

Application of Fuzzy Analytic Hierarchy Process Method to predict Some Properties Soil Data

Safa Muhammad Ibrahim and Ghanim Mahmood Dhaher

University of Mosul/College of Education for Pure Sciences/ Department of Mathematics/ Iraq, Mosul

Author Emails

Corresponding author: safa.as423@student.uomosul.edu.iq

Article History:

Received: 10-01-2025

Revised: 27-02-2025

Accepted: 07-03-2025

Abstract: This research deals with the selection of the best analysis and the selection of the best compound for high-quality soil data based on multiple specific criteria of quantitative and qualitative nature and in order to address the uncertainty property. In this research, we used the fuzzy analytical hierarchy process (F-AHP). In order to make the right decision to know the level of importance of each soil compound and its properties. This method is based on an effective algorithm through comparisons between the properties of soil compounds and their weight. F-AHP was applied in this work to choose one criterion out of five criteria in decision-making using fuzzy numbers. Through the results, we found that soil compounds with small weights are sometimes more effective and flexible than others. Through this study, we note that the method helps in reducing bias in decision-making and the method is converted into numerical values that are processed and compared for evaluation in the hierarchy associated with fuzzy logic with the uncertainty property. The direct impact on the accuracy of the data and all the results we obtained through the evaluation appears as the method of the analytical hierarchy process was combined in this research with the theory of fuzzy logic to become the method as the fuzzy analytical hierarchy process. All the calculations applied were done in Matlab.

Keywords: Analytical Hierarchy Process, Fuzzy AHP, Uncertainty, Soil Properties.

1. Introduction

In 1965, Zadeh was the first to develop the fuzzy theory through his published research (Fuzzy Sets). He showed that the idea of fuzzy sets provides us with the ability to build many concepts parallel to regular sets and provides a wider range of vision in the field of application, especially in the fields of pattern classification and information processing, and the use of fuzzy sets basically provides an important means to deal with the property of uncertainty.

The analytical sequence process is a systematic method for decision-making used to help decision makers evaluate options based on a set of criteria and give them priority. This method was developed by the Iraqi professor Thomas Al-Saati from the city of Mosul/Iraq in the seventies. The analytical sequence process is a common tool in areas that require multi-criteria evaluations such as strategic planning in project management and choosing between different projects.[5]

Due to the fluctuations within the field of environmental data, especially soil data, and the demand for value-added products, the study of soil components has been viewed as a major cost and source of uncertainty. Soil components retain their utmost importance for a number of reasons, such as

allowing flexible production schedules and taking advantage of the economics of the properties of the components involved in the soil. In making the right decision sometimes, a single criterion alone is not a very effective measure for decision making. Therefore, multi-criteria decision-making methods are used [3], [6]. Apart from other criteria such as importance of parts and availability, other costs such as ordering cost, scarcity, replaceability, repairability, etc. have been taken into account. More studies have been conducted on multi-criteria inventory classification in the past twenty years. Therefore, many different methods have been used and developed to classify inventory items considering multiple criteria [2], [4], [7], and criteria matrix approach [11], through a common criteria matrix and another criterion. Although this approach is interesting, it is accompanied by some limitations. The analytic hierarchy process (AHP) developed by Nguyen LD and Tran [9] has been successfully applied to multi-criteria inventory classification [10]. The advantage of the analytical hierarchy process is that it can include many criteria and is easy to use in a large accounting and measurement system, but its disadvantage is that a great deal of subjectivity is involved in pairwise comparisons.

2.Methodology

2-1 Classical logic and Fuzzy logic

In the traditional logic (dual) things are classified as either true or false with giving value to false and values to correct, but in real life there are many cases that cannot be classified accurately within this binary because there are different degrees between right and wrong, for example when classifying a degree as cold or hot (hot), it is difficult to set precise boundaries between cold and hot, this is where Fuzzy logic comes in. Where a set of scores is used to classify a specific case, where each element can partially belong to several groups, that is, true and false cases can be represented by degrees between 0 and 1, such as 0.3 or 0.8, and so on, which expresses the degree of belonging of this element to a certain category. Fuzzy area is known as fuzzy logic, a type of logic that allows dealing with information that lies in a gray area between true and false, i.e., unconfirmed information, developed by the scientist Lotifazole in the sixties of the twentieth century as a way to apply mathematics and logic to areas that require vague or approximate estimates such as robotics and control systems. [12].

2-2 Fuzzy logic features

Fuzzy logic allows flexibility in dealing with uncertain or ambiguous data that has the property of uncertainty. Fuzzy logic is used in designing systems that require approximate evaluations such as car controls and intelligent adaptive systems, and it also helps in building models that simulate human thinking, such as complex human classifications. Fuzzy logic includes rules that are the basis of the fuzzy system, as they link inputs and outputs based on a set of rules that describe the relationship between variables that cannot be described in traditional logic, but rather are flexible and express uncertainty or ambiguity in values.[13]

2-3 Analytic Hierarchy process (AHP)

In the analytical hierarchy method, the work of collecting data and arranging it in a hierarchical form is an important and basic priority for making the right decision. Therefore, this method has been integrated to support the decision-making process and is called the analytical hierarchy process (AHP) as a model for the best comparison of relevant criteria. There are many tools for making multiple decisions to achieve maximum goals, such as linear programming and the binary system. The multi-criteria hierarchical decision analysis method is considered one of the most important methods used in decision analysis in the case of ensuring decision problems that include multiple criteria and qualitative factors that cannot be included in the analysis of traditional discussion data. It is the main entrance to the analytical hierarchy method. However, there is a characteristic of uncertainty. In order to make a good decision, sometimes a single criterion is not an effective measure. Therefore, multiple criteria methods are followed to make a final and irreversible decision. More studies have been conducted on the topics of stock classification and considering multiple criteria. Several studies have been conducted to develop the analytical hierarchy process (AHP) through a common criteria matrix with another criterion. Although this approach is interesting, it comes with some limitations. [1].

Steps to the process of Hierarchical analysis

1- Determine the problem and build the hierarchy: -

- The main goal that the decision makers seek to achieve.
- Determine the criteria that will affect the decision.
- Determine the available alternatives or options to be evaluated

2- Conducting marital comparisons

In this step, decision-makers make an even comparison between the criteria where the extent to which each criterion is preferred over the other or vice versa is estimated using a preferential scale according to the importance of each criterion.

3-Account of relative weights

After conducting marital comparisons, the relative weights of the standards and alternatives are calculated based on the comparison matrix. These weights reflect the importance of each standard or advantage of each alternative that calculates weights using sports methods such as calculating the self -values of the matrix.

4- Verification of consistency

It is necessary to verify the extent of coordination of the assessments to ensure that the preferences presented are logical and interconnected. If the consistency rate is greater than (0.1), marital comparisons should be re -evaluated because they may not be sufficiently consistent.

5-Final evaluation account and decision -making

After calculating the weights, it is collected to determine the final evaluation of each alternative. The alternative to the highest final evaluation is the best and can be relied upon as an ideal option to achieve the main goal.

2-4 Fuzzy Analytical Hierarchy Process. (F-AHP)

In (1977-1980) the Analytical Hierarchy Process (AHP), proposed by Saaty, has found wide applications in many fields. The main advantage of AHP is that it can be combined with other methods, including linear programming, fuzzy logic, etc. In order to model uncertainty, the AHP method is combined with pairwise comparison of fuzzy sets. F-AHP facilitates decision-making procedures through precise definitions, were the first researchers in F-AHP, who used the trigonometric membership function to represent pairwise fuzzy ratios to find out the partial fuzzy priority through the least square's method, their selection tool used resources, benefits, and impacts as the main characteristics of their F-AHP project selection tool [9]. studied the use of F-AHP in construction projects for site selection, contractor selection, construction methods, risk assessment, and other areas related to construction projects. There are other examples in the literature that use F-AHP methodology in project selection [8].

3. Data Analysis

The analytical hierarchy process has been combined with the fuzzy Logic theory to become the method as the fuzzy analytical hierarchy process, where the fuzzy analytical process is used similarly to AHP method, whereas the fuzzy AHP or F-AHP method]. determines to reach the best decision-making method.

Step (1): Determine the problem statement According to the criteria that are used in this research, in order to choose the soil data form through soil properties. We have used the following chemical properties (PH, EC, K⁺, CL⁻, PO₄⁻, No₃⁻, So₄⁻) Its importance has been determined in the Table (1)

Table (1): Importance of soil data

Standards	PH	EC	K ⁺	CL ⁻	PO ₄ ⁻	NO ₃ ⁻	SO ₄ ⁻
Importance	9	3	7	4	1	6	5

Step (2): Calculating relative weights

After conducting marital comparisons, the relative weights of the standards and alternatives are calculated based on the comparison matrix. These weights reflect the importance of each criterion or advantage of each alternative that calculates weights using sports methods such as calculating the self -values of the matrix and the matrix of the binary comparisons is formed in the following formula: -

$$A = \begin{bmatrix} \frac{w_1}{w_1} & \dots & \frac{w_n}{w_1} \\ \frac{w_1}{w_1} & \dots & \frac{w_1}{w_1} \\ \vdots & \ddots & \vdots \\ \frac{w_1}{w_1} & \dots & \frac{w_n}{w_1} \\ \frac{w_m}{w_m} & \dots & \frac{w_m}{w_m} \end{bmatrix}$$

If w is the relative weight or the degree of relative importance for each standard

The weight $\frac{w_i}{w_j}$ represents the importance of the standard i compared to the standard j

Since $a_{ij} = \frac{w_i}{w_j}$ the matrix A it becomes as follows, and since $a_{ij}, ij = 1,2,\dots,n$ it corresponds to a_{ji} and the comparison of the alternative with itself is an equal preference so we see all the elements of the country in the matrix equal to one and therefore the matrix can be expressed as the following :-

$$A = \begin{bmatrix} 1 & a_{12} & \dots & a_{1m} \\ \frac{1}{a_{21}} & 1 & \dots & a_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{1}{a_{n1}} & \frac{1}{a_{n2}} & \dots & 1 \end{bmatrix}$$

The data matrix that we deal with in our research is as follows: -

Table (2): Cererians data of soil properties

Standards	Average	Relative weight
PH	2.5714	0.198
EC	2.75	0.212
K ⁺	2.4285	0.187
CL ⁻	1.7976	0.138
PO ₄ ⁻	1.576	0.1215
NO ₃ ⁻	1.469	0.1133
SO ₄ ⁻	0.3704	0.028
Total	12.96	0.997

Table (2) show the Cererians data of soil properties (PH, EC, K⁺, CL⁻, PO₄⁻, No₃⁻, So₄⁻)

Table (3): Average and Relative weight of soil properties

SO ₄ ⁻	NO ₃ ⁻	PQ ₄ ⁻	CL ⁻	K ⁺	EC	PH	Standards
7	1/6	5	4	1/3	1/2	1	PH
6	5	4	3	2	1	2	EC
5	4	3	2	1	1/2	3	K ⁺
4	3	2	1	1/2	1/3	1/4	CL ⁻
3	2	1	1/2	1/3	1/4	1/5	PO ₄ ⁻
2	1	1/2	1/3	1/4	1/5	6	NO ₃ ⁻
1	1/2	1/3	1/4	1/5	1/6	1/7	SO ₄ ⁻

Table (3) Illiterate the Average and Relative weight of soil properties

Step (3) Calculate the value of Priority and that by finding the value of the Geometric Mean for each row first and then we calculate the value of the Priority for each row

Geometric Mean is calculated using the following law

$$Geometric \quad Row_i = \sqrt[n]{a_{ij}} \quad i, j = 1, 2, \dots, n \quad (1)$$

Whereas n is the number of data values

Geometric Row =(1.214 1.9019 2.0998 1 0.6518 0.71968 0.29585)

Now, we calculate Priority using the following law

$$Priority \quad (Row_i) = \frac{Geometric \quad Row_i}{Total \quad Geometric \quad Mean} \quad (2)$$

Priority Row =(0.1540 0.2413 0.2664 0.1269 0.0827 0.0913 0.0375)

Sum of priority =1

Step (4) Find the Eigen value as a symbol λ_{max1}

$$By \quad \lambda_{max1} = \sum_{i=1}^n (total \quad column_i * priority \quad Row_i) \quad , i = 1, 2, \dots, n \quad (3)$$

$\lambda_{max1} = (12.5928 \quad 2.9500 \quad 4.6166 \quad 11.0833 \quad 15.8333 \quad 15.6666 \quad 28)$

$\lambda_{max} = 8.7671$

Step (5) Consistency check

It is necessary to verify the extent of the coordination of technologies to ensure that the preferences presented are logical and interconnected.

If the consistency rate is greater than 0.1, marital comparisons should be reassessed because they may not be sufficiently consistent, and that is to calculate the consistency rate, which we symbolize with the symbol CR

$$It \quad is \quad calculated \quad in \quad the \quad following \quad relationship \quad CR = \frac{CI}{RI} \quad (4)$$

CR: - The consistency rate

CI: - The consistency index or stability index is calculated in the following form

$$CI = \frac{\lambda_{max} - n}{n - 1} \quad (5)$$

RI: - The comparison of consistency indicator in the random generator (RI) as the results of following in table (3):

Table (3): Ratio index

n	1	2	3	4	5	6	7	8	9
RI	0	0	0.58	0.90	1.120	1.2400	1.320	1.4200	1.4500

Where the value of Ratio index relies on the matrix order.

λ_{max} : Self -value of the self -tragedy that was calculated in step (4)

n: The number of elements is the comparison

$$CI = 0.2945$$

$$CR = 0.2231$$

Step (6) Triangular Furry Number (TFN) We used F – AHP scale for Triangular fuzzy Number contents three values (K, m, n) where lowest value, middle value and highest value, respectively, see figure (1)

TFN uses in this paper through (K, m, n) and membership function $\mu_m(x)$ as the follow:

$$\mu_m(x) = \begin{cases} \frac{x}{m-K} - \frac{K}{m-K}, & x \in [k \ m] \\ \frac{x}{m-n} - \frac{n}{m-n} & x \in [m \ n] \end{cases} \quad (6)$$

Let $(k_1 \ m_1 \ n_1)$ and $(k_2 \ m_2 \ n_2)$ then the fuzzy arithmetic operations are defined as:

- Addition $(k_1 \ m_1 \ n_1) + (k_2 \ m_2 \ n_2) = (k_1 + k_2 \ m_1 + m_2 \ n_1 + n_2)$
- Multiplication $(k_1 \ m_1 \ n_1) \cdot (k_2 \ m_2 \ n_2) = (k_1 k_2 \ m_1 m_2 \ n_1 n_2)$
- Inverse $(k_1 \ m_1 \ n_1)^{-1} \approx (\lambda k_1 \ \lambda m_1 \ \lambda n_1)$, [2].

Step (7): Calculating weights of fuzzy vector

(TFN) is of the most uses technique to calculate weights from fuzzy comparison matrices (TFN) required, we have $X = \{x_1, x_2, \dots, x_n\}$ is object set, and $G = \{g_1, g_2, \dots, g_n\}$ goal set for each object set can be analysis procedure of each g_i . Can be calculate value of S_i as: [14].

$$S_i = \sum_{j=1}^m M_{qi}^j \times \left[\sum_{i=1}^n \sum_{j=1}^m M_{qi}^j \right]^{-1} \quad (7)$$

$$S_i = \begin{bmatrix} 0.7450 & 0.2804 & 0.1777 & 0.0679 & 0.0772 & 0.0204 \\ 0.6966 & 0.2804 & 0.1777 & 0.0679 & 0.0772 & 0.0204 \\ 0.6966 & 0.2917 & 0.1777 & 0.0679 & 0.0772 & 0.0204 \\ 0.6966 & 0.2917 & 0.2966 & 0.0679 & 0.0772 & 0.0204 \\ 0.6966 & 0.2917 & 0.2966 & 0.1033 & 0.0772 & 0.0204 \\ 0.6966 & 0.2917 & 0.2966 & 0.1033 & 0.0547 & 0.0204 \\ 0.6966 & 0.2917 & 0.2966 & 0.1033 & 0.0547 & 0.0808 \end{bmatrix}$$

$$\text{Where } \sum_{j=1}^m M_{qi}^j = \left(\sum_{j=1}^m l_j, \sum_{j=1}^m m_j, \sum_{k=1}^m u_j \right) \quad (8)$$

In the origin method of AHP, we use the scale (1 to 9) to Calculate the final weights through the (TFN). Fuzzy weight with the others fuzzy and we used the lower set of this probability for each scale, and fuzzy matrix.

$$\text{fuzzy matrix} = \begin{pmatrix} 2 & 1 & 2 & 3 & 0.25 & \dots & 4 & 5 & 0.166 & 7 \\ 3 & 0.5 & 1 & 2 & 3 & \dots & 3 & 0.25 & 5 & 6 \end{pmatrix}$$

0.25 0.33 0.5 1 2 ... 2 3 4 5
 0.20 0.25 0.33 0.5 1 ... 1 2 3 4
 0.142 0.166 0.20 0.25 0.33 ... 0.33 0.5 1 2)

$$\begin{aligned}
 V(M_2 \geq M_1) &= hgt(M_1 \cap M_2) \\
 &= M_{m_2}(d) \\
 &= 1 \quad \text{if } m_2 \geq m_1 \\
 &= 0 \quad \text{if } k_1 \geq n_2 \\
 &= \frac{k_1 - n_2}{(m_2 - n_2) - (m_1 - k_1)} \text{ otherwise}
 \end{aligned}
 \tag{9}$$

See figure (2):

We note the degree of possibility of fuzzy number (convex) is greater than of convex fuzzy numbers and it is given as:

$$\begin{aligned}
 V(M \geq M_1, M_2, \dots, M_k) &= \\
 V(M \geq M_1) \text{ and } (M \geq M_2), \dots, (M \geq M_k) &= \\
 = \min(M \geq M_i), i = 1, \dots, k &= \\
 \text{assuming } W_i^l = \min V(M_i \geq M_k) &=
 \end{aligned}
 \tag{10}$$

Then weight value given by $W^l = W_1^l, W_2^l, \dots, W_n^l$

Step (8): Ranking and selection of decision after using the alternative value S_i

Where $S_i (i = 1, 2, \dots, n)$ is n elements of the matrix. Also, we are getting the normalized weights vector as the following:

The vector of normalized weights are

$$w = (d(A_1), d(A_2), \dots, d(A_n))^T \tag{11}$$

$W = (0 \ 0 \ 1 \ 0.978 \ 0.885 \ 1 \ 1)$

Where w is non fuzzy number, we can get the final weights of each criterion by multiplying the criteria with the matrix. We also obtain the matrix of fuzzy written as:

Table (4):- Comparison matrix for three soil data

Soil	PO ₄	NO ₃	SO ₄
PO ₄	1	4	3
NO ₃	1/4	1	7

SO ₄	1/3	1/7	1
-----------------	-----	-----	---

Geometric= (2.2894 1.205071 0.362460)

Priority = (0.59358 0.31244 0.093975)

CI =0.2902

CR = 0.5004

Si =

(0.025864 1.5742128

0.05499 2.597451

0.08037 2.030734)

W= (0.5975 1 0.7770)

$\lambda_{max} = 3.58041$

$\lambda_{max1} = 1.5833333333 \quad 5.14285714285714 \quad 11$

Table (5):- Comparison matrix for three soil data

Soil	EC	CL	NO ₃
EC	1	1	2
CL	1	1	7
NO ₃	1/2	1/7	1

Geometric = (1.25992104989487 1.91293118277239 0.41491326683122)

Priority = (0.351171516121385 0.533181776545528 0.11564670733087)

CI= 0.0885

CR=0.1525

Si = (0.421393841166937 0.0405172413793104

0.680713128038898 0.181034482758621

0.777957860615883 0.310344827586207)

$\lambda_{max1} = 2.500 \quad 2.1429 \quad 10$

$\lambda_{max} = 3.1769$

W= (0 0 1) where W the weights.

Conclusions

Analytic Hierarchy Process (AHP) is a decision support method that can be developed by dividing the solution problems, grouping them, and then arranging them in a hierarchical structure with criteria that have levels of importance. To obtain the priority criteria, this method uses the comparison of criteria associated with a specified scale. The main approach of the Analytic Hierarchy Method also takes into account the validity of the data with the number of criteria. However, the large uncertainty and doubt in giving the evaluation will have an impact on the accuracy of the data and the results obtained. Accordingly, the Analytic Hierarchy Process (AHP) method was combined and linked to the theory of fuzzy logic. The Fuzzy Analytic Hierarchy Method is used similarly to the AHP method. The Fuzzy AHP method only assigns the AHP metric in the fuzzy triangle metric that should have priority in reaching it. This paper summarizes the application of the analytical hierarchy process which can help us in the selection and evaluation of organic compounds selection for high-quality soil data by F-AHP. The single-step accuracy shows that there is no error when applying the F-AHP method in Matlab language which reduces the uncertainty property. The high-quality organic compound selection model is based on the relative weights of the criteria and the optimal weight is selected by comparing the criteria of the soil data.

References

- [1] Audey Be-liveau , Georgia Salanti (2025) , Adriani Nikolakopoulou, Precision of Treatment Hierarchy: A Metric for Quantifying Certainty in Treatment Hierarchies from Network Meta-Analysis. <https://www.researchgate.net>
- [2] Enrica M, Purba HH, and Purba A (2021). Risks leading to cost overrun in construction projects: A systematic literature review. *Advance Researches in Civil Engineering*, 3(1): 43-60.
- [3] Erdogan SA, Šaparauskas J, and Turskis Z (2019). A multi-criteria decision-making model to choose the best option for sustainable construction management. *Sustainability*, 11(8):2239. <https://doi.org/10.3390/su11082239>
- [4] Fernanda A., João G., Marcia T. (2022)."A Fuzzy Analytic Hierarchy Process Model to Evaluate Logistics Service Expectations and Delivery Methods in Last-Mile Delivery in Brazil".*Open Access. Sustainability* 2022, 14(10),5753; Published: 10 May 2022. <https://doi.org/10.3390/su14105753>.
- [5] Hatefi SM and Tamošaitienė J (2018). Construction projects assessment based on the sustainable development criteria by an integrated fuzzy AHP and improved GRA model. *Sustainability*, 10(4): 991.<https://doi.org/10.3390/su10040991>
- [6] Kwon TJ, Fu L, and Melles SJ (2017). Location optimization of road weather information system (RWIS) network considering the needs of winter road maintenance and the traveling public. *Computer-Aided Civil and Infrastructure Engineering*, 32(1):57-71. <https://doi.org/10.1111/mice.12222>.
- [7] Lilane A., Irenilza A. and Pedro L.(2021). " Sustainable Development of Gem Prediction: An application of the Analytical Hierarchy Process".[International Journal of the Analytic Hierarchy Process](https://doi.org/10.13033/ijahp.v13i3.880) 13(3) 2021, DOI: 10.13033/ijahp.v13i3.880
- [8] Masoumi S, Hadji Molana SM, Javadi M, and Azizi A (2022). Designing integrated model of decision-making-robust optimisation to manage the maintenance of inter-urban routes under

- uncertainty. *International Journal of Pavement. Engineering*, 23(10): 3522-3535. <https://doi.org/10.1080/10298436.2021.1904238>
- [9] Nguyen LD and Tran DQ (2017). FAHP-based decision making framework for construction projects. In: Emrouznejad A and Ho W (Eds.), *Fuzzy analytic hierarchy process*: 327-346. Chapman and Hall/CRC, New York, USA. <https://doi.org/10.1201/9781315369884-14>
- [10] Reffas Nouredine, Mustafa Belboula, (2024), *The Other as Language: from the Hierarchy of Languages to the Hierarchy of Nations* <https://www.researchgate.net>
- [11] Roche M (2017). Road maintenance—patching a hole in mobilities- roading research: A case study of the Long Beach road board, Canterbury, New Zealand, 1911–1938. *New Zealand Geographer*, 73(2): 119-128. <https://doi.org/10.1111/nzg.12160>
- [12] Saeid Jafari, (2025) *Research on Fuzzy Logic and Mathematics with Applications*, <https://www.researchgate.net>
- [13] Theodoros A. Kyriazos, Mary Poga, (2024), *Exploring Fuzzy Logic as an Alternative Approach in Psychological Scoring*. <https://www.researchgate.net>
- [14] Wang, G., et al. (2020). "Approximation of fuzzy numbers by using multi-knots piecewise linear fuzzy numbers." *Journal of Intelligent & Fuzzy Systems* **39**(3): 3597-3615. <https://doi.org/10.3390/math10010145>