

An Approach for Solving Fuzzy Sequencing Problem with Triangular Fuzzy Neutrosophic Numbers Using Ranking Function

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Abstract:

This study presents a modified approach to address the sequencing problem, specifically when the values are expressed using Triangular Fuzzy Neutrosophic Numbers. First, we have to convert Triangular Neutrosophic fuzzy numbers into crisp one by using Score function. A numerical example has been considered and solved for illustration purpose.

Keywords: Triangular Neutrosophic Fuzzy Number (TNFN), Optimal sequence, Minimum total elapsed time, Idle time, Score function.

1. Introduction

Flow shop problems are those that involve allocating a certain number of tasks to a sequence of machines in a precise order on each machine with the goal of optimizing a set of goals and minimizing a set of y constraints. In 1954, Johnson [3] presented a precise approach for determining the minimal make span in a two-machine flow shop situation. Moreover, Johnson's approach has been expanded to the issue of "m" machines with the goal of doing all the tasks in the shortest amount of time. However, it is challenging to apply such common-sense strategies to actual life circumstances. Indeed, it is evident that the supplied knowledge is inherently imperfect, leading to a significant degree of ambiguity in the situation.

Envision a scenario in real life where 'j' tasks need to be processed on m machines. A laborious and time-consuming workout may take care of them. In such a scenario, (j!)m distinct sequences would be needed. We do, however, have a procedure that is applicable if none of the above stated requirements is fulfilled and no transfer of work is allowed. Consider a scenario where there are n tasks that need to be processed using k machines, say M1, M2, Mk in the order M1, M2, ... Mk. An optimal solution to this issue may be achieved if either one or both of the following requirements are met

i) $\min t_{ij} \geq \max t_{ij}$ for $i=1,2,3,..k-1$ (or)

ii) $\min t_{kj} \geq \max t_{ij}$ for $i=1,2,3,...k-1$

Smarandache (1999) [8] first suggested the neutrosophic set (NS) to address issues with inconsistent and ambiguous data. Applying NS to actual issues may be challenging. To address this, Wang (2010) [10] introduced the notion of a single-valued neutrosophic set, which is specifically designed for use in realistic scientific and technical situations. The preliminary

section 2 provides fundamental definitions of TNFNs, as well as explanations of arithmetic operations, Score functions, and Accuracy functions. The method for addressing the neutrosophic fuzzy sequencing issue is presented in section 3. A numerical standard demonstrating the procedure is provided in section 4. Section 5 provides a final analysis of the study and outlines potential future research.

2. Preliminaries

This portion provides a comprehensive overview of the fundamental ideas and terminology related to neutrosophic sets, single valued neutrosophic sets, and triangular neutrosophic numbers, as documented in the literature.

Definition 2.1

Consider X as a set of points or objects, and let x be an element of X . A NS A in X is characterized by three functions: the truth-membership function $T_A(x)$, the indeterminacy-membership function $I_A(x)$, and the falsity-membership function $F_A(x)$. $T_A(x)$, $I_A(x)$ and $F_A(x)$ are real standard or real nonstandard subsets of $] -0, 1+ [$.

That is $T_A(x) : X \rightarrow] -0, 1+ [$, $I_A(x) : X \rightarrow] -0, 1+ [$ and $F_A(x) : X \rightarrow] -0, 1+ [$. There are no restrictions on the sum $T_A(x)$, $I_A(x)$ of and $F_A(x)$, hence $0 - \leq \sup T_A(x) + \sup I_A(x) + \sup F_A(x) \leq 3 +$

Definition 2.2

Consider X as the universe of opinions. A single-valued NS A over X is an entity that may be represented as $A = \{ \langle x, T_A(x), I_A(x), F_A(x) \rangle : x \in X \}$, where $T_A(x) : X \rightarrow [0, 1]$, $I_A(x) : X \rightarrow [0, 1]$ and $F_A(x) : X \rightarrow [0, 1]$ with $0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3$ for all $x \in X$. The intervals $T_A(x)$, $I_A(x)$ and $F_A(x)$ indicate the degree to which something is true, The degree of indeterminacy of x 's membership to A , as well as the degree of untruth of x 's membership to A , respectively. The notation A is used to represent a single valued neutrosophic number for the sake of simplicity. $A = (a, b, c)$, where $a, b, c \in [0, 1]$ and $a+b+c \leq 3$.

Definition 2.3

Let $\alpha_{\tilde{a}}, \theta_{\tilde{a}}, \beta_{\tilde{a}} \in [0, 1]$ and $a_1, a_2, a_3 \in \mathbf{R}$ such that $a_1 \leq a_2 \leq a_3$. Then a single valued triangular neutrosophic number, $\tilde{a} = \langle (a_1, a_2, a_3); \alpha_{\tilde{a}}, \theta_{\tilde{a}}, \beta_{\tilde{a}} \rangle$ is a special NS on the real line set \mathbf{R} , whose truth-membership, indefinity membership, and falseness-membership functions is presented in the following manner:

$$T_{\tilde{a}}(x) = \left\{ \begin{array}{l} \alpha_{\tilde{a}} \left(\frac{x - a_1}{a_2 - a_1} \right), a_1 \leq x \leq a_2 \\ \alpha_{\tilde{a}}, x = a_2 \\ \alpha_{\tilde{a}} \left(\frac{a_3 - x}{a_3 - a_2} \right), a_2 < x \leq a_3 \\ 0, otherwise \end{array} \right\} \text{ -----(1)}$$

$$\tilde{I}_{\tilde{a}}(x) = \begin{cases} \left(\frac{a_2 - x + \theta_a^-(x - a_1)_1}{a_2 - a_1} \right), a_1 \leq x \leq a_2 \\ \theta_a^-, x = a_2 \\ \left(\frac{x - a_2 + \theta_a^-(a_3 - x)}{a_3 - a_2} \right), a_2 < x \leq a_3 \\ 1, \text{otherwise} \end{cases} \quad \text{--- (2)}$$

$$\tilde{F}_{\tilde{a}}(x) = \begin{cases} \left(\frac{a_2 - x + \beta_a^-(x - a_1)_1}{a_2 - a_1} \right), a_1 \leq x \leq a_2 \\ \beta_a^-, x = a_2 \\ \left(\frac{x - a_2 + \beta_a^-(a_3 - x)}{a_3 - a_2} \right), a_2 < x \leq a_3 \\ 1, \text{otherwise} \end{cases} \quad \text{----- (3)}$$

Where α_a^- , θ_a^- and β_a^- Indicate the highest degree of truth membership, the lowest degree of indeterminacy membership, and the lowest degree of falsehood membership, respectively. A single valued TNFN $\bar{a} = \langle (a_1, a_2, a_3); \alpha_a^-, \theta_a^-, \beta_a^- \rangle$ may indicate an imprecise amount about a, which is roughly equivalent to a.

Definition 2.4

Let $\bar{a} = \langle (a_1, a_2, a_3); \alpha_a^-, \theta_a^-, \beta_a^- \rangle$ and $\bar{b} = \langle (b_1, b_2, b_3); \alpha_b^-, \theta_b^-, \beta_b^- \rangle$ be two single valued triangular neutrosophic numbers and $\gamma \neq 0$ be any non-negative number. Afterwards

$$\begin{aligned} \bar{a} + \bar{b} &= \langle (a_1 + b_1, a_2 + b_2, a_3 + b_3); \alpha_a^- \wedge \alpha_b^-, \theta_a^- \vee \theta_b^-, \beta_a^- \vee \beta_b^- \rangle \\ \bar{a} - \bar{b} &= \langle (a_1 - b_3, a_2 + b_2, a_3 - b_1); \alpha_a^- \wedge \alpha_b^-, \theta_a^- \vee \theta_b^-, \beta_a^- \vee \beta_b^- \rangle \\ \frac{\bar{a}}{\bar{b}} &= \begin{cases} \left(\frac{a_1}{b_3}, \frac{a_2}{b_2}, \frac{a_3}{b_3} \right); \alpha_a^- \wedge \alpha_b^-, \theta_a^- \vee \theta_b^-, \beta_a^- \vee \beta_b^- \text{ if } a_3 > 0, b_3 > 0 \\ \left(\frac{a_1}{b_3}, \frac{a_2}{b_2}, \frac{a_3}{b_3} \right); \alpha_a^- \wedge \alpha_b^-, \theta_a^- \vee \theta_b^-, \beta_a^- \vee \beta_b^- \text{ if } a_3 > 0, b_3 > 0 \\ \left(\frac{a_3}{b_1}, \frac{a_2}{b_2}, \frac{a_1}{b_3} \right); \alpha_a^- \wedge \alpha_b^-, \theta_a^- \vee \theta_b^-, \beta_a^- \vee \beta_b^- \text{ if } a_3 < 0, b_3 < 0 \end{cases} \\ \bar{a}\bar{b} &= \begin{cases} (a_1b_1, a_2b_2, a_3b_3); \alpha_a^- \wedge \alpha_b^-, \theta_a^- \vee \theta_b^-, \beta_a^- \vee \beta_b^- \text{ if } a_3 > 0, b_3 > 0 \\ a_1b_3, a_2b_2, a_3b_1; \alpha_a^- \wedge \alpha_b^-, \theta_a^- \vee \theta_b^-, \beta_a^- \vee \beta_b^- \text{ if } a_3 < 0, b_3 > 0 \\ a_3b_3, a_2b_2, a_1b_1; \alpha_a^- \wedge \alpha_b^-, \theta_a^- \vee \theta_b^-, \beta_a^- \vee \beta_b^- \text{ if } a_3 < 0, b_3 < 0 \end{cases} \end{aligned}$$

$$\gamma \bar{a} = \left\{ (\gamma a_1, \gamma a_2, \gamma a_3); \alpha_{\bar{a}}, \theta_{\bar{a}}, \beta_{\bar{a}}, \gamma > 0 \right\} \\ \left\{ (\gamma a_3, \gamma a_2, \gamma a_1); \alpha_{\bar{a}}, \theta_{\bar{a}}, \beta_{\bar{a}}, \gamma < 0 \right\}$$

Definition 2.5

In order to get a precise model of the neutrosophic sequencing issue, it is necessary to Utilize the below expressions: We have devised a technique to compare any two single-valued TNFNs using the scoring function and the accuracy function.

Let $\bar{a} = \langle (a, b, c); \alpha_{\bar{a}}, \theta_{\bar{a}}, \beta_{\bar{a}} \rangle$ be a single valued triangular neutrosophic number, then

$$S(\bar{a}) = \frac{1}{16} [a + b + c] x [2 + \alpha_{\bar{a}} - \beta_{\bar{a}} + \chi_{\bar{a}}] \text{ ----- (4)}$$

$$\text{and } A(\bar{a}) = \frac{1}{16} [a + b + c] x [2 + \alpha_{\bar{a}} - \beta_{\bar{a}} - \chi_{\bar{a}}] \text{ ----- (5)}$$

The degree is referred to as the score and accuracy of \bar{a} respectively. The neutrosophic sequencing problem preserve be symbolized by a crisp model with truth membership, indefinity membership and falseness membership function and the score & accuracy degree of \bar{a} using the relations (1), (2), (3) and (4), (5) respectively.

3. Fuzzy Sequencing Problem

The difficulty of sequencing with unpredictable handling time is referred to as the fuzzy sequencing problem. Algorithms are suggested for various forms of fuzzy sequencing issues in order to arrange the tasks to be executed on separate machines with the least possible overall processing time. The theories made for the conventional dispute are likewise valid for the fuzzy sequencing problem.

Algorithm for solving different fuzzy sequencing problem**Processing n jobs in two machines**

Consider A_1, A_2, \dots, A_n as the durations of 'n' tasks on machinery 1, and B_1, B_2, \dots, B_n as the durations of tasks on machinery 2. The objective is to determine the optimal sequence for processing 'n' tasks across two machines in order to achieve the highest total elapsed time.

Procedure:

Step-1: Utilize the Score function to determine the least duration of processing from the provided list of administering durations A_1, A_2, \dots, A_n and B_1, B_2, \dots, B_n .

Step-2: If the minimum administering time is A_X (i.e., job number 'X' on machine 1 then do the X^{th} job first in the sequence. If the minimum handling time is B_Y (i.e., job number 'Y' on machinery 2) then do the Y^{th} job last in the sequence.

Step- 3: (i) If there is a tie in minimum processing of both machineries (i.e., $A_X = B_Y$). Process the X^{th} job first and Y^{th} job last in the sequence.

(ii) If the tie for the minimum occurs among the processing time on Machine 1.

Choose the job corresponding to the minimum of processing time on Machine 2 and process it first.

(iii) If the tie for the minimum occurs among the processing time on Machine 2.

Choose the job corresponding to the minimum of processing time on Machine 1 and process it last.

Step – 4: Terminate the currently allocated tasks and repeat steps 2 to 4 until all jobs have been allocated. The resultant arrangement will minimize the overall elapsed time and is referred to as the ideal sequence.

Step – 5: After obtaining an optimal sequence as stated above, the minimum total elapsed time and also the idle time on machinery 1 and 2 are calculated as follows:

Minimum Total elapsed time = Time out of the last job on machine 2

Idle time on Machine 1 = Total elapsed time – time when the last job is out of machinery 1

Idle time on Machine 2 = Time at which the first job on machinery 1 ends in a progression.

Processing n jobs on three machines

Let A_1, A_2, A_3 be the three machines. Let the order of operations be $A_1 A_2 A_3$. This issue may be simplified into a two-machine problem if any one of the following conditions is met.

(i) $\text{Min } A_{i1} \geq \text{Max } A_{i2}$

(ii) $\text{Min } A_{i3} \geq \text{Max } A_{i2}$

The method fails if none of these conditions is satisfied. If one of the conditions is met, we establish two machines, H and K, such that the processing time on H and K is specified

$$H_i = A_{i1} + A_{i2}, i = 1, 2, \dots, n \quad \text{and} \quad K = A_{i2} + A_{i3}, i = 1, 2, \dots, n.$$

4. Numerical Examples

Problem-1:

There are a total of nine tasks that need to be handled sequentially via two machines, M_1 and M_2 , in the specific sequence of M_1 followed by M_2 . The duration of processing is provided below:

Jobs	Machine (M_1)	Machine (M_2)
A	(1,2,3 ; 0.8, 0.5, 0.3)	(3,5,7 ; 0.8, 0.5, 0.7)
B	(2,3,8 ; 0.6, 0.3, 0.5)	(2,3, 4 ; 0.6, 0.3, 0.4)
C	(1,3,10 ; 0.9, 0.7, 0.6)	(1,4, 8 ; 0.8, 0.6, 0.4)
D	(1,2,6 ; 0.5, 0.6, 0.4)	(2,6,8 ; 0.6, 0.4, 0.2)
E	(2,5,11 ; 0.8, 0.6, 0.7)	(6,8,10 ; 0.6, 0.4, 0.4)

F	(1,4,8 ; 0.4, 0.6, 0.8)	(3,4,6 ; 0.3, 0.5, 0.4)
G	(3,5, 20 ; 0.8, 0.3, 0.2)	(1,2,5 ; 0.5, 0.2, 0.4)
H	(4,6,10 ; 0.8, 0.5, 0.3)	(3,5,6 ; 0.4, 0.3, 0.2)
I	(5,7,15 ; 0.3, 0.5, 0.4)	(1,5,7 ; 0.8, 0.6, 0.7)

Retrieve the most efficient sequence and calculate the minimal total elapsed time and idle time for each machine.

Solution:

Order of Cancellation

Jobs	A	B	C	D	E	F	G	H	I
Order of Cancellation	1	6	4	3	7	5	2	8	9

Optimal Sequence:

A	D	C	F	B	E	H	I	G
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The Total Elapsed Time:

Jobs	Machine(M ₁)		Machine(M ₂)	
	Time in	Time out	Time in	Time out
A	(0,0,0 ; 0,0,0)	(1,2,3 ; 0.8,0.5,0.3)	(1,2,3 ; 0.8,0.5,0.3)	(4,7,10 ; 0.8,0.5,0.7)
D	(1,2,3 ; 0.8,0.5,0.3)	(2,4,9 ; 0.5, 0.6,0.4)	(4,7,10 ; 0.8,0.5,0.7)	(6,13,18; 0.6, 0.5, 0.7)
C	(2,4,9 ; 0.5, 0.6,0.4)	(3,7,19 ; 0.5,0.7,0.6)	(6,13,18; 0.6, 0.5, 0.7)	(7,17,26 ; 0.6,0.6,0.7)
F	(3,7,19 ; 0.5,0.7,0.6)	(4,11,27; 0.4,0.7,0.8)	(7,17,26 ; 0.6,0.6,0.7)	(10,21,32; 0.3,0.6,0.7)
B	(4,11,27; 0.4,0.7,0.8)	(6,14,35 ; 0.4,0.7,0.8)	(10,21,32; 0.3,0.6,0.7)	(12,24,36;0.3,0.6,0.7)
E	(6,14,35 ; 0.4,0.7,0.8)	(8,19,46; 0.4,0.7,0.8)	(12,24,36;0.3,0.6,0.7)	(18,32,46; 0.3,0.6,0.7)
H	(8,19,46; 0.4,0.7,0.8)	(12,25,56; 0.4,0.7,0.8)	(18,32,46; 0.3,0.6,0.7)	(21,37,52; 0.3,0.6,0.7)
I	(12,25,56;0.4,0.7,0.8)	(17,32, 41; 0.3,0.7,0.8)	(21,37,52; 0.3,0.6,0.7)	(22,42,59; 0.3,0.6,0.7)
G	(17,32, 41;0.3,0.7,0.8)	(20,37, 61; 0.3,0.7,0.8)	(22,42,59; 0.3,0.6,0.7)	(23,44,64; 0.3,0.6,0.7)

Minimum Total Elapsed time = (23,44,64; 0.3,0.6,0.7) = 19.65 hours

Idle time on Machine 1 = (20,7,3; 0.3,0.7,0.8) = 4.5 hours

Idle time on Machine 2 = (1,2,3; 0.8, 0.5, 0.3) = 0.97 hours.

Problem-2:

There are a total of nine tasks that need to be handled sequentially via three machines, namely M₁ M₂ M₃, following the sequence M₁ M₂ M₃. The duration of the processing is provided below

Jobs	Machine (M ₁)	Machine (M ₂)	Machine (M ₃)
A	(1,2,6 ; 0.5, 0.6, 0.4)	(2,3,8 ; 0.6, 0.3, 0.5)	(3,5,10 ; 0.8,0.3, 0.2)
B	(1,4,8 ; 0.4, 0.6, 0.8)	(1,2,34 ; 0.8, 0.5, 0.3)	(4,6,10 ; 0.6, 0.5, 0.3)
C	(2,5,11 ; 0.8, 0.6, 0.7)	(1,4, 8 ; 0.4, 0.6, 0.8)	(5,6,7 ; 0.7,0.5,0.3)
D	(1,3,10 ; 0.9, 0.7, 0.8)	(3,5,10 ; 0.8, 0.3, 0.2)	(4,7,8 ; 0.7,0.4,0.3)

Retrieve the most efficient sequence and calculate the minimal total elapsed time and idle time for each machine.

Solution:

Given that the issue involves three machines, we simplify it by transforming it into a two-machine problem. In order to meet the requirements, it must fulfil either one or both of the following conditions:

- (i) $\text{Min}(M_1) \geq \text{Max}(M_2)$
- (ii) $\text{Min}(M_3) \geq \text{Max}(M_2)$

Here, $\text{Min}(M_3) \geq \text{Max}(M_2) = (3,5,10; 0.8,0.3, 0.2)$

Thus, the second condition is fulfilled.

We transform the issue into a dual-machine problem, denoted as H and K. The processing time for the four tasks on machines H and K is as follows:

Jobs	H	K	Order of Cancellation
I	(3,5,14 ; 0.5,0.6,0.5)	(5,8,18 ; 0.6,0.3, 0.5)	(2)
II	(2,6,11 ; 0.4,0.6,0.8)	(5,8,13 ; 0.6, 0.5, 0.3)	(1)
III	(3,9,19 ; 0.4,0.6, 0.8)	(6,10,15 ; 0.4, 0.6, 0.8)	(3)
IV	(4,8,25 ; 0.8,0.7,0.8)	(10,12,18 ; 0.7,0.4,0.3)	(4)

Optimal Sequence

II	I	III	IV
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The Total Elapsed Time

Jobs	Machine(M ₁)	
	Time in	Time out
II	(0,0,0 ; 0,0,0)	(1,4,8 ; 0.4,0.6,0.8)
I	(1,4,8 ; 0.4,0.6,0.8)	(2,6,14 ; 0.4,0.6,0.8)
III	(2,6,14 ; 0.4,0.6,0.8)	(4,11,25 ; 0.4,0.6,0.8)
IV	(4,11,25 ; 0.4,0.6,0.8)	(5,14,35 ; 0.4,0.7,0.8)

Jobs	Machine(M ₂)	
	Time in	Time out
II	(1,4,8 ; 0.4,0.6,0.8)	(2,6,11 ; 0.4,0.6,0.8)
I	(2,6,11 ; 0.4,0.6,0.8)	(4,9,19 ; 0.4,0.6,0.8)
III	(4,9,19 ; 0.4,0.6,0.8)	(5,13,27 ; 0.4,0.6,0.8)
IV	(5,13,27 ; 0.4,0.6,0.8)	(8,18,37 ; 0.4,0.6,0.8)

Jobs	Machine(M ₃)	
	Time in	Time out
II	(2,6,11 ; 0.4,0.6,0.8)	(6,12,21 ; 0.4,0.6,0.8)
I	(6,12,21 ; 0.4,0.6,0.8)	(9,17,31 ; 0.4,0.6,0.8)
III	(9,17,31 ; 0.4,0.6,0.8)	(14,23,38 ; 0.4,0.6,0.8)
IV	(14,23,38 ; 0.4,0.6,0.8)	(18,30,46 ; 0.4,0.6,0.8)

Minimum Total Elapsed time = (18,30,46 ; 0.4,0.6,0.8) = 15.27 hrs.

Idle time on Machine 1 = (13,16,11 ; 0.4,0.7,0.8) = 6.25 hr.

Idle time on Machine 2 = (11,16,17 ; 0.4, 0.6, 0.8) = 3.08 hrs.

5. Conclusion

In this paper, we have successfully addressed various sequencing problems by utilizing triangular neutrosophic fuzzy numbers. Through our research, we were able to achieve an optimal sequence and accurately measure the total elapsed time using this algorithm. Another approach to solving the sequencing problem is by transforming it into a scenario involving multiple machines and two jobs. The concept of fuzzy sequencing problem offers a highly effective framework for addressing real-life problems.

References

- [1] McCahon C. S. and Lee E. S. (1990), Job sequencing with fuzzy processing times, Computers and Mathematics with Applications, 19(7), 31-41.
- [2] V. Jeyanthi, Minimol, A.R.(2018), International Journal of Science Technology & Engineering ,5(4),56-59.
- [3] Johnson, S. M. (1954). Optimal two and three stage production schedules with setup times included, Naval Research Logistics Quarterly, 1, 61-68.
- [4] Morton T. E., and Pentico D. W (1993)., Heuristic Scheduling Systems with Applications to Production Systems and Project Management, New York: John Wiley and Sons Inc,
- [5] Nirmala, G and Anju, R (2014)., An Application of fuzzy quantifier in sequencing problem with fuzzy ranking method,
- [6] Aryabhata Journal of Mathematics and Informatics, 6(1), Prade H (1979)., Using fuzzy set theory in a scheduling problem: A case study, Fuzzy sets and systems 2,153-165.
- [7] Radhakrishnan, S. and Saikethana, D. (2020). Single Machine Sequencing Problem Using Fuzzy Parameters. The International Journal of Analytical and Experimental Modal Analysis. XII (XI): 1521-1537.

- [8] Smarandache. F (1999), A unifying field in logics: neutrosophic logic, in Philosophy, American Research Press, pp 1-141.
- [9] M. Shanmugasundari, (2017), Fuzzy sequencing problem- a novel approach, International Journal of Pure and Applied Mathematics, 113 (13), 56 – 64.
- [10] H. Wang, F. Smarandache, Y. Zhang, and R. Sunder Raman, Single valued Neutrosophic Sets, Multispace and Multistructure 4, 2010, 410-413.
- [11] Zadeh, L.A. (1965). Fuzzy sets, Information and Control, John Wiley and Sons, New York, 8: 338 – 353.