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On Strongly Ɣ-Clean Rings

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Abstract:
For any element z of a ring $\mathcal{R}$ is said to be Ɣ-clean if $z = a + e$, where a is γ-regular and e is an idempotent, further if $ae = ea$, the element z is called strongly Ɣ-clean. If all the elements of a ring $\mathcal{R}$ *is* Ɣ-clean (resp. strongly Ɣ-clean), then $\mathcal{R}$ is called an Ɣ-clean (strongly Ɣ-clean). In this paper, some examples, and properties of Ɣ-clean and strongly Ɣ-clean are introduced and invest gules the behavior of these properties and discuss some relations between Ɣ-clean (resp., strongly Ɣ-clean) rings and other rings.

Keywords: regular ring, clean ring, Ɣ-regular, strongly regular.

1. Introduction

Throughout this article, all rings have an identity element, and it is associative. let $\mathcal{R}$ be a ring, then an element $a \in \mathcal{R}$ is regular (strongly regular) if $a \in a\mathcal{R}a$ $(a \in a^2\mathcal{R})$[1]. $\mathcal{R}$ is called Ɣ-regular (resp. strongly Ɣ-regular) if $\mathfrak{r} = \mathfrak{r}\, b^{Ɣ}\mathfrak{r}$ (resp. $\mathfrak{r} = \mathfrak{r}^2 b^{Ɣ}$) for some $b \in \mathcal{R}$ and $1 \neq Ɣ \in Z^+$ [2]. An element $a \in \mathcal{R}$ is known as clean if a equal the sum of idempotent elements and unit element. A ring $\mathcal{R}$ is known as clean if for all $a \in R$ clean [3][4][5]. An element z of a ring $\mathcal{R}$ is Ɣ-clean if $z = \mathfrak{r} + e$, when $\mathfrak{r}$ is regular and e is idempotent [6]. Any regular ring and clean rings must be Ɣ-clean clearly. In general, Ɣ-clean rings may not be regular and not be clean [7]. The symbols $Id(\mathcal{R}), J(\mathcal{R}), reg(\mathcal{R})$ *and* $U(\mathcal{R})$ stand respectively for of all idempotent elements, a Jacobson radical, a set of regular elements, and a unit element.

2. Ɣ-clean rings

Some properties over Ɣ-clean rings are given in this section, and with discussing some relations over Ɣ-clean rings and other types of rings.

Definition 2.1. [8]

A ring $\mathcal{R}$ is said to be Ɣ-clean if for any $x \in R, x = \mathfrak{r} + e$ where $\mathfrak{r} \in$ Ɣ-regular ring and $e \in Id(\mathcal{R})$.

Example 2.2.

1-Any Boolean ring is Ɣ-clean ring.

2-All fields are Ɣ-clean rings, in addition Z_p, for each prime number p.

The following proposition investigates a condition on ideal I of a ring $\boldsymbol{\mathcal{R}}$ which gives the guarantee that the $\boldsymbol{\mathcal{R}}$ is Ɣ-clean when $\boldsymbol{\mathcal{R}}/I$ is Ɣ-clean.

Proposition 2.3.

Suppose $\boldsymbol{\mathcal{R}}$ be a ring & I is a regular ideal over $\boldsymbol{\mathcal{R}}$. So $\boldsymbol{\mathcal{R}}/I$ is γ-clean if and only if $\boldsymbol{\mathcal{R}}\ is$ Ɣ-clean.

Proof: Assume that $\boldsymbol{\mathcal{R}}\ is$ Ɣ-clean ring, then by [5, Theorem 4.1], $\boldsymbol{\mathcal{R}}/I$ is γ-clean. Now, suppose that $\boldsymbol{\mathcal{R}}/I$ is Ɣ-clean ring and $x \in \boldsymbol{\mathcal{R}}$. Now, by choose $x' \in (\boldsymbol{\mathcal{R}}/I)\ and$ Ɣ$'$ is Ɣ-regular element in $(\boldsymbol{\mathcal{R}}/I) and\ e' \in Id(\boldsymbol{\mathcal{R}}/I)$, such that $x' = \mathfrak{r}' + e'$ .Since I is regular ideal and $I \in \boldsymbol{\mathcal{R}}$ and I is lifting idempotent modulo ideal and by [9, Lemma 2.4], then the Ɣ-regular element is also lifting idempotent modulo I.Thus we can assume that $\mathfrak{r}$ is Ɣ-regular element in $\boldsymbol{\mathcal{R}}$. Now $x - \mathfrak{r}$ is also idempotent modulo I and I is regular, that is $x - \mathfrak{r}$ is idempotent element, therefore $\boldsymbol{\mathcal{R}}\ is$ Ɣ-clean ring.□

Lemma 2.4. [7, The. 2.4]

The center of Ɣ-regular ring is also Ɣ-regular. □

Lemma 2.5. [8, The. 4.4]

Suppose $\boldsymbol{\mathcal{R}}$ is Ɣ-clean ring and it have only idempotents 0 and 1. So the center of $\boldsymbol{\mathcal{R}}$ is just also Ɣ-clean. □

Theorem 2.6.

Suppose $\boldsymbol{\mathcal{R}}$ is ring in which 2 is invertible, then $\boldsymbol{\mathcal{R}}$ is Ɣ-clean iff each element of $\boldsymbol{\mathcal{R}}$ can be written as the sum of a Ɣ-regular and the square root of 1.

Proof: Suppose that $\boldsymbol{\mathcal{R}}$ is Ɣ-clean and $x \in \boldsymbol{\mathcal{R}}$, then $\frac{x+1}{2} \in \boldsymbol{\mathcal{R}}$. Now, we take $\frac{x+1}{2} = \mathfrak{r} + e$, where $\mathfrak{r}$ is Ɣ-regular element and $e \in Id(\boldsymbol{\mathcal{R}})$ so $x = (2e - 1) + 2\mathfrak{r}$. But there exists $b \in \boldsymbol{\mathcal{R}}$ and Ɣ $\in Z^+\ with$ Ɣ $\neq 1$ such that $\mathfrak{r}\, b^{Ɣ}\mathfrak{r} = \mathfrak{r}$. Thus $(\mathfrak{r}+\mathfrak{r})\frac{b^{Ɣ}}{2}(\mathfrak{r}+\mathfrak{r}) = \frac{\mathfrak{r}b^{Ɣ}\mathfrak{r}}{2} + \frac{\mathfrak{r}b^{Ɣ}\mathfrak{r}}{2} + \frac{\mathfrak{r}b^{Ɣ}\mathfrak{r}}{2} + \frac{\mathfrak{r}b^{Ɣ}\mathfrak{r}}{2} = \frac{1}{2}\,(4\,\mathfrak{r}b^{Ɣ}\mathfrak{r}) = 2\mathfrak{r}$. Thus $2\mathfrak{r}$ is Ɣ-regular element since

$$(2e-1)^2 = (2e-1)(2e-1) = 4e^2 - 2e - 2e + 1$$
$$= 4e - 4e + 1 = 1$$

So x can be written as the sum of a regular and the square root of 1.

Conversely, if $x \in \boldsymbol{\mathcal{R}}$, then $2x - 1 = t + \mathfrak{r}$, where $t^2 = 1$and $\mathfrak{r} = \mathfrak{r}\, b^{Ɣ}\mathfrak{r}\, , b \in \boldsymbol{\mathcal{R}}\ and\ \emptyset \neq$ Ɣ $\in Z^+$

Thus, $x = \frac{t+1}{2} + \frac{\mathfrak{r}}{2}$ Now, $\left(\frac{t+1}{2}\right)^2 = \left(\frac{t+1}{2}\right)\left(\frac{t+1}{2}\right) = \frac{t^2+t+t+1}{4} = \frac{1+2t+1}{4} = \frac{2(t+1)}{4} = \frac{t+1}{2}$. Thus $\frac{t+1}{2} \in Id(\boldsymbol{\mathcal{R}})$ since $\frac{\mathfrak{r}}{2}\,(y^{Ɣ} + y^{Ɣ})\frac{\mathfrak{r}}{y} = \frac{\mathfrak{r}\,y^{Ɣ}\mathfrak{r}}{4} + \frac{\mathfrak{r}\,y^{Ɣ}\mathfrak{r}}{4} = \frac{\mathfrak{r}}{2}$, it follows that $\frac{\mathfrak{r}}{2}$ is Ɣ-clean. Hence x is Ɣ-clean. Therefore $\boldsymbol{\mathcal{R}}$ is Ɣ-clean. □

Proposition 2.7.

Let $\mathcal{R}$ is ring. So $x \in \mathcal{R}$ be Ɣ-clean iff $1 - x$ be a Ɣ-clean, when $Ɣ = 2n + 1$ and $n \in Z^{+}$.

Proof: Suppose $x \in \mathcal{R}$ is Ɣ-clean. So $x = \mathfrak{r} + e$, where $\mathfrak{r} = \mathfrak{r}\, b^{Ɣ}\mathfrak{r}\ and\ e \in Id(\mathcal{R})$ .Thus $1 - x = -r + (1 - e)$, then there exists $b \in \mathcal{R}$ such that $= \mathfrak{r}\, b^{Ɣ}\mathfrak{r}$. Hence $(-\mathfrak{r})(-b)^{Ɣ}(-\mathfrak{r}) = -\mathfrak{r}$ since Ɣ is odd positive integer and $-\mathfrak{r}$ is Ɣ- regular and $(1 - e) \in Id(\mathcal{R})$. Therefore $1 - x$ is Ɣ-clean.

Conversely: Let $1 - x$ is Ɣ-clean, then $1 - x = e + \mathfrak{r}$, where $e \in Id(\mathcal{R})$ and $\mathfrak{r}\ is\ Ɣ -$ regular.Then, $x = 1 - (e + \mathfrak{r}) = (1 - e) + (-\mathfrak{r})$, as aprevious part we have $(1 - e) \in Id(\mathcal{R})\ and\ (-r)$ is $Ɣ - regular$, which implies that x is a sum of idempotent & $Ɣ - regular$. Therefore x be a Ɣ-clean. □

In [2], a ring $\mathcal{R}$ is known as "quasi-commutative" if for any $a, b \in \mathcal{R}$, where $1 \neq a$, there is natural number m so that $ab = b^{m}a$.

Corollary 2.8.

Let $\mathcal{R}$ be a quasi-commutative ring and $x \in J(\mathcal{R})$. Then x is Ɣ-clean element.

Proof: Let $x \in J(\mathcal{R})$. Then $1 - x \in U(\mathcal{R})$ so, $1 - x$ is regular element in $\mathcal{R}$. Hence $1 - x$ is Ɣ-clean. Since $\mathcal{R}$ is quasi-commutative ring. Then by Proposition (2.6), we have x is Ɣ-clean element.

Now, to give the relation between Ɣ-clean and $\mathfrak{r}$-clean.

Remark 2.9.[5]

Every Ɣ-clean ring be $\mathfrak{r}$-clean ring. Conversely is not always true as the following

Example:

$(Z_4, +, .)$ be $\mathfrak{r}$-clean and it is not Ɣ-clean, because $2 \in Z_4$ is not Ɣ-regular.

In [8, Theorem 3.5], proved that every $\mathfrak{r}$-clean is Ɣ-clean, if $\mathcal{R}$ is quasi-commutative ring.

Proposition 2.10.

Suppose $\mathcal{R}$ is ring and $Id(\mathcal{R}) = \{0,1\}$. So $\mathcal{R}$ is Ɣ-clean iff its clean ring.

Proof: Suppose $\mathcal{R}$ is clean ring, $x \in \mathcal{R}$. Then $x = e + u$, where $e \in Id(\mathcal{R})\ and\ u \in U(\mathcal{R})$. To prove x is Ɣ- clean, we must prove that u is Ɣ-regular, since $u \in U(\mathcal{R})$, then there is $v \in \mathcal{R}$ such that $uv = vu = 1.1 \in \mathcal{R}$. Now $u = u.1.u = u^{2}$, that is $u \in Id(\mathcal{R})$ thus $u \in reg(\mathcal{R})$ and hence u is Ɣ- regular.

Conversely, let $\mathcal{R}$ be Ɣ- clean ring and $x \in \mathcal{R}$. Then x is Ɣ-clean and there exists $e \in Id(\mathcal{R})$ and is $\mathfrak{r}$ is Ɣ-regular element such that $x = e + \mathfrak{r}$. Now if $\mathfrak{r} = 0$, then $x = e = (1 - e) + (2e - 1)$, since $(1 - e)^{2} = 1 - 2e + e^{2} = 1 - e$, then$(1 - e) \in Id(\mathcal{R})$ and since $(2e - 1)^{2} = 4e^{2} - 4e + 1 = 1$, then $(2e - 1) \in U(\mathcal{R})$ and hence x is clean element. Therefore $\mathcal{R}$ is clean ring, now, assume that $\mathfrak{r} \neq 0$, $\mathfrak{r} = \mathfrak{r}\, b^{Ɣ}\mathfrak{r}$ for some $b \in R\ and\ Ɣ \neq 1 \in Z^{+}$. Hence

$(\mathfrak{r}\, b^{Ɣ})^2 = (\mathfrak{r}\, b^{Ɣ})(\mathfrak{r}\, b^{Ɣ}) = \mathfrak{r}\, b^{Ɣ} \in Id(\boldsymbol{\mathcal{R}})$ so by hypothesis $\mathfrak{r}\, b^{Ɣ} = 0$ *or* $\mathfrak{r}\, b^{Ɣ} = 1$, if $\mathfrak{r}\, b^{Ɣ} = 0$, then $\mathfrak{r} = \mathfrak{r}\, b^{Ɣ}\mathfrak{r}$ contradiction

Thus, $\mathfrak{r}\, b^{Ɣ} = 1$, similarly $b^{Ɣ}\mathfrak{r} = 1$, that is $\mathfrak{r} \in U(\boldsymbol{\mathcal{R}})$. Hence x is clean element and therefore $\boldsymbol{\mathcal{R}}$ is clean. □

3. Strongly Ɣ-clean rings:

We start the section by recalling the definition of the strongly Ɣ-clean ring.

Definition 3.1.

Each element z in a ring $\boldsymbol{\mathcal{R}}$ is known as strongly Ɣ-clean if z can be written as $z = \mathfrak{r} + e$, where $\mathfrak{r}$ is Ɣ-clean element & $e \in Id(\boldsymbol{\mathcal{R}})$ and $\mathfrak{r}e = e\mathfrak{r}$.

Example3.2.

Every Ɣ-clean commutative ring is strongly Ɣ-clean.

Proposition3.3.

If $\boldsymbol{\mathcal{R}}$ be a commutative strongly Ɣ-clean ring, then, $x - x^2 = a + 2\mathfrak{r}e$, where $\mathfrak{r}$ is Ɣ- regular element and $e \in Id(\boldsymbol{\mathcal{R}})$. and $a = (\mathfrak{r} - \mathfrak{r}^2)$.

Proof: Assume that $= \mathfrak{r} + e$, where $\mathfrak{r}$ is Ɣ- regular element and $\in Id(\boldsymbol{\mathcal{R}})$, that is $x = \mathfrak{r}\, b^{Ɣ}\mathfrak{r} + e$, $1 \neq Ɣ \in Z^{+}, with\ e\mathfrak{r} = \mathfrak{r}e$

$$= \mathfrak{r}^2 b^{Ɣ} + e = \mathfrak{r} + e$$

Now, $x^2 = (\mathfrak{r}b^{Ɣ}\mathfrak{r} + e)^2 = (\mathfrak{r}^2 b^{Ɣ} + e)^2$

$$= (\mathfrak{r}^2 b^{Ɣ})^2 + 2\,\mathfrak{r}^2 b^{Ɣ} e + e^2$$

$$= \mathfrak{r}^2 + 2\mathfrak{r}e + e^2$$

Then $x - x^2 = \mathfrak{r} + e - \mathfrak{r}^2 - 2\mathfrak{r}e - e^2$

$$= (\mathfrak{r} - \mathfrak{r}^2) + 2\mathfrak{r}e = a + 2\mathfrak{r}e.\ \square$$

Proposition 3.4.

A direct product $\boldsymbol{\mathcal{R}} = \pi_{i\in I}\boldsymbol{\mathcal{R}}_i$ is strongly Ɣ-clean if and only if so each of $\{\boldsymbol{\mathcal{R}}_i\}_{i\in I}$.

Proof: Let $\boldsymbol{\mathcal{R}}_i$ be strongly Ɣ-clean for each $i \in I$ set $x = (x_i)_{i\in I} \in \pi_{i\in I}\boldsymbol{\mathcal{R}}_i$ for each i, write $x_i = e_i + \mathfrak{r}_i$, where $e_i \in Id(\boldsymbol{\mathcal{R}}), \mathfrak{r}_i = \mathfrak{r}_i b^{Ɣ}\mathfrak{r}_i$ and $e_i\mathfrak{r}_i = \mathfrak{r}_i e_i$, $Ɣ \neq 1 \in Z^{+}$. Thus $x = (\mathfrak{r}_i b^{Ɣ}\mathfrak{r}_i)_{i\in I} + (e_i)_{i\in I}$ that is $(\mathfrak{r}_i b^{Ɣ}\mathfrak{r}_i)$ is Ɣ-regular for all $i \in I$ and $(e_i)_{i\in I} \in Id(\pi_{i\in I}\boldsymbol{\mathcal{R}}_i)$. Therefore $\pi_{i\in I}\boldsymbol{\mathcal{R}}_i$ is strongly Ɣ- clean ring . The conversely is true. □

Proposition 3.5.

Let $\boldsymbol{\mathcal{R}}$ is ring. So $\boldsymbol{\mathcal{R}}$ is strongly Ɣ-clean iff eRe is strongly Ɣ- clean.

Proof: Assume that $a \in e\boldsymbol{\mathcal{R}}e \subseteq \boldsymbol{\mathcal{R}}$, then there exists Ɣ-regular element $\mathfrak{r}$ and $e \in Id(\boldsymbol{\mathcal{R}})$ such that $a = \mathfrak{r} + e$. Now, $eae = e\mathfrak{r}e + eee = e\mathfrak{r}e + e$

$$= e(\,\mathfrak{r}\, b^{Ɣ}\mathfrak{r})e + e$$

$$eae = e\mathfrak{r}b^{\text{Ɣ}}\mathfrak{r}e + e = e\mathfrak{r}b^{\text{Ɣ}}e\mathfrak{r} + e = e\mathfrak{r}eb^{\text{Ɣ}}e^2\mathfrak{r} + e$$
$$= e\mathfrak{r}eb^{\text{Ɣ}}e\mathfrak{r}e + e$$

and $(e\mathfrak{r}eb^{\text{Ɣ}}e\mathfrak{r}e)e = e(e\mathfrak{r}eb^{\text{Ɣ}}e\mathfrak{r}e)$. That is $e\mathfrak{r}e$ is Ɣ-regular element in $e\boldsymbol{\mathcal{R}}e, and\ e \in Id(\boldsymbol{\mathcal{R}})$. Therefore $e\boldsymbol{\mathcal{R}}e$ is strongly Ɣ-clean element.

Conversely: Suppose that $e\boldsymbol{\mathcal{R}}e$ is strongly Ɣ-clean, then let $x \in e\boldsymbol{\mathcal{R}}e$, as $x = e + a\,; e \in Id(\boldsymbol{\mathcal{R}})$ and a is Ɣ-regular and $ea = ae$.

Then $e \in e\boldsymbol{\mathcal{R}}e$ and $a \in e\boldsymbol{\mathcal{R}}e$, Thus $aw = e = wa$, where $w \in e\boldsymbol{\mathcal{R}}e$ since $ab^{\text{Ɣ}}a \in e\boldsymbol{\mathcal{R}}e \subset \boldsymbol{\mathcal{R}}$. Hence $ab^{\text{Ɣ}}a \in R$, and therefore $\boldsymbol{\mathcal{R}}$ is strongly Ɣ-clean. □

Proposition 3.6.

Let $\boldsymbol{\mathcal{R}}$ be strongly Ɣ-clean ring. Then $\boldsymbol{\mathcal{R}}$ is Ɣ-regular, where

$\text{Ɣ} = 2n + 1\,, n \in Z^+$.

Proof: Let $\boldsymbol{\mathcal{R}}$ be a strongly Ɣ-clean ring. Then for every $x \in \boldsymbol{\mathcal{R}}$, we can write $x = a + e$, where a is Ɣ-regular element and $e \in Id(\boldsymbol{\mathcal{R}})$ with

$ae = ea$. Assume $e = ab^n = b^n a$, where $n \neq 1, n \in Z^+$, $ae = ea$, then $e^2(ab^n ab^n) = e\ and\ ea = ae = a$ and hence $x = a + e = ab^n a + ab^n = ab^n(a + 1) = e(a + 1)$

Since $ae = ae = a$, then $a - ae = 0$. So $a(1 - e) = 0$. But $a \neq 0$, then $1 - e = 0$. Thus $e = 1, and\ x = e(a + 1) = a + 1$

Now, $1 - x = -a = -(ab^{\text{Ɣ}}a)$, Thus $1 - x$ is Ɣ-regular and therefore x is Ɣ-regular. □

Proposition 3.7.

In a ring R, if x is strongly Ɣ-clean element in R, then x^m is strongly Ɣ-clean.

Proof: Assume that $x \in \boldsymbol{\mathcal{R}}$ be strongly Ɣ-clean. then $x = \mathfrak{r} + e$, where $\mathfrak{r}$ is Ɣ-regular and $\mathfrak{r}e = e\mathfrak{r}$, to prove x^m is strongly Ɣ-clean by the mathematical induction.

$$x^2 = (\mathfrak{r} + e)^2 = \mathfrak{r}^2 + e\mathfrak{r} + \mathfrak{r}e + e^2$$
$$= (\mathfrak{r}\, b^{\text{Ɣ}}\mathfrak{r})^2 + 2e\mathfrak{r} + e^2$$
$$= (\mathfrak{r}\, b^{\text{Ɣ}}\mathfrak{r})^2 + e\Big(2(\mathfrak{r}\, b^{\text{Ɣ}}\mathfrak{r} + e)\Big)$$

Now $(\mathfrak{r}\, b^{\text{Ɣ}}\mathfrak{r})^2$ is Ɣ-regular and $e\Big(2(\mathfrak{r}\, b^{\text{Ɣ}}\mathfrak{r} + e)\Big) \in Id(\boldsymbol{\mathcal{R}})$

That is x^2 is strongly Ɣ-clean. Suppose that x^{m-1} is strongly Ɣ-clean, that is $x^{m-1} = (\mathfrak{r} + e)^{m-1} = \sum_{k=0}^{m-1} \binom{m}{k} \mathfrak{r}^k e^{m-k} = \sum_{k=0}^{m-1} \binom{m}{k} \mathfrak{r}^k e$

$$= \mathfrak{r}^{m-1} + e((e-1)\mathfrak{r}^{m-2} + \frac{e(e-1)(e-2)}{2!}\, \mathfrak{r}^{m-3}e + \cdots + e^{m-2}) = \mathfrak{r}^{m-1} + e^{m-1}$$

Hence, $x^m = xx^{m-1} = r^m + e\left(\mathfrak{r}^{m-1} + \frac{e(e-1)}{2!}\, r^{m-2}e + \cdots + e^{m-1}\right) = \mathfrak{r}^m + e^m$. Thus $\mathfrak{r}^m$ is Ɣ-regular and $e^m \in Id(\mathcal{R})$ and since $\mathfrak{r}e = e\mathfrak{r}$ so $\mathfrak{r}^m e^m = e^m \mathfrak{r}^m$. Therefor x^m is strongly Ɣ-clean. □

4. Conclusions

The main results of this work are:

- When $\mathcal{R}$ is ring with 2 is invertible. Then $\mathcal{R}$ is Ɣ-clean if and only if each element of $\mathcal{R}$ can be written as the sum of a Ɣ-regular and the square root of 1.
- When $\mathcal{R}$ is ring. So $x \in \mathcal{R}$ be Ɣ-clean if and only if $\mathbf{1} - x$ be a Ɣ-clean with Ɣ= $\mathbf{2n} + \mathbf{1}$ and $n \in Z^+$.
- When $\mathcal{R}$ is ring and $Id(\mathcal{R}) = \{\mathbf{0}, \mathbf{1}\}$. Then $\mathcal{R}$ is Ɣ-clean if and only if its clean ring.
- When $\mathcal{R}$ is ring. Then $\mathcal{R}$ is strongly Ɣ-clean if and only if $e\mathcal{R}e$ is strongly Ɣ-clean.
- When $\mathcal{R}$ is a ring. If x is strongly Ɣ-clean element in , then x^m is strongly Ɣ-clean.

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