-13
0.15+0.15i	0.148872471463748	0.148872471462054	1.694311357880451e-12
0.18+0.18i	0.178049711166519	0.178049711157778	8.740869139600704e-12
0.21+0.21i	0.206899414953700	0.206899414918707	3.499303624643346e-11
. . .	. . .	. . .	. . .
0.87+0.87i	0.634496201642770	0.634483786444727	1.241519804295876e-05
0.90+0.90i	0.638093029296765	0.638093029296646	1.682786819368065e-05
0.93+0.93i	0.639669049450652	0.639646468436996	2.258101365570919e-05
0.96+0.96i	0.639131701215711	0.639101683854775	3.001736093599661e-05
0.99+0.99i	0.636386352316713	0.636346800476202	3.955184051107619e-05

In order to gain more convergence, $N = 8$ was applied.as demonstrated in Tables (3 and 4) for the real and imaginary values of z, respectively, it is feasible to observe the rapid convergence of the approximate.

Table 3. For the real(z) value, the table shows the exact, approximate solutions (where $N = 8$) and the absolute error.

z	Exact solution	Approximate solution	$\lvert w(z)exact - w(z)approximate\rvert$
0+0i	0	0	0
0.03+0.03i	0.030008999189965	0.030008999189965	3.469446951953614e-18
0.06+0.06i	0.060071974075557	0.060071974075557	6.938893903907228e-18
0.09+0.09i	0.090242803094097	0.090242803094097	0
0.12+0.12i	0.120575169991469	0.120575169991469	2.775557561562891e-17
0.15+0.15i	0.151122466039642	0.151122466039642	0
0.18+0.18i	0.181937691730973	0.181937691730973	2.775557561562891e-17
0.21+0.21i	0.213073357776342	0.213073357776342	0
. . .	. . .	. . .	. . .

0.87+0.87i	1.072300913875244	1.072300913875176	6.750155989720952e-14
0.90+0.90i	1.122575129542809	1.122575129542688	1.205702204742920e-13
0.93+0.93i	1.173997609557486	1.173997609557276	2.107203300738547e-13
0.96+0.96i	1.226571176871459	1.226540136305225	3.614886168179510e-13
0.99+0.99i	1.280294850063048	1.280294850062438	6.104006189389111e-13

Table 4. For the imaginary (z) value, the table shows the exact, approximate solutions (where $N = 8$) and the absolute error.

z	**Exact solution**	**Approximate solution**	$\|w(z)exact - w(z)approximate\|$
0+0i	0	0	0
0.03+0.03i	0.029990999190035	0.029990999190035	3.469446951953614e-18
0.06+0.06i	0.059927974084444	0.059927974084443	0
0.09+0.09i	0.089756803245937	0.089756803245937	1.387778780781446e-17
0.12+0.12i	0.119423171128986	0.119423171128986	1.387778780781446e-17
0.15+0.15i	0.148872471463748	0.148872471463748	2.775557561562891e-17
0.18+0.18i	0.178049711166519	0.178049711166519	2.775557561562891e-17
0.21+0.21i	0.206899414953700	0.206899414953700	0
. . .	. . .	. . .	. . .
0.87+0.87i	0.634496201642770	0.634483786444727	6.716849298982197e-14
0.90+0.90i	0.638093029296765	0.638093029296646	1.192379528447418e-13
0.93+0.93i	0.639669049450652	0.639669049450443	2.084998840246044e-13
0.96+0.96i	0.639131701215711	0.639131701215353	3.576028362317629e-13
0.99+0.99i	0.636386352316713	0.636386352316109	6.032951915813101e-13

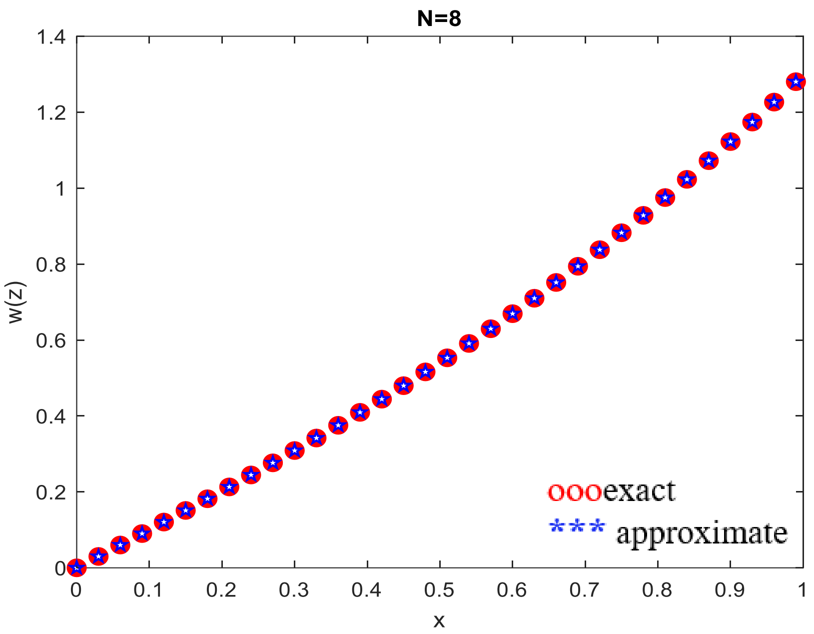

Figure 1. comparison of the exact and approximate solutions.

3.2. Example 2:

This example,[24] , to solve the second order LCDE as follows:

$$w''(z) + zw(z) = e^z + ze^z \tag{17}$$

$w(0)$=1 , $w'(0) = 1$ as initial conditions , and the exact solution is $w(z) = e^z$.

Let $w(z) = \sum_{m=0}^{k} c_m z^m$ (18)

$$w'(z) = \sum_{m=0}^{k} m c_m z^{m-1} \tag{19}$$

$$w''(z) = \sum_{m=0}^{k} m(m-1) c_m z^{m-2} \tag{20}$$

$$Re(0) = \sum_{m=0}^{k} m(m-1) c_m z^{m-2} + z\sum_{m=0}^{k} c_m z^m - e^z - ze^z \tag{21}$$

For $K = 2$

$Re(0) = 2c_2 - 1$, $c_2 = \frac{1}{2}$

$0 = 6c_3 + \sum_{m=0}^{3} c_m z^m$ $c_3 = -\frac{1}{6}$ if $k = 3$

And if $k = 4$, $Re'(0) = 24c_4 + c_1$ implies that $c_4 = \frac{1}{24}$, we obtain the approximate solution

$$w(z) = 1 + z + \frac{z^2}{2!} + \frac{z^3}{3!} + \frac{z^4}{4!} + \cdots$$

$w(z) \cong \sum_{n=0}^{N} \frac{z^n}{n!}$, n=0,1,2,…,N

Table 5. Displays the exact, approximate solutions (where $N = 4$) and the absolute error.

For the real(z) value

z	Exact solution	Approximate solution	$\lvert w(z)exact - w(z)approximate \rvert$
0+0i	1	1	0
0.03+0.03i	1.029990864190035	1.029991000000000	1.358124292977058e-07
0.06+0.06i	1.059925814084510	1.059928000000000	2.185915489505419e-06
0.09+0.09i	1.089745868247646	1.089757000000000	1.113175235456509e-05
0.12+0.12i	1.119388611146049	1.119424000000000	3.538885395104607e-05
0.15+0.15i	1.148788096565450	1.148875000000000	8.690343455008964e-05
0.18+0.18i	1.177874751603819	1.178056000000000	1.812483961809441e-04
0.21+0.21i	1.206575281454606	1.206913000000000	3.377185453936615e-04
. . .	. . .	. . .	. . .
0.87+0.87i	1.539143484247743	1.650499000000000	0.113511028466702
0.90+0.90i	1.528913811884699	1.657000000000000	0.128086188115301
0.93+0.93i	1.515215714798987	1.661881000000000	0.146665285201013
0.96+0.96i	1.497860125063111	1.665088000000000	0.167227874936889
0.99+0.99i	1.476653067425679	1.666567000000000	0.189913932574321

Table 6. Displays the exact, approximate solutions (where $N = 4$) and the absolute error.

For the imaginary (z) value

z	Exact solution	Approximate solution	$\lvert w(z)exact - w(z)approximate \rvert$
0+0i	1	1	0
0.03+0.03i	0.030908999181865	0.030909000000000	8.181347092850455e-10
0.06+0.06i	0.063671973557157	0.063672000000000	2.644284298469746e-08
0.09+0.09i	0.098342797189197	0.098343000000000	2.028108027352094e-07
0.12+0.12i	0.134975136813874	0.134976000000000	8.631861257990536e-07
0.15+0.15i	0.173622339477193	0.173625000000000	2.660522806979815e-06
0.18+0.18i	0.214337313817688	0.214344000000000	6.686182312254374e-06
0.21+0.21i	0.257172404820913	0.257187000000000	1.459517908691810e-05
. . .	. . .	. . .	. . .
0.87+0.87i	1.824385035448851	1.846401000000000i	0.022015964551149
0.90+0.90i	1.926673303972716	1.953000000000000	0.026326696027283
0.93+0.93i	2.031713097131415	2.063019000000000	0.031305902868584
0.96+0.96i	2.139479729989670	2.176512000000000	0.037032270010330
0.99+0.99i	2.249941933389248	2.293533000000000	0.043591066610752

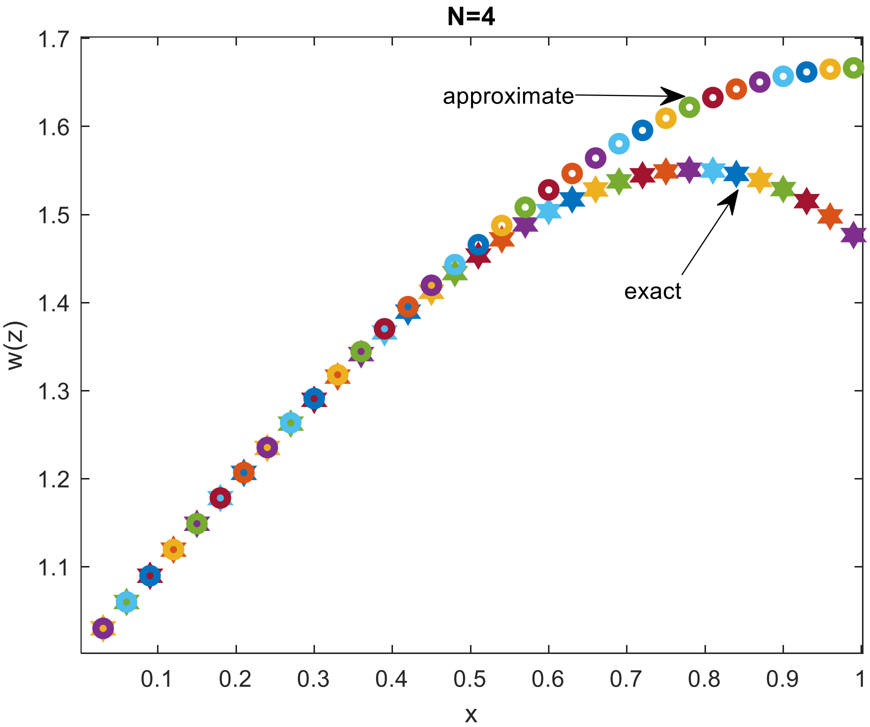

Figure 2. shows the exact solutions and the approximate results when $N = 4$.

Table 7. shows the result of the exact, approximate (where $N = 8$) and the absolute error.

For the real (z) value

z	Exact solution	Approximate solution	$\|w(z)exact - w(z)approximate\|$
0+0i	1	1	0
0.03+0.03i	1.029990864190035	1.029990864190035	4.440892098500626e-16
0.06+0.06i	1.059925814084510	1.059925814084443	6.705747068735946e-14
0.09+0.09i	1.089745868247646	1.089745868245920	1.725286580267493e-12
0.12+0.12i	1.119388611146049	1.119388611128759	1.729016929630234e-11
0.15+0.15i	1.148788096565450	1.148788096462053	1.033964025509704e-10
0.18+0.18i	1.177874751603819	1.177874751157778	4.460407598827487e-10
0.21+0.21i	1.206575281454606	1.206575279918707	1.535899407656416e-09
. . .	. . .	. . .	. . .
0.87+0.87i	1.539143484247743	1.539000851444727	1.426328030165180e-04
0.90+0.90i	1.528913811884699	1.528726201428571	1.876104561275849e-04
0.93+0.93i	1.515215714798987	1.514971133436996	2.445813619911252e-04
0.96+0.96i	1.497860125063111	1.497543923854775	3.162012083359755e-04
0.99+0.99i	1.476653067425679	1.476247465476202	4.056019494771412e-04

Table 8. Shows the result of the exact, approximate (where $N = 8$) and the absolute error.

For the imaginary (z) value

z	Exact solution	Approximate solution	$\|w(z)exact - w(z)approximate\|$
0+0i	1	1	0
0.03+0.03i	0.030908999181865	0.030908999181865	6.938893903907228e-18
0.06+0.06i	0.063671973557157	0.063671973557157	4.440892098500626e-16
0.09+0.09i	0.098342797189197	0.098342797189180	1.740274591099933e-14
0.12+0.12i	0.134975136813874	0.134975136813641	2.330358128688204e-13
0.15+0.15i	0.173622339477193	0.173622339475446	1.746602862340296e-12
0.18+0.18i	0.214337313817688	0.214337313808622	9.066025707937797e-12
0.21+0.21i	0.257172404820913	0.257172404784393	3.652012026122975e-11
. . .	. . .	. . .	. . .
0.87+0.87i	1.824385035448851	1.824370083275173	1.495217367808443e-05
0.90+0.90i	1.926673303972716	1.926652898571428	2.040540128800394e-05
0.93+0.93i	2.031713097131415	2.031685527126904	2.757000451136804e-05
0.96+0.96i	2.139479729989670	2.139442827530825	3.690245884557797e-05
0.99+0.99i	2.249941933389248	2.249892972314898	4.896107434948505e-05

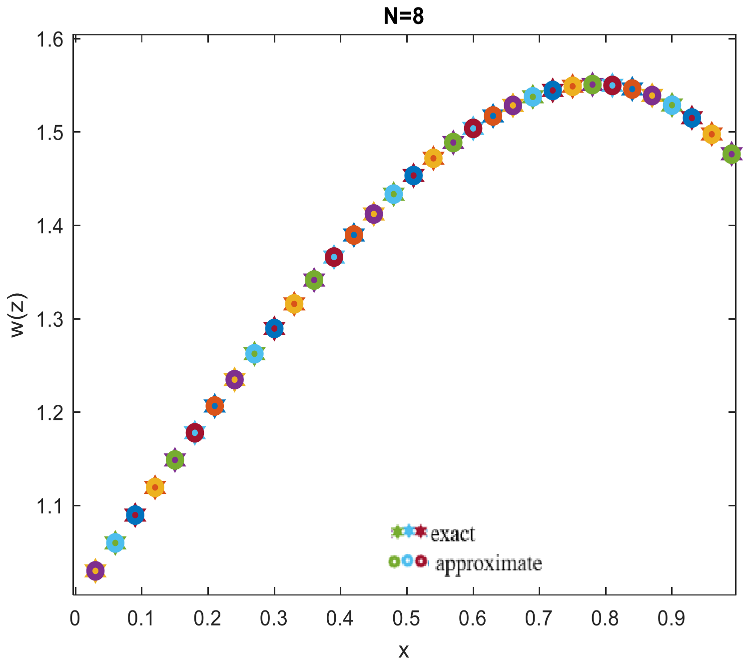

Figure 3. the exact and approximate solutions when $N = 8$.

3.3. Example 3:

For last example, [25], consider the LCDE :

$$w''(z) - 2w'(z) + 3w(z) = 2e^z + 3z - 2 \tag{22}$$

$w(0)=1$, $w'(0) = 2$ as initial conditions . With the exact solution $w(z) = z + e^z$.

By repeating the same sequential steps applied in the two examples (1 and 2) we reach the following solution:

Let $w(z) = \sum_{m=0}^{k} c_m z^m$ (23)

$$w'(z) = \sum_{m=0}^{k} m c_m z^{m-1} \tag{24}$$

$$w''(z) = \sum_{m=0}^{k} m(m-1) c_m z^{m-2} \tag{25}$$

$$Re = \sum_{m=0}^{k} m(m-1) c_m z^{m-2} - 2\sum_{m=0}^{k} m c_m z^{m-1} + 3\sum_{m=0}^{k} c_m z^m - 2e^z - 3z + 2$$
(26)

When $k = 2$

$Re(0) = 2c_2 - 2c_1 + 3c_0 - 2 + 2$

$Re(0) = 2c_2 - 1$, $c_2 = \frac{1}{2}$

If k=3

$Re'(0) = 6c_3 - 4c_2 + 3c_1 - 2 - 3$, so $c_3 = \frac{1}{6}$,

we continue to implement the steps until we get the approximate solution, as in the following:

$$w(z) = 1 + 2z + \frac{(z)^2}{2!} + \frac{(z)^3}{3!} + \frac{(z)^4}{4!} + \cdots$$

$$w(z) = z + e^z$$

$w(z) \cong z + \sum_{n=0}^{N} \frac{z^n}{n!}$, n=0,1,2,…,N

Table 9. Shows the result of the exact, approximate (where $N = 4$) and the absolute error, for the real (z) value

z	Exact solution	Approximate solution	$\lvert w(z)exact - w(z)approximate \rvert$
0+0i	1	1	0
0.03+0.03i	1.059990864190035	1.059991000000000	1.358099650516920e-07
0.06+0.06i	1.119925814084511	1.119928000000000	2.185915489505419e-06
0.09+0.09i	1.179745868247646	1.179757000000000	1.113175235456509e-05
0.12+0.12i	1.239388611146049	1.239424000000000	3.538885395082403e-05
0.15+0.15i	1.298788096565450	1.298875000000000	8.690343455008964e-05
0.18+0.18i	1.357874751603819	1.358056000000000	1.812483961809441e-04
0.21+0.21i	1.416575281454606	1.416913000000000	3.377185453936615e-04
. . .	. . .	. . .	. . .
0.87+0.87i	2.409143484247743	2.520499000000000	0.111355515752257
0.90+0.90i	2.428913811884699	2.557000000000000	0.128086188115301
0.93+0.93i	2.445215714798987	2.591881000000000	0.146665285201013
0.96+0.96i	2.457860125063111	2.625088000000000	0.167227874936889
0.99+0.99i	2.466653067425679	2.656567000000000	0.189913932574321

Table 10. Shows the result of the exact, approximate (where $N = 4$) and the absolute error, for the imaginary (z) value

z	Exact solution	Approximate solution	$\lvert w(z)exact - w(z)approximate \rvert$
0+0i	1	1	0
0.03+0.03i	0.060908999181865	0.060909000000000	8.181347058155986e-10
0.06+0.06i	0.123671973557157	0.123672000000000	2.644284298469746e-08
0.09+0.09i	0.188342797189197	0.188343000000000	2.028108027352094e-07
0.12+0.12i	0.254975136813874	0.254976000000000	8.631861257990536e-07
0.15+0.15i	0.323622339477193	0.323625000000000	2.660522806952059e-06
0.18+0.18i	0.394337313817688	0.394344000000000	6.686182312254374e-06
0.21+0.21i	0.467172404820913	0.467187000000000	1.459517908697361e-05
. . .	. . .	. . .	. . .
0.87+0.87i	2.694385035448851	2.716401000000000	0.022015964551148
0.90+0.90i	2.826673303972716	2.853000000000000	0.026326696027283
0.93+0.93i	2.961713097131415	2.993018999999999	0.031305902868584
0.96+0.96i	3.099479729989670	3.136512000000000	0.037032270010330
0.99+0.99i	3.239941933389248	3.283533000000000	0.043591066610753

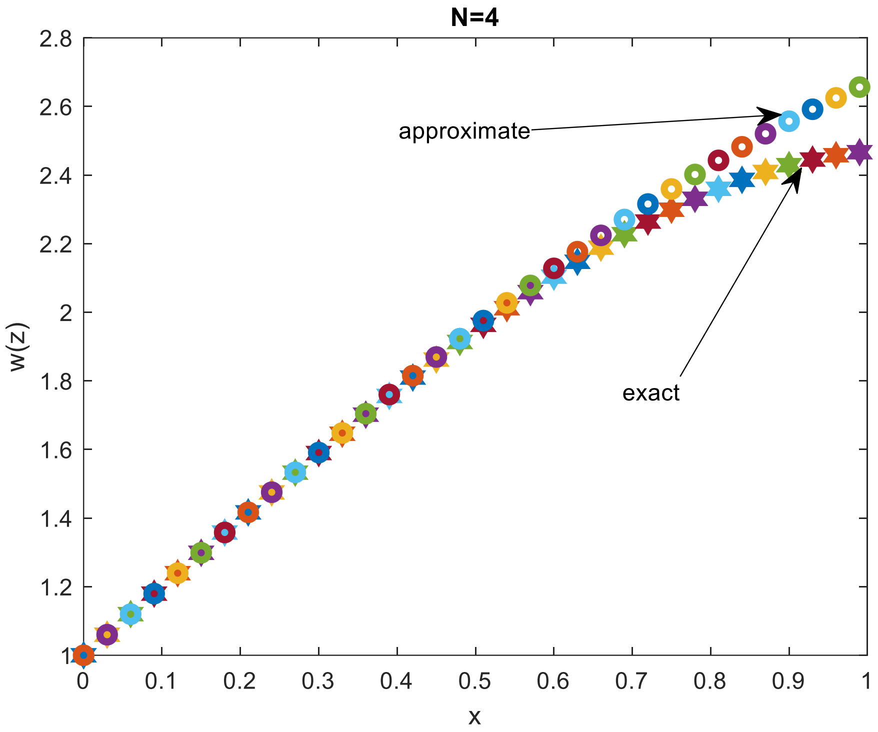

Figure 4. the exact and approximate solutions when $N = 4$.

Table 11. Shows the result of the exact, approximate (where $N = 8$) and the absolute error, for the real (z) value

z	Exact solution	Approximate solution	$\lvert w(z)exact - w(z)approximate \rvert$
0+0i	1	1	0
0.03+0.03i	1.059990864190035	1.059990864190035	4.440892098500626e-16
0.06+0.06i	1.119925814084511	1.119925814084443	6.705747068735946e-14
0.09+0.09i	1.179745868247646	1.179745868245920	1.725286580267493e-12
0.12+0.12i	1.239388611146049	1.239388611128759	1.729016929630234e-11
0.15+0.15i	1.298788096565450	1.298788096462053	1.033964025509704e-10
0.18+0.18i	1.357874751603819	1.357874751157778	4.460407598827487e-10
0.21+0.21i	1.416575281454606	1.416575279918707	1.535899407656416e-09
. . .	. . .	. . .	. . .
0.87+0.87i	2.409143484247743	2.409000851444727	1.426328030165180e-04
0.90+0.90i	2.428913811884699	2.428726201428571	1.876104561273628e-04
0.93+0.93i	2.445215714798987	2.444971133436996	2.445813619909032e-04
0.96+0.96i	2.457860125063111	2.457543923854775	3.162012083359755e-04
0.99+0.99i	2.466653067425679	2.466247465476202	4.056019494775853e-04

Table 12. Shows the result of the exact, approximate (where $N = 8$) and the absolute error, for the imaginary (z) value

z	Exact solution	Approximate solution	$\lvert w(z)exact - w(z)approximate \rvert$
0+0i	1	1	0
0.03+0.03i	0.060908999181865	0.060908999181865	1.387778780781446e-17
0.06+0.06i	0.123671973557157	0.123671973557157	4.440892098500626e-16
0.09+0.09i	0.188342797189197	0.188342797189180	1.740274591099933e-14
0.12+0.12i	0.254975136813874	0.254975136813641	2.330358128688204e-13
0.15+0.15i	0.323622339477193	0.323622339475446	1.746602862340296e-12
0.18+0.18i	0.394337313817688	0.394337313808622	9.066025707937797e-12
0.21+0.21i	0.467172404820913	0.467172404784393	3.652012026122975e-11
⋮	⋮	⋮	⋮
0.87+0.87i	2.694385035448851	2.694370083275173	1.495217367830648e-05
0.90+0.90i	2.826673303972716	2.826652898571428	2.040540128822599e-05
0.93+0.93i	2.961713097131415	2.961685527126904	2.757000451136804e-05
0.96+0.96i	3.099479729989670	3.099442827530825	3.690245884557797e-05
0.99+0.99i	3.239941933389248	3.239892972314898	4.896107434948505e-05

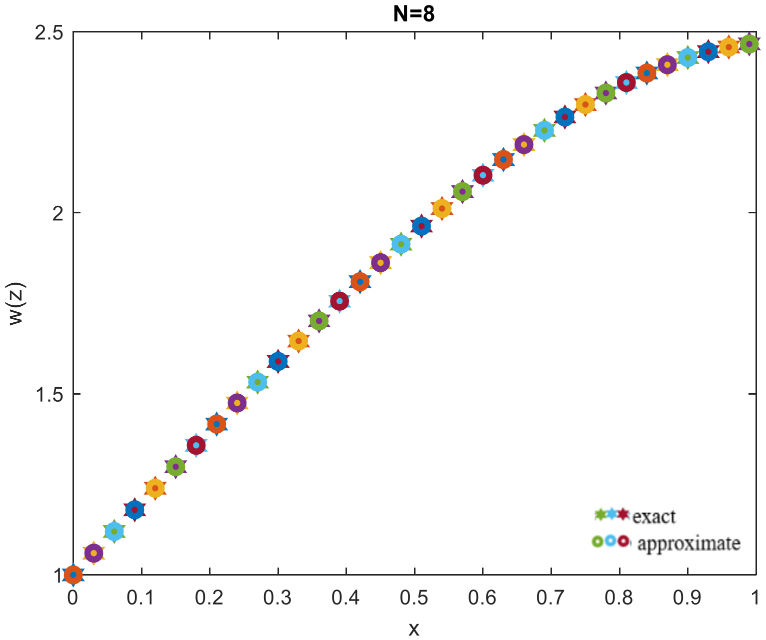

Figure 5. the exact and approximate solutions when $N = 8$.

4. Conclusion

A suitable semi-analytic method for solving differential equations is the residual power series method (RPSM). In order to solve challenging differential equations, this method was utilized in this paper. As illustrated in the resulting tables and figures, the method proved to be effective in obtaining very close approximations to exact solutions. We conclude that this technology is easy to use, useful, and has achieved great results at a low computing cost and in less time.

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