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n-Cylindrical Neutrosophic $\delta\beta$-open Sets

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Abstract

The purpose of this paper is to define and study a new class of sets called n-Cylindrical neutrosophic δ (resp. δ pre, δ semi, $\delta\alpha$ and $\delta\beta$)-open sets in n-Cylindrical neutrosophic topological spaces. Basic properties of n-Cylindrical neutrosophic δ (resp. δ pre, δ semi, $\delta\alpha$ and $\delta\beta$)-open (resp. closed) sets are analysed. We also used them to introduce the new notions like n-Cylindrical neutrosophic δ (resp. δ pre, δ semi, $\delta\alpha$ and $\delta\beta$)-closure (resp. interior) and their relations with already existing well known sets are also investigated.

Keywords and phrases: n-$CyN\delta\beta$-open sets, n-$CyN\delta\beta$-closed sets, n-$CyN\delta\beta int$, and n-$CyN\delta\beta cl$.
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1 Introduction

Following Zadeh's introduction of fuzzy set (denoted as fs) in 1965 [22], Chang [3] developed the notion of fuzzy topological spaces (fts), which led to the adaptation of classical topological concepts within the framework of fuzzy topology by various researchers. A significant generalization of fuzzy sets, known as intuitionistic fuzzy set (ifs), was introduced by Atanassov in 1986 [2]. Building on this, Coker [4] introduced the concept of intuitionistic fuzzy topological spaces ($ifts$) based on ifs's. Jeon et al. [7] further investigated intuitionistic fuzzy continuity and pre-continuity within this framework.

With the advent of neutrosophy and neutrosophic sets by Smarandache [16, 15], a new direction in uncertainty modeling emerged. Salama and Alblowi [10] introduced neutrosophic crisp set and neutrosophic topological spaces (Nts), extending $ifts$ and incorporating degrees of positive membership, neutral membership, and negative membership for each element. Neutrosophic has formed the foundation for a broader class of theories that generalize both crisp and fuzzy structures. Smarandache also introduced the concept of dependence degrees between fuzzy and neutrosophic components. Later, Arokiarani et al. [1] introduced the neutrosophic set (NS), wherein the sum of

the three membership values does not exceed 3. In the same year, Veereswari [21] proposed the notion of neutrosophic topological spaces (Nts) and studied fundamental operations on them.
Saranya et al. [11] introduced the concept of n-Cylindrical neutrosophic set (abbreviated as n-$CyNS$), characterized by α and γ as dependent components and β as an independent component. Apart from neutrosophic set (NS), n-$CyNS$ represents the most extensive generalization of fuzzy sets. In this framework, the membership functions positive (α), neutral (β), and negative (γ) satisfy the conditions $0 \leq \beta_A \leq 1$ and $0 \leq \alpha_A^n(x) + \gamma_A^n(x) \leq 1, n > 1$, where $n > 1$ is an integer.
Later, Saranya et al. [12] introduced the notion of n-CyN continuity for functions between two n-Cylindrical neutrosophic topological spaces (n-$CyNts$). They also defined the n-CyN interior (n-$CyNint$) and n-CyN closure (n-$CyNcl$) of subsets within n-$CyNts$.
In this paper some preliminary concepts required in our work are briefly recalled in section 2. In section 3, we introduce the concept of n-Cylindrical neutrosophic δ (resp. δ pre, δ semi, $\delta\alpha$ and $\delta\beta$)-open sets and studied some of their properties. Also, we discuss on n-Cylindrical neutrosophic $\delta\beta$-interior and n-Cylindrical neutrosophic $\delta\beta$-closure operators in n-Cylindrical neutrosophic topological spaces.

2 Preliminaries

This section covers some basic definitions and examples that will be useful in subsequent discussions.

Definition 2.1 [22] A fuzzy set (briefly, fs) A in X is defined by membership function $\mu_A: A \rightarrow [0,1]$ whose membership value $\mu_A(x)$ shows the degree to which $x \in X$ includes in the fuzzy set A for all $x \in X$.

Definition 2.2 [3] A fuzzy topological space (briefly, fts) is a pair (X, τ), where X is any set and τ is a family of fuzzy sets in X satisfying following axioms:

1. $\phi, X \in \tau$,
2. If $A, B \in \tau$ then $A \cap B$ in τ,
3. If $A_i \in \tau$ for each $i \in I$, then $\cup A_i \in \tau$.

Definition 2.3 [2] An intuitionistic fuzzy set (briefly, ifs) A on X is an object of the form $A = \{\langle x, \alpha_A(x), \gamma_A(x): x \in X\rangle\}$ where $\alpha_A(x) \in [0,1]$ is called the degree of positive membership of x in A, $\gamma_A(x) \in [0,1]$ is called the degree of negative membership of x in A, and where $\alpha_A(x)$ and $\gamma_A(x)$ satisfy (for all $x \in X$) $(\alpha_A(x) + \gamma_A(x) \leq 1)$ ifs(X) denotes the set of all ifs's on X.

Definition 2.4 [16] An neutrosophic set (briefly, NS) A on X is an object of the form $A = \{\langle x, \alpha_A(x), \beta_A(x), \gamma_A(x)\rangle: x \in X\}$, where $\alpha_A(x), \beta_A(x), \gamma_A(x) \in [0,1], 0 \leq \alpha_A(x) + \beta_A(x) + \gamma_A(x) \leq 3$, for all $x \in X$. $\alpha_A(x)$ is the degree of positive membership, $\beta_A(x)$ is the degree of neutral membership and $\gamma_A(x)$ is the degree of negative membership. Here, $\alpha_A(x)$ and $\gamma_A(x)$ are dependent components and $\beta_A(x)$ is an independent component.

Definition 2.5 [10] An neutrosophic topology (Nt) on a non-empty set X is a family τ_N of neutrosophic subsets in X satisfying the following axioms:

1. $0_N, 1_N \in \tau_N$,
2. $G_1 \cap G_2 \in \tau_N$ for any $G_1, G_2 \in \tau_N$,
3. $\cup G_i \in \tau_N$, for all $\{G_i : i \in J\} \subseteq \tau_N$.

In this case the pair (X, τ_N) is called a neutrosophic topological spaces (briefly, Nts) and any neutrosophic set in τ_N is known as neutrosophic open set (briefly, Nos) in X. The elements of τ_N are called neutrosophic open sets. A neutrosophic set F is closed if and only if C(F) is neutrosophic open.

Definition 2.6 [11] An n-Cylindrical neutrosophic set (briefly, n-CyNS) A on X is an object of the form $A = \{\langle x, \alpha_A(x), \beta_A(x), \gamma_A(x)\rangle : x \in X\}$, where $\alpha_A(x) \in [0,1]$ called the degree of positive membership of x in A, $\beta_A(x) \in [0,1]$ called the degree of neutral membership of x in A, and $\gamma_A(x) \in [0,1]$ called the degree of negative membership of x in A, which satisfies the condition: (for all $x \in X$) $(0 \leq \beta_A(x) \leq 1)$ and $0 \leq \alpha_A^n(x) + \gamma_A^n(x) \leq 1, n > 1$, is an integer. Here, $\alpha_A(x)$ and $\gamma_A(x)$ are dependent neutrosophic components and $\beta_A(x)$ is 100% independent.

For the convenience, $\langle \alpha_A(x), \beta_A(x), \gamma_A(x)\rangle$ is called as n-Cylindrical neutrosophic number (briefly, n-CyNN) and is denoted as $A = \{\langle \alpha_A, \beta_A, \gamma_A\rangle\}$.

Definition 2.7 [11] Let $\{A_i : i \in I\}$ be an arbitrary family of n-CyNS in X. Then, $\cap A_i = \{\langle x, \inf(\alpha_{A_i}(x)), \inf(\beta_{A_i}(x)), \sup(\gamma_{A_i}(x))\rangle : x \in X\}$.

$\cup A_i = \{\langle x, \sup(\alpha_{A_i}(x)), \sup(\beta_{A_i}(x)), \inf(\gamma_{A_i}(x))\rangle : x \in X\}$.

Definition 2.8 [11] $0_{CyN} = \{\langle x, 0,0,1\rangle : x \in X\}$ and $1_{CyN} = \{\langle x, 1,1,0\rangle : x \in X\}$.

Definition 2.9 [11] **(The Basic Connectives)** Let $\tau_{CyN}(X)$ denote the family of all n-CyNS's on X.

Inclusion: For every two $A, B \in \tau_{CyN}(X)$, the inclusion of two n-CyNS's A and B is $A \subseteq B$ iff (for all $x \in X$, $\alpha_A(x) \leq \alpha_B(x)$ and $\beta_A(x) \leq \beta_B(x)$ and $\gamma_A(x) \geq \gamma_B(x)$) and ($A \subseteq B$ and $B \subseteq A$).

Union: For every two $A, B \in \tau_{CyN}(X)$, the union of two n-CyNS's A and B is $A \cup B(x) = \{\langle x, \max(\alpha_A(x), \alpha_B(x)), \max(\beta_A(x), \beta_B(x)), \min(\gamma_A(x), \gamma_B(x))\rangle : x \in X\}$.

Intersection: For every two $A, B \in \tau_{CyN}(X)$, the intersection of two n-CyNS's A and B is $A \cap B(x) = \{\langle x, \min(\alpha_A(x), \alpha_B(x)), \min(\beta_A(x), \beta_B(x)), \max(\gamma_A(x), \gamma_B(x))\rangle : x \in X\}$.

Complementary: For every $A \in \tau_{CyN}(X)$, the complement of an n-CyNS A is $A^c = \{\langle x, \gamma_A(x), 1 - \beta_A(x), \alpha_A(x)\rangle : x \in X\}$.

Sum: For every two $A, B \in \tau_{CyN}(X)$, the sum of two n-CyNS's A and B is $A \oplus B(x) = \{\langle x, (\frac{\alpha_A(x) \cdot \alpha_B(x)}{\alpha_A(x) + \alpha_B(x)}), \max(\beta_A(x), \beta_B(x)), \min(\gamma_A(x), \gamma_B(x))\rangle : x \in X\}$.

Difference: For every two $A, B \in \tau_{CyN}(X)$, the difference of two n-CyNS's A and B is $A \ominus B(x) = \{\langle x, \max(\alpha_A(x), \alpha_B(x)), \min(\beta_A(x), \beta_B(x)), (\frac{\gamma_A(x) \cdot \gamma_B(x)}{\gamma_A(x) + \gamma_B(x)})\rangle : x \in X\}$.

Product: For every two $A, B \in \tau_{CyN}(X)$, the product of two n-CyNS's A and B is $A \otimes B(x) = \{\langle x, (\alpha_A(x) \cdot \alpha_A(x)), (\beta_A(x) \cdot \beta_B(x)), (\gamma_A(x) \cdot \gamma_B(x))\rangle : x \in X\}$.

Division: For every two $A, B \in \tau_{CyN}(X)$, the division of two n-CyNS's A and B is
$A \oslash B(x) = \{\langle x, \min(\alpha_A(x), \alpha_B(x)), (\beta_A(x) \cdot \beta_B(x)), \max(\gamma_A(x), \gamma_B(x))\rangle : x \in X\}$.

Remark 2.1 [11]

1. If $A \subseteq B$ and $B \subseteq C$ then $A \subseteq C$,
2. $A \cup B = B \cup A$ and $A \cap B = B \cap A$,
3. $(A \cup B) \cup C = A \cup (B \cup C)$ and $(A \cap B) \cap C = A \cap (B \cap C)$,
4. $(A \cup B) \cap C = (A \cap C) \cup (B \cap C)$ and $(A \cap B) \cup C = (A \cup C) \cap (B \cap C)$,
5. $A \cap A = A$ and $A \cup A = A$,
6. De Morgan's Law for A and B ie., $(A \cup B)^c = A^c \cap B^c$ and $(A \cap B)^c = A^c \cup B^c$,
7. $(A \oplus B) = (B \oplus A)$,
8. $(A \otimes B) = (B \otimes A)$.

Definition 2.10 [12] An n-Cylindrical neutrosophic topology (briefly, n-CyNt) on a non-empty set X is a family, τ_{CyN}, of n-CyNS's in X which satisfies the following conditions:

1. $0_{CyN}, 1_{CyN} \in \tau_{CyN}$,
2. $A_1 \cap A_2 \in \tau_{CyN}$,
3. $\cup A_i \in \tau_{CyN}$, for any arbitrary family $A_i \in \tau_{CyN}, i \in I$.

The pair (X, τ_{CyN}) is called an n-Cylindrical neutrosophic topological Spaces (briefly, n-CyNts) and any n-CyNS belongs to τ_{CyN} is called an n-Cylindrical neutrosophic open set (briefly, n-CyNos) and the complement of n-CyNos is called n-Cylindrical neutrosophic closed set (briefly, n-CyNcs) in X. Like classical topological spaces and fuzzy topological spaces, the family $\{0_{CyN}, 1_{CyN}\}$ is called indiscrete n-CyNts and the topology containing all the n-CyN subsets is called discrete n-CyNts.

Remark 2.2 [12] Obviously any fuzzy topological spaces or intuitionistic fuzzy topological spaces or Pythagorean fuzzy topological spaces is an n-CyNts as any subsets of the fuzzy spaces, intuitionistic fuzzy space, and Pythagorean fuzzy space can be viewed as n-CyN subsets.

Definition 2.11 [12] Let A and B be two n-Cylindrical neutrosophic subsets of an n-CyNts. B is called neighbourhood of A if there exists an n-CyNos, O such that $A \subset O \subset B$.

Proposition 2.1 [12] $A \subset X$ is n-Cylindrical neutrosophic open in (X, τ_{CyN}) if and only if it carries a neighbourhood of its subsets.

Definition 2.12 [12] Let (X, τ_{CyN}) be an n-CyNts and let $A = \{\langle x, \alpha_A(x), \beta_A(x), \gamma_A(x)\rangle : x \in X\}$ is an n-CyNS in X. Then, the n-Cylindrical neutrosophic interior (briefly, n-CyNint) is defined as the n-CyN union of all n-CyN open subsets of X. ie, $\text{n-CyNint}(A) = \bigcup \{G : G \in \tau_{CyN} \text{ and } G \subseteq A\}$.

Clearly, n-CyNint(A) is the biggest n-CyNos that is contained by A.

Definition 2.13 [12] Let (X, τ_{CyN}) be an n-CyNts and let $A = \{\langle x, \alpha_A(x), \beta_A(x), \gamma_A(x)\rangle : x \in X\}$ is an n-CyNS in X. Then, the n-Cylindrical neutrosophic closure (briefly, n-CyNcl) is defined as the n-CyN intersection of all n-CyN closed subsets of X. ie, $\text{n-CyNcl}(A) = \cap \{K : K \in \tau_{CyN} \text{ and } A \subseteq K\}$.

Clearly, n-CyNcl(A) is the smallest n-CyNcs that contains A.

3 n-Cylindrical neutrosophic δβ-open sets

In this section, we introduce the concept n-CyNδβos and investigate some related properties.

Definition 3.1 Let (X, τ_{CyN}) be an n-CyNts and A be an n-CyNS. Then, A is said to be an n-CyN

1. regular open set (briefly, n-CyNros), if A = n-CyNint(n-CyNcl(A)),
2. regular closed set (briefly, n-CyNrcs), if A = n-CyNcl(n-CyNint(A)).

Definition 3.2 Let (X, τ_{CyN}) be an n-CyNts and $A = \{\langle x, \alpha_A(x), \beta_A(x), \gamma_A(x)\rangle : x \in X\}$ be an n-CyNS in X. Then, the n-Cylindrical neutrosophic δ-interior of A and the n-Cylindrical neutrosophic δ-closure of A are denoted by n-CyNδint(A) and n-CyNδcl(A) are defined as follows:

1. $\text{n-CyN}\delta\text{int}(A) = \cup \{G \mid G \text{ isan n-CyNros and } G \subseteq A\}$,
2. $\text{n-CyN}\delta\text{cl}(A) = \cap \{K \mid K \text{ isan n-CyNrcs and } A \subseteq K\}$.

Definition 3.3 Let (X, τ_{CyN}) be an n-CyNts and $A = \{\langle x, \alpha_A(x), \beta_A(x), \gamma_A(x)\rangle : x \in X\}$ be an n-CyNS in X. A set A is said to be n-CyN

1. open set (briefly, n-CyNos), if A = n-CyNint(A),
2. δ-open set (briefly, n-CyNδos), if A = n-CyNδint(A),
3. δ-pre open set (briefly, n-CyNδ$\mathcal{P}$os), if A ⊆ n-CyNint(n-CyNδcl(A)),
4. δ-semi open set (briefly, n-CyNδ$\mathcal{S}$os), if A ⊆ n-CyNcl(n-CyNδint(A)),
5. δα-open set (briefly, n-CyNδαos), if A ⊆ n-CyNint(n-CyNcl(n-CyNδint(A))),
6. δβ or e^*-open set (briefly, n-CyNδβos or e^*os), if A ⊆ n-CyNcl(n-CyNint(n-CyNδcl(A))).

The complement of a n-CyNδos (resp. n-CyNδ$\mathcal{P}$os, n-CyNδ$\mathcal{S}$os, n-CyNδαos, and n-CyNδβos) is called a n-CyNδ (resp. n-CyNδ$\mathcal{P}$, n-CyNδ$\mathcal{S}$, n-CyNδα, and n-CyNδβ) closed set (briefly, n-CyNδcs (resp. n-CyNδ$\mathcal{P}$cs, n-CyNδ$\mathcal{S}$cs, n-CyNδαcs, and n-CyNδβcs)) in X.

The family of all n-CyNδos (resp. n-CyNδcs, n-CyNδ$\mathcal{P}$os, n-CyNδ$\mathcal{P}$cs, n-CyNδ$\mathcal{S}$os, n-CyNδ$\mathcal{S}$cs, n-CyNδαos, n-CyNδαcs, n-CyNδβos, and n-CyNδβcs) of X is denoted by n-CyNδOS(X), (resp. n-CyNδCS(X), n-CyNδ$\mathcal{P}$OS(X), n-CyNδ$\mathcal{P}$CS(X), n-CyNδ$\mathcal{S}$OS(X), n-CyNδ$\mathcal{S}$CS(X), n-CyNδαOS(X), n-CyNδαCS(X), n-CyNδβOS(X), and n-CyNδβ CS(X)).

Definition 3.4

Let (X, τ_{CyN}) be an n-CyNts and $A = \{\langle x, \alpha_A(x), \beta_A(x), \gamma_A(x)\rangle : x \in X\}$ be an n-CyNS in X. Then, the n-Cylindrical neutrosophic

1. δ$\mathcal{P}$-interior (resp. n-CyNδ$\mathcal{S}$-interior, n-CyNδα-interior, and n-CyNδβ-interior or n-

CyNe*-interior) of A (briefly, n-CyNδ$\mathcal{P}$int(A) (resp. n-CyNδ$\mathcal{S}$int(A), n-CyNδαint(A), and n-CyNδβint(A) or n-CyNe*int(A)) is defined by n-CyNδ$\mathcal{P}$int(A) (resp. n-CyNδ$\mathcal{S}$int(A), n-CyNδαint(A), and n-CyNδβint(A) or n-CyNe*int(A))= ⋃ {G: G ⊆ A and G is a n-CyNδ$\mathcal{P}$os (resp. n-CyNδ$\mathcal{S}$os, n-CyNδαos and n-CyNδβos or n-CyNe*os) in X}.

2. δ$\mathcal{P}$-closure (resp. n-CyNδ$\mathcal{S}$-closure, n-CyNδα-closure and n-CyNδβ-closure or n-CyNe*-closure) of A (briefly, n-CyNδ$\mathcal{P}$cl(A) (resp. n-CyNδ$\mathcal{S}$cl(A), n-CyNδαcl(A) and n-CyNδβcl(A) or n-CyNe*cl(A)) is defined by n-CyNδ$\mathcal{P}$cl(A) (resp. n-CyNδ$\mathcal{S}$ cl(A), n-CyNδαcl(A) and n-CyNδβcl(A) or n-CyNe*cl(A))= ⋂ {K: K ⊇ A and K is a n-CyNδ$\mathcal{P}$cs (resp. n-CyNδ$\mathcal{S}$cs, n-CyNδαcs and n-CyNδβcs or n-CyNe*cs) in X}.

Proposition 3.1 Let (X, τ_{CyN}) be a n-CyNts. Then, the following statements are hold but the converse does not true.

1. Every n-CyNδos (resp. n-CyNδcs) is a n-CyNos (resp. n-CyNcs),
2. Every n-CyNδos (resp. n-CyNδcs) is a n-CyNδ$\mathcal{S}$os (resp. n-CyNδ$\mathcal{S}$cs),
3. Every n-CyNδos (resp. n-CyNδcs) is a n-CyNδ$\mathcal{P}$os (resp. n-CyNδ$\mathcal{P}$cs),
4. Every n-CyNδ$\mathcal{S}$os (resp. n-CyNδ$\mathcal{S}$cs) is a n-CyNδβos (resp. n-CyNδβcs),
5. Every n-CyNδ$\mathcal{P}$os (resp. n-CyNδ$\mathcal{P}$cs) is a n-CyNδβos (resp. n-CyNδβcs),
6. Every n-CyNδαos (resp. n-CyNδαcs) is a n-CyNδ$\mathcal{S}$os (resp. n-CyNδ$\mathcal{S}$cs),
7. Every n-CyNδαos (resp. n-CyNδαcs) is a n-CyNδ$\mathcal{P}$os (resp. n-CyNδ$\mathcal{P}$cs).

Proof.

1. If A is a n-CyNδos in X, then A = n-CyNδint(A). So, A = n-CyNδint(A) ⊆ n-CyNint(A). Therefore, A is a n-CyNos.
2. If A is a n-CyNδos in X, then A = n-CyNδint(A). So, A = n-CyNδint(A) ⊆ n-CyNcl(n-CyNδint(A)). Therefore, A is a n-CyNδ$\mathcal{S}$os.
3. If A is a n-CyNδos in X, then A = n-CyNδint(A). So, A = n-CyNδint(A) ⊆ n-CyNint(n-CyNδcl(A)). Therefore, A is a n-CyNδ$\mathcal{P}$os.
4. If A is a n-CyNδ$\mathcal{S}$os in X, then A ⊆ n-CyNcl(n-CyNδint(A)) ⊆ n-CyNcl(n-CyNint(n-CyNδcl(A))). Therefore, A is a n-CyNδβos.
5. If A is a n-CyNδ$\mathcal{P}$os in X, then A ⊆ n-CyNint(n-CyNδcl(A)) ⊆ n-CyNcl(n-CyNint(n-CyNδcl(A))). Therefore, A is a n-CyNδβos.
6. If A is a n-CyNδαos in X, then A ⊆ n-CyNint(n-CyNcl(n-CyNδint(A))). So A ⊆ n-CyNint(n-CyNcl(n-CyNδint(A))) ⊆ n-CyNcl(n-CyNδint(A)). Therefore, A is a n-CyNδ$\mathcal{S}$os.
7. If A is a n-CyNδαos in X, then A ⊆ n-CyNint(n-CyNcl(n-CyNδint(A))). So, A ⊆ n-CyNint(n-CyNcl(n-CyNδint(A))) ⊆ n-CyNint(n-CyNδcl(A)). Therefore, A is a n-CyNδ$\mathcal{P}$os.

It is also true for their respective closed sets.

Example 3.1 Let $X = \{x_1, x_2\}$ and the n-Cylindrical neutrosophic sets A_1, A_2, A_3, A_4 and A_5 in X are defined as

$$A_1 = \{\langle x_1, 0.1250732, 0.50, 0.1850738\rangle, \langle x_2, 0.1450734, 0.50, 0.1650736\rangle\}$$
$$A_2 = \{\langle x_1, 0.1150731, 0.50, 0.1950739\rangle, \langle x_2, 0.1350733, 0.50, 0.1750737\rangle\}$$

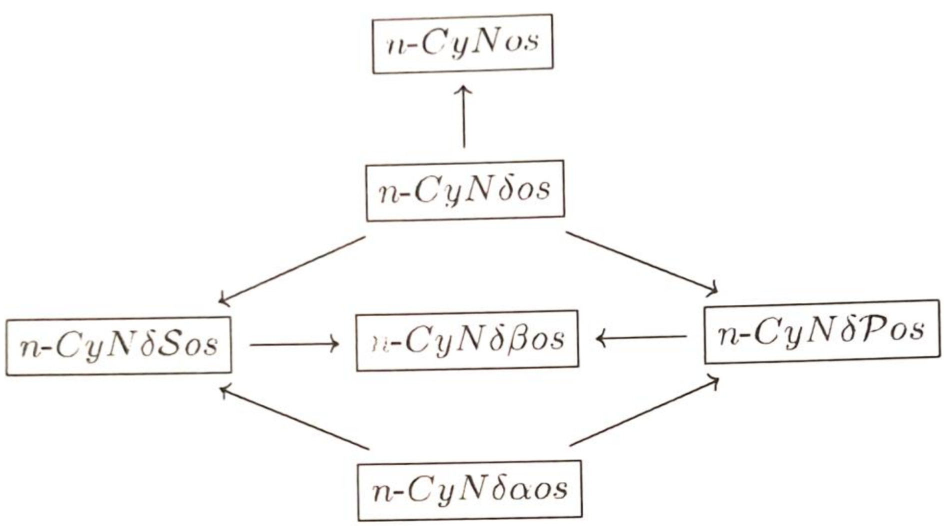
$A_3 = \{\langle x_1, 0.1950739, 0.50, 0.1150731\rangle, \langle x_2, 0.1750737, 0.50, 0.1350733\rangle\}$
$A_4 = \{\langle x_1, 0.1250732, 0.50, 0.1850738\rangle, \langle x_2, 0.1350733, 0.50, 0.1750737\rangle\}$
$A_5 = \{\langle x_1, 0.1850738, 0.50, 0.1250732\rangle, \langle x_2, 0.1650736, 0.50, 0.1450734\rangle\}$

Then, we have $\tau_{CyN} = \{0_{CyN}, 1_{CyN}, A_1, A_2, A_3, A_4\}$ be a n-Cylindrical neutrosophic topology on X. Then,

1. A_4 is a n-CyNos (resp.n-CyNδ$\mathcal{P}$os) but not n-CyNδos.
2. A_5 is a n-CyNδ$\mathcal{S}$os but not n-CyNδos.
3. A_4 is a n-CyNδαos but not n-CyNδ$\mathcal{P}$os.
4. A_5 is a n-CyNδαos but not n-CyNδ$\mathcal{S}$os.
5. A_5 is a n-CyNδ$\mathcal{P}$os but not n-CyNδβos.
6. A_4 is a n-CyNδ$\mathcal{S}$os but not n-CyNδβos.

Remark 3.1 The diagram shows that n-CyNδβos's in n-CyNts.

Figure 3.1 From the above Proposition 3.1 and Example 3.1, the following implications are hold.



Note: A → B denotes A implies B, but not conversely.

Theorem 3.1 Let (X, τ_{CyN}) be a n-CyNts and let A and B be n-CyNS's. Then, the following statements are hold.

1. n-CyNδβint$(0_{CyN}) = 0_{CyN}$ and n-CyNδβint$(1_{CyN}) = 1_{CyN}$,
2. A is a n-CyNδβos, iff A = n-CyNδβint(A),
3. n-CyNδβint(A) is the greatest n-CyNδβos containing A,
4. n-CyNδβint(n-CyNδβint(A)) = n-CyNδβint(A),

5. $A \subseteq B$ implies that n-CyNδβint(A) ⊆ n-CyNδβint(B),
6. n-CyNδβint(A ∩ B) = n-CyNδβint(A) ∩ n-CyNδβint(B),
7. n-CyNδβint(A ∪ B) ⊇ n-CyNδβint(A) ∪ n-CyNδβint(B),
8. n-CyNδβint(A) ⊆ A,
9. $(1_{\text{CyN}} -$ n-CyNδβint(A)) = n-CyNδβcl$(1_{\text{CyN}} - A)$.

Proof.

1. The proofs are directly from Definition 3.2.
2. Suppose, A is any n-CyNδβos of X. Then, the greatest n-CyNδβos containing in A is itself. Therefore n-CyNδβint(A) = A.
3. If G is any n-CyNδβos contained in A, then A ⊆ n-CyNδβint(A). Hence, n-CyNδβint(A) is the greatest n-CyNδβos containing A.
4. By (iii) the greatest n-CyNδβos containing n-CyNδβint(A) is itself. Hence, n-CyNδβint(n-CyNδβint(A)) = n-CyNδβint(A).
5. n-CyNδβint(B) =∪ {G|G isa n-CyNros and G ⊆ B} ⊇∪ {G|G isa n-CyNros and G ⊆ A} = n-CyNδβint(A). Then, n-CyNδβint(A) ⊆ n-CyNδβint(B).
6. n-CyNδβint(A ∩ B) =∪ {G|G isa n-CyNros and G ⊆ A ∩ B} = (∪ {G|G isa n-CyNros and G ⊆ A}) ∩ (∪ {G|G isa n-CyNros and G ⊆ B}) = n-CyNδβint(A) ∩ n-CyNδβint(B).
7. A ⊆ A ∪ B or B ⊆ A ∪ B. Hence, n-CyNδβint(A) ⊆ n-CyNδβint(A ∪ B) or n-CyNδβint(B) ⊆ n-CyNδβint(A ∪ B). Therefore, n-CyNδβint(A ∪ B) ⊇ n-CyNδβint(A) ∪ n-CyNδβint(B).
8. n-CyNδβint(A) =∪ {G|G isa n-CyNros and G ⊆ A}. Thus, n-CyNδβint(A) ⊆ A.
9. n-CyNδβint(A) is the greatest n-CyNδβos containing A. The complement is the smallest n-CyNδβcs contained in $1_{\text{CyN}} - A$. Therefore, $1_{\text{CyN}} -$ n-CyNδβint(A) = n-CyNδβcl$(1_{\text{CyN}} - A)$.

Theorem 3.2 Let (X, τ_{CyN}) be a n-CyNts and let A and B be n-CyNS's. Then, the following statements are hold.

1. n-CyNδβcl$(0_{\text{CyN}}) = 0_{\text{CyN}}$ and n-CyNδβcl$(1_{\text{CyN}}) = 1_{\text{CyN}}$,
2. A is a n-CyNδβcs iff A = n-CyNδβcl(A),
3. n-CyNδβcl(A) is the smallest n-CyNδβcs contained in A,
4. n-CyNδβcl(n-CyNδβcl(A)) = n-CyNδβcl(A),
5. A ⊆ B implies that n-CyNδβcl(A) ⊆ n-CyNdeltacl(B),
6. n-CyNδβcl(A ∩ B) ⊆ n-CyNδβcl(A) ∩ n-CyNδβcl(B),
7. n-CyNδβcl(A ∪ B) = n-CyNδβcl(A) ∪ n-CyNδβcl(B),
8. A ⊆ n-CyNδβcl(A),
9. $(1_{\text{CyN}} -$ n-CyNδβcl(A)) = n-CyNδβint$(1_{\text{CyN}} - A)$,
10. x ∈ n-CyNδβcl(A) iff $A \cap B \neq 0_{\text{CyN}}$ for every n-CyNδβos B containing x,
11. n-CyNδβcl(A) = A iff A is a n-CyNδβcs.

Proof. (x) Suppose, $x \in$ n-CyNδβcl(A). Let B be a n-CyNδβos containing x. If $A \cap B = 0_{CyN}$, then $1_{CyN} - B$ is a n-CyNδβcs containing A and so $x \notin$ n-CyNδβcl(A), a contradiction. Therefore, $A \cap B \neq 0_{CyN}$. If $x \notin$ n-CyNδβcl(A), then there exists a n-CyNδβcs C containing $A \ni x \notin C$. Then, $D = 1_{CyN} - C$ is a n-CyNδβos containing x such that $A \cap D = 0_{CyN}$, a contradiction. Therefore, $x \in$ n-CyNδβcl(A). The other cases are follows from Theorem 3.1.

Remark 3.2 The Theorems 3.1 and 3.2 are also true for n-CyNδ𝒫os, n-CyNδ𝒮os, n-CyNδαos, and n-CyNδβos of their respective interior and closure operators.

Theorem 3.3 Let (X, τ_{CyN}) be a n-CyNts and $\{A_\alpha | \alpha \in I\}$ be a family of n-CyNδβo (resp. n-CyNδβc) sets in (X, τ_{CyN}). Then, $\cup \{A_\alpha | \alpha \in I\}$ (resp. $\cap \{A_\alpha | \alpha \in I\}$) is a n-CyNδβos (resp. n-CyNδβcs).

Proof. Let $\{A_\alpha | \alpha \in I\}$ be a family of n-CyNδβo sets in (X, τ_{CyN}). Then, for each α, $A_\alpha \subseteq$ n-CyNcl(n-CyNint(n-CyNδcl(A_α))).

Since, $A_\alpha \subseteq \cup A_\alpha$, n - CyNcl(n - CyNint(n - CyNδcl(A_α))) ⊆ n - CyNcl(n - CyNint(n - CyNδcl($\cup A_\alpha$))) and also $A_\alpha \subseteq$ n-CyNcl(n-CyNint(n-CyNδcl($\cup A_\alpha$))).

Hence, $\cup A_\alpha \subseteq$ n-CyNcl(n-CyNint(n-CyNδcl($\cup A_\alpha$))) which shows that $\cup A_\alpha$ is an n-CyNδβos. The other case is similar.

Remark 3.3 The Theorem 3.3 is also true for n-CyNδ𝒮os(X), n-CyNδ𝒫os(X), and n-CyNδαo(X).

Proposition 3.2 If A is a n-CyNδos and B is a n-CyNδβos, then $A \cap B$ is a n-CyNδβos.

Proof. $A \cap B \subseteq A \cap$ n - CyNcl(n - CyNint(n - CyNδcl(B))) ⊆ n - CyNcl($A \cap$ n - CyNint(n - CyNδcl(B))) ⊆ n-CyNcl(n-CyNint(n-CyNδcl($A \cap B$))). Therefore $A \cap B$ is a n-CyNδβos.

Remark 3.4 The Proposition 3.2 is also true if B is a n-CyNδ𝒫os, n-CyNδ𝒮os, and n-CyNδαos.

Proposition 3.3 If A is a n-CyNδ𝒫os and B is a n-CyNδαos. Then, $A \cap B$ is a n-CyNδ𝒫os.

Proof. $\cap B \subseteq$ n-CyNint(n-CyNδcl(A))) ∩ n-CyNint(n-CyNcl(n-CyNδint(B)) ⊆ n-CyNint(n-CyNint(n-CyNδcl(A))) ∩ n-CyNcl(n-CyNδint(B)) ⊆ n-CyNint(n-CyNcl(n-CyNδcl(A))) ∩ n-CyNcl(n-CyNδint(B)) ⊆ n-CyNint(n-CyNcl(n-CyNδcl($A \cap B$))) = n-CyNint(n-CyNδcl($A \cap B$)). Therefore, $A \cap B$ is a n-CyNδ𝒫os.

Corollary 3.1 If A is a n-CyNδ𝒫cs and B is a n-CyNδαos. Then, $A \cup B$ is a n-CyNδ𝒫cs.

Proposition 3.4 If A is a n-CyNS of X and B is a n-CyNδ𝒫os of X ∋ $B \subseteq A \subseteq$ n-CyNcl(n-CyNδint(B)). Then, A is a n-CyNδβos.

Proof. Since B is a n-CyNδ𝒫os, $B \subseteq$ n-CyNint(n-CyNδcl(B)). Now, $A \subseteq$ n-CyNcl(n-CyNδint(B)) ⊆ n-CyNcl(n-CyNδint(n-CyNint(n-CyNδcl(B)))) = n-CyNcl(n-CyNint(n-CyNδcl(A))). Hence, $A \subseteq$ n-CyNcl(n-CyNint(n-CyN δcl(A))). Therefore, A is a n-CyNδβos.

Proposition 3.5 If each B is a n-CyNδβos which is a n-CyNδ𝒮cs is also a n-CyNδ𝒮os.

Proof. Let B be a n-CyNδβos and n-CyNδ𝒮cs. Then, $B \subseteq$ n-CyNcl(n-CyNint(n-CyNδcl(B))) and n-CyNint(n-CyNδcl(B)) ⊆ B. Therefore, n-CyNint(n-CyNδcl(B)) ⊆ B and so, n-CyNcl(n-CyNint(n-CyNδcl(B))) ⊆ n-CyNcl(n-CyNδint(B)). Hence, $B \subseteq$ n-CyNcl(n-

CyNint(n-CyNδcl(B))) ⊆ n-CyNcl(n-CyNδint(B))). Therefore, B is a n-CyNδ$\mathcal{S}$os.

Proposition 3.6 If B is a n-CyNδβcs and n-CyNδ$\mathcal{S}$os. Then, B is a n-CyNδcs.

Proof. Since, B is a n-CyNδβcs and n-CyNδ$\mathcal{S}$os. Then, $1_{\tau_{CyN}} - B$ is a n-CyNδβos and n-CyNδ$\mathcal{S}$cs and so by Proposition 3.5, $1_{\tau_{CyN}} - B$ is a n-CyNδ$\mathcal{S}$os. Therefore, B is a n-CyNδcs.

Proposition 3.7 If each B is a n-CyNδβos which is a n-CyNδαcs is a n-CyNδcs.

Proof. Let B be a n-CyNδβos and n-CyNδαcs. Then, B ⊆ n-CyNcl(n-CyNint(n-CyNδcl(B))) and n-CyNcl(n-CyNint(n-CyNδcl(B)) ⊆ B. Therefore, n-CyNcl(n-CyNint(n-CyNδcl(B)) ⊆ B ⊆ n-CyNcl(n-CyNint(n-CyNδcl(B))). So, B = n-CyNcl(n-CyNint(n-CyNδcl(B))). Therefore, B is a n-CyNδcs.

Corollary 3.2 If each A is a n-CyNδβcs, which is n-CyNδαos, is also a n-CyNδos.

Theorem 3.4 Let (X, τ_{CyN}) be a n-CyNts, A be a n-CyNδ$\mathcal{S}$os and B be a n-CyNS ∋ B ⊆ A ⊆ n-CyNδ$\mathcal{S}$cl(B). Then, B is also a n-CyNδ$\mathcal{S}$os.

Proof. A ⊆ n-CyNδ$\mathcal{S}$cl(B), implies that n-CyNδ$\mathcal{S}$cl(A) ⊆ n-CyNδ$\mathcal{S}$cl(B) and so A ⊆ n-CyNcl(n-CyNδint(A)) ⊆ n-CyNcl(n-CyNδint(B)) and so B ⊆ n-CyNcl(n-CyNδint(B)). Therefore, B is a n-CyNδ$\mathcal{S}$os.

Remark 3.5 Theorem 3.4 is also true for n-CyNδ$\mathcal{P}$os, n-CyNδαos and n-CyNδβos.

Theorem 3.5 Let (X, τ_{CyN}) be a n-CyNts and B be an n-CyNS. Then, the following statements are hold.

1. n-CyNδ$\mathcal{P}$int(B) ⊆ B ∩ n-CyNint(n-CyNδcl(B),
2. n-CyNδ$\mathcal{P}$cl(B) ⊇ B ∪ n-CyNcl(n-CyNδint(B),
3. n-CyNδ$\mathcal{S}$int(B) ⊆ B ∩ n-CyNcl(n-CyNδint(B)),
4. n-CyNδ$\mathcal{S}$cl(B) ⊇ B ∪ n-CyNint(n-CyNδcl(B)),
5. n-CyNδαint(B) ⊆ B ∩ n-CyNint(n-CyNcl(n-CyNδint(B))),
6. n-CyNδαcl(B) ⊇ B ∪ n-CyNcl(n-CyNint(n-CyNδcl(B))),
7. n-CyNδβint(B) ⊆ B ∩ n-CyNcl(n-CyNint(n-CyNδcl(B))),
8. n-CyNδβcl(B) ⊇ B ∪ n-CyNint(n-CyNcl(n-CyNδint(B))).

Proof. (ii) Since n-CyNδ$\mathcal{P}$cl(B) is n-CyNδ$\mathcal{P}$cs, we have n-CyNcl(n-CyNδint(B)) ⊆ n-CyNcl(n-CyNδint(n-CyNδ cl(B))) ⊆ n-CyNδ$\mathcal{P}$cl(B)). Thus, B ∪ n-CyNcl(n-CyNδint(B)) ⊆ n-CyNδ$\mathcal{P}$cl(B). The other cases are similar.

Theorem 3.6 Let (X, τ_{CyN}) be a n-CyNts and let A be a n-CyNS. Then, the following statements are equivalent.

1. A is n-CyNδβ dense,
2. n-CyNδβcl(A) = $1_{\tau_{CyN}}$,
3. If B is any n-CyNδβcs such that A ⊂ B, then B = $1_{\tau_{CyN}}$,
4. Every non-empty n-CyNδβos has a non-empty intersection with A,
5. n-CyNδint($1_{\tau_{CyN}}$ − A) = $0_{\tau_{CyN}}$.

Proof. (i) ⇒ (ii): Suppose x ∉ n-CyNδβcl(A). Then, there exists an n-CyNδβosC containing

$x \ni C \cap A \neq 0_{\tau_{CyN}}$. Since C is a non-empty n-CyNδβos, there is a non-empty n-CyNos $H \ni H \subseteq C$ and so $H \cap A = 0_{\tau_{CyN}}$, a contradiction. Therefore, $n\text{-}CyN\delta cl(A) = 1_{\tau_{CyN}}$.

(ii) ⇒ (iii): If B is any n-CyNδβcs $\ni A \subset B$, then $X = n\text{-}CyN\delta\beta cl(A) \subset n\text{-}CyN\delta\beta cl(B) = B$ which implies that $B = 1_{\tau_{CyN}}$.

(iii) ⇒ (iv): If C is a non-empty n-CyNδβos $\ni C \cap A = 0_{\tau_{CyN}}$, then $A \subset 1_{\tau_{CyN}} - C$ and $1_{\tau_{CyN}} - C$ is n-CyNδβcs. By (iii), it follows that $C = 0_{\tau_{CyN}}$, a contradiction.

(iv) ⇒ (v): Suppose that $n\text{-}CyN\delta\beta int(1_{\tau_{CyN}} - A) \neq 0_{\tau_{CyN}}$. Then, $n\text{-}CyN\delta\beta int(1_{\tau_{CyN}} - A)$ is a non-empty n-CyNδos $\ni n\text{-}CyN\delta int(1_{\tau_{CyN}} - A) \cap A \neq 0_{\tau_{CyN}}$, a contradiction to the hypothesis.

(v) ⇒ (i): $n\text{-}CyN\delta int(1_{\tau_{CyN}} - A) \neq 0_{\tau_{CyN}}$ implies that $1_{\tau_{CyN}} - n\text{-}CyN\delta\beta int(1_{\tau_{CyN}} - A) = 1_{\tau_{CyN}}$. Thus $n\text{-}CyN\delta\beta cl(A) = 1_{\tau_{CyN}}$. Hence, $n\text{-}CyN\delta\beta cl(A) = 1_{\tau_{CyN}}$ which shows that A is n-CyNδβ-dense.

4 Conclusion

In this paper, we have studied the concepts of n-CyNδβos and n-CyNδβcs, along with their corresponding interior and closure operators within n-CyNts. Fundamental properties of these structures have been explored and illustrated through examples in the same setting. Additionally, we have discussed near open sets of n-CyNδβos in n-CyNts. As a potential direction for future research, these results may be extended to n-Cylindrical neutrosophic δβ-continuous mappings in n-CyNts.

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