Advances in Nonlinear Variational Inequalities
ISSN: 1092-910X
Vol 28 No. 7s (2025)

On Vertex, Edge Matrix Cordial Labeling

A. Sudha Rani [1], N. Ramalingam[2], R. Vanaja[3], R. Shanthi[4], H. A. Bhavithra[5], S. Sindu Devi[6]

[1] Department of Mathematics, SRMIST, Ramapuram, Chennai-89, India

sudharaa@srmist.edu.in

[2]Department of Mathematics, SRMIST, Ramapuram, Chennai-89, India

Lingam.rama5@gmail.com

[3]Department of Mathematics, SIMATS Engineering, Chennai-105 ,India

vanajar.sse@saveetha.com

[4]Department of Mathematics, Meenakshi Academy of Higher Education and Rsearch, Chennai- 78 ,India

shanthir@maherfhs.ac.in

[5]Department of Mathematics, SRMIST, Ramapuram, Chennai-89, India

bhavith1@srmist.edu.in

[6]Department of Mathematics, SRMIST, Ramapuram, Chennai-89, India

sindudes@srmist.edu.in

Article History:

Received: 21-12-2024

Revised: 29-1-2025

Accepted:9-2-2025

Abstract:

Let G be a simple undirected graph whose vertices are from non cyclic abelian group and there is an edge between any two vertices iff vertices form a non singular matrix. Here we assign the Matrix Cordial Labeling to the vertices. In this paper we introduce the concept of Matrix Cordial labeling for some splitting graphs We shall discuss Matrix Cordial Labeling for path, cycle .

Conclusions: In this paper we investigated three new matrix cordial graphs. The findings in this paper are original. For a better understanding of the labeling pattern specified in each theorem, illustrations are given at the conclusion of each theorem.

Keywords: Cordial labeling, singular matrix and non-singular matrix, Splitting graph

1. Introduction

Sampathkumar and Walikar introduced the splitting graph. new vertex v1 is added to a graph G in order to create the splitting graph S1(G) of that graph, where N(v)=N(v1) and N1(v) are the neighbourhood sets of vertex v and vertex v1, respectively. We start with an undirected, connected, simple, finite graph with vertices and edges. We adhere to Gross and Yellen [1]'s nomenclature and notational guidelines. We refer Beineke and Hegde [2]'s strongly multiplicative graphs . For further graphs, we refer Gallian[3]'s "Dynamic Survey of Graph Labeling." We also refer Cahit[4], "Cordial Graphs: A Weaker Version of Graceful and Harmonious Graphs for codial labeling. Numbers and graph structure have a tight relationship thanks to graph labelling. Gallian provides a helpful overview of the various graph labelling techniques [7]Combining the cordial labelling [6] notion in graph labelling with the relatively prime concept from number theory.

2.Preliminaries

This section contains basic defisnitions that are needed throughout this paper

Definition 2.1

Let g : V(G) $\rightarrow K_4$ in such a way, the edge uv is labeled as |g(u)–g(v)|. g is called cordial labeled if the difference between the number of vertices labeled 0 and the number of vertices labeled 1 is atmost 1.

Definition 2.2

An $m\ x\ n$ matrix A is an array of mn numbers a_{ij} where $1 \leq i \leq m, 1 \leq j \leq n$ arranged in m rows and n columns as follows.

$$\begin{pmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & \ddots & \vdots \\ a_{m1} & \cdots & a_{mn} \end{pmatrix}$$

We shall denote this matrix by the symbol (a_{ij}). If m=n , A is called a square matrix of order n.

Definition 2.3

A square matrix A is said to be singular if |A|=0.

A is called a non-singular matrix if |A|≠0.

Definition 2.4

For a graph G the splitting graph $S_1(G)$ of graph G is obtained by adding a new vertex v1 corresponding to each vertex v of G such that $N(v)=N(v_1)$ and $N_1(v)$ are the neighbourhood sets of v and v1 respectively.

Theorem 1:

All Splitting path $S^1(P_n)$,n≥ 2 is vertex matrix cordial labeling .

Proof:-

Let $u_1,u_2,u_3,\ldots\ldots,u_n$ & $v_1,v_2,v_3,\ldots\ldots..,v_n$ be the vertices of $S^1(P_n)$ .Edge set of this is given by

$$E(S^1(P_n)) = \begin{cases} u_i u_{i+1}, 1 \leq i \leq n-1 \\ u_i v_{i+1}, 1 \leq i \leq n-1 \\ u_{i+1} v_i, 1 \leq i \leq n-1 \end{cases}$$

Case 1: $n \equiv 0\ (mod 4)$

Let $n = 4k, k \geq 1$

$$f(u_i) = \begin{cases} (2,0), i \equiv 1 \text{ (mod 4)} \\ (1,1), i \equiv 2 \text{(mod 4)} \\ (1,2), i \equiv 3 \text{ (mod 4)} \\ (0,1), i \equiv 4 \text{(mod 4)} \end{cases}$$

$$f(v_i) = \begin{cases} (1,0), i \equiv 1 \text{ (mod 4)} \\ (2,2), i \equiv 2 \text{(mod 4)} \\ (2,1), i \equiv 3 \text{ (mod 4)} \\ (0,2), i \equiv 4 \text{(mod 4)} \end{cases}$$

$u_f(2,0) = u_f(1,1) = u_f(1,2) = u_f(0,1) = k$,

$v_f(2,1) = v_f(2,2) = v_f(1,0) = v_f(0,2) = k$

Hence $|n_f(0) - n_f(1)| \leq 1$

Case 2: $n \equiv 1\ (mod4)$

Let $n = 4k + 1, k \geq 1$ for $1 \leq i \leq 4k$ we labeled as in case (i),for remaining vertices we define

$f(u_i) = \{(2,0); i = 4k + 1$, $f(v_i) = \{(1,0); i = 4k + 1$

$u_f(1,1) = u_f(1,2) = u_f(0,1) = k$, $u_f(2,0) = k+1$

$v_f(2,1) = v_f(2,2) = v_f(0,2) = k$, $v_f(1,0) = k+1$

Hence $|n_f(0) - n_f(1)| \leq 1$

Case 3: $n \equiv 2\ (mod4)$

Let $n = 4k + 2, k \geq 1$ for $1 \leq i \leq 4k$ we labeled as in case(i), for remaining vertices we define

$$f(u_i) = \begin{cases} (1,1); i = 4k + 1 \\ (0,1); i = 4k + 2 \end{cases},$$

$$f(v_i) = \begin{cases} (2,2); i = 4k + 1 \\ (2,1); i = 4k + 2 \end{cases}$$

$u_f(2,0) = u_f(1,2) = k$, $u_f(1,1) = u_f(0,1)=k+1$

$v_f(1,0) = v_f(0,2) = k$, $v_f(2,2)= v_f(2,1)=k+1$

Hence $|n_f(0) - n_f(1)| \leq 1$

Case 4: $n \equiv 3\ (mod4)$

Let $n = 4k + 3, k \geq 1$ for $1 \leq i \leq 4k$ we labeled as in case(i),for remaining vertices we define

$$f(u_i) = \begin{cases} (1,1); i = 4k + 1 \\ (0,1); i = 4k + 2 \\ (2,0); i \equiv 4k + 3 \end{cases},$$

$$f(v_i) = \begin{cases} (2,2); i = 4k + 1 \\ (2,1); i = 4k + 2 \\ (1,0); i \equiv 4k + 3 \end{cases}$$

$u_f(1,2) = k$, $u_f(2,0) = u_f(1,1) = u_f(0,1) =k+1$

$v_f(0,2) = k$, $v_f(2,1) = v_f(2,2) = v_f(1,0) =k+1$

Hence $|n_f(0) - n_f(1)| \leq 1$

Hence all **Splitting path $S^1(P_n)$, n>1 is matrix cordial labeling**

From the below table we can easily prove that all path $P_n, n > 1$is vertex matrix cordial labeling

Nature of n	$u_f(2,0)$ & $v_f(2,1)$	$u_f(1,1)$ & $v_f(2,2)$	$u_f(1,2)$ & $v_f(1,0)$	$u_f(0,1)$ & $v_f(0,2)$
4k	k & k	k & k	k & k	k & K
4k+1	k+1 &	k &	k &	k &

	k	k	k+1	K
4k+2	k & k+1	k+1 & k+1	k & k	k+1 & K
4k+3	k+1 & k+1	k+1 & k+1	k & k+1	k+1 & K

Theorem 2 :

All Splitting cycle $S^1(C_n)$,n ≥3 matrix cordial labeling.

Proof:

Let $u_1,u_2,u_3,\ldots\ldots,u_n$ & $v_1,v_2,v_3,\ldots\ldots..,v_n$ be the vertices of $S^1(C_n)$ .Edge set of this is given by

$$E(S^1(C_n))=\begin{cases} u_iv_{i+1}, 1 \le i \le n-1 \\ v_iu_{i+1}, 1 \le i \le n-1 \\ u_iv_{i+1}, 1 \le i \le n-1 \end{cases}$$

Case 1: $n \equiv 0\ (mod4)$

Let $n = 4k, k \ge 1$

$$f(u_i) = \begin{cases} (1,0), i \equiv 1 \text{ (mod 4)} \\ (2,2), i \equiv 2 \text{(mod 4)} \\ (1,2), i \equiv 3 \text{ (mod 4)} \\ (0,2), i \equiv 4 \text{(mod 4)} \end{cases}$$

$$f(v_i) = \begin{cases} (2,0), i \equiv 1 \text{ (mod 4)} \\ (1,1), i \equiv 2 \text{(mod 4)} \\ (2,1), i \equiv 3 \text{ (mod 4)} \\ (0,1), i \equiv 4 \text{(mod 4)} \end{cases}$$

$u_f(1,0) = u_f(2,2) = u_f(1,2) = u_f(0,2) = k$,

$v_f(2,0) = v_f(1,1) = v_f(2,1) = v_f(0,1) = k$

Hence$|n_f(0) - n_f(1)| \le 1$

Case 2: $n \equiv 1\ (mod4)$

Let $n = 4k + 1, k \ge 1$ for $1 \le i \le 4k$ we labeled as in case (i),for remaining vertices we define

$f(u_i) = \{(2,2); i = 4k + 1,\ \ f(v_i) = \{(1,1); i = 4k + 1$

$u_f(1,0) = u_f(1,2) = u_f(0,2) = k$, $u_f(2,2)=k+1$

$v_f(2,0) = v_f(2,1) = v_f(0,1) = k$, $v_f(1,1)$

Hence $|n_f(0) - n_f(1)| \le 1$

Case 3: $n \equiv 2\ (mod4)$

Let $n = 4k + 2, k \ge 1$ for $1 \le i \le 4k$ we labeled as in case(i), for remaining vertices we define

$$f(u_i) = \begin{cases} (1,0); i = 4k + 1 \\ (2,2); i = 4k + 2 \end{cases}$$,

$$f(v_i) = \begin{cases} (2,0); i = 4k+1 \\ (2,1); i = 4k+2 \end{cases}$$

$u_f(1,2) = u_f(0,2) = k$, $u_f(1,0) = u_f(2,2)=k=1$

$v_f(1,1) = v_f(0,1) = k$, $v_f(2,0)= v_f(2,1)=k+1$

Hence $|n_f(0) - n_f(1)| \le 1$

Case 4: $n \equiv 3\ (mod4)$

Let $n = 4k+3, k \ge 1$ for $1 \le i \le 4k$ we labeled as in case(i),for remaining vertices we define

$$f(u_i) = \begin{cases} (1,2); i = 4k+1 \\ (1,0); i = 4k+2 \\ (2,2); i \equiv 4k+3 \end{cases},$$

$$f(v_i) = \begin{cases} (2,1); i = 4k+1 \\ (2,0); i = 4k+2 \\ (1,1); i \equiv 4k+3 \end{cases}$$

$u_f(0,2) = k$, $u_f(1,0) = u_f(2,2) = u_f(1,2) =k+1$

$v_f(0,1) = k$, $v_f(2,0) = v_f(1,1) = v_f(2,1) =k+1$

Hence $|n_f(0) - n_f(1)| \le 1$

Hence all **Splitting path $S^1(P_n)$, n>1 is matrix cordial labeling**

From the below table we can easily prove that all path $P_n, n > 1$is K_4 matrix cordial labeling

Nature of n	$u_f(2,0)$ & $v_f(2,1)$	$u_f(1,1)$ & $v_f(2,2)$	$u_f(1,2)$ & $v_f(1,0)$	$u_f(0,1)$ & $v_f(0,2)$
4k	k & k	k & k	k & k	k & K
4k+1	k & k	k+1 & k+1	k & k	k & K
4k+2	k+1 & k+1	k & k+1	k & k+1	k & K
4k+3	k+1 & k+1	k+1 & k+1	k & k+1	k & K

Theorem 3:

All Splitting path $S^1(P_n)$,n≥ 2 is edge matrix cordial labeling .

Let $u_1,u_2,u_3,\ldots\ldots,u_n$ & $v_1,v_2,v_3,\ldots\ldots..,v_n$ be the vertices of $S^1(P_n)$ .Edge set of this is given by

$$E(S^1(P_n)) = \begin{cases} u_i u_{i+1}, 1 \le i \le n-1 \\ u_i v_{i+1}, 1 \le i \le n-1 \\ u_{i+1} v_i, 1 \le i \le n-1 \end{cases}$$

Case 1: $n \equiv 0\ (mod4)$

Let $n = 4k, k \ge 1$

$$f(u_i) = \begin{cases} (1,0), i \equiv 1 \text{ (mod 4)} \\ (1,1), i \equiv 2 \text{(mod 4)} \\ (0,1), i \equiv 3 \text{ (mod 4)} \\ (1,1), i \equiv 4 \text{(mod 4)} \end{cases}$$

$$f(v_i) = \begin{cases} (2,2), i \equiv 1 \text{ (mod 4)} \\ (0,2), i \equiv 2 \text{(mod 4)} \\ (2,2), i \equiv 3 \text{ (mod 4)} \\ (0,2), i \equiv 4 \text{(mod 4)} \end{cases}$$

$ne^{(0)} = 6k - 1, ne^{(1)} = 6k - 2$

Hence $|n_f(0) - n_f(1)| \le 1$

Case 2: $n \equiv 1\ (mod4)$

Let $n = 4k + 1, k \ge 1$ for $1 \le i \le 4k$ we labeled as in case (i),for remaining vertices we define

$f(u_i) = \{(0,1); i = 4k + 1, \quad f(v_i) = \{(2,1); i = 4k + 1$

$ne^{(0)} = 6k, ne^{(1)} = 6k$

Hence $|n_f(0) - n_f(1)| \le 1$

Case 3: $n \equiv 2\ (mod4)$

Let $n = 4k + 2, k \ge 1$ for $1 \le i \le 4k$ we labeled as in case(i), for remaining vertices we define

$$f(u_i) = \begin{cases} (0,1); i = 4k + 1 \\ (1,2); i = 4k + 2 \end{cases},$$

$$f(v_i) = \begin{cases} (2,2); i = 4k + 1 \\ (2,1); i = 4k + 2 \end{cases}$$

$ne^{(0)} = 6k + 2, ne^{(1)} = 6k + 1$

Hence $|n_f(0) - n_f(1)| \le 1$

Case 4: $n \equiv 3\ (mod4)$

Let $n = 4k + 3, k \ge 1$ for $1 \le i \le 4k$ we labeled as in case(i),for remaining vertices we define

$$f(u_i) = \begin{cases} (1,0); i = 4k + 1 \\ (1,1); i = 4k + 2 \\ (1,0); i \equiv 4k + 3 \end{cases},$$

$$f(v_i) = \begin{cases} (2,2); i = 4k + 1 \\ (2,0); i = 4k + 2 \\ (2,1); i \equiv 4k + 3 \end{cases}$$

$ne^{(0)} = 6k + 3, ne^{(1)} = 6k + 3$

Hence $|n_f(0) - n_f(1)| \le 1$

Hence all **Splitting path $S^1(P_n)$, n>1 is matrix cordial labeling**

From the below table we can easily prove that all path $P_n, n > 1$is K_4 matrix cordial labeling

Nature of n	$ne^{(0)}$	$ne^{(1)}$
4k	$6k-1$	$6k-2$
4k+1	$6k$	$6k$
4k+2	$6k+2$	$6k+1$
4k+3	$6k+3$	$6k+3$

Theorem 4:

All Splitting cycle $S^1(C_n)$,n$\geq$ 2 is edge matrix cordial labeling .

Let $u_1,u_2,u_3,\ldots,u_n$ & $v_1,v_2,v_3,\ldots,v_n$ be the vertices of $S^1(C_n)$ .Edge set of this is given by

$$E(S^1(C_n))=\begin{cases} u_iu_{i+1}, 1\leq i\leq n-1 \\ v_iu_{i+1}, 1\leq i\leq n-1 \\ u_iv_{i+1}, 1\leq i\leq n-1 \end{cases}$$

Case 1: $n\equiv 0\ (mod4)$

Let $n=4k, k\geq 1$

$$f(u_i)=\begin{cases}(1,1), i\equiv 1\ (\text{mod } 4)\\ (0,2), i\equiv 2(\text{mod } 4)\\ (0,2), i\equiv 3\ (\text{mod } 4)\\ (1,1), i\equiv 4(\text{mod } 4)\end{cases}$$

$$f(v_i)=\begin{cases}(0,1), i\equiv 1\ (\text{mod } 4)\\ (2,2), i\equiv 2(\text{mod } 4)\\ (0,1), i\equiv 3\ (\text{mod } 4)\\ (2,1), i\equiv 4(\text{mod } 4)\end{cases}$$

$ne^{(0)}=6k, ne^{(1)}=6k$

Hence$|n_f(0)-n_f(1)|\leq 1$

Case 2: $n\equiv 1\ (mod4)$

Let $n=4k+1, k\geq 1$ for $1\leq i\leq 4k$ we labeled as in case (i),for remaining vertices we define

$f(u_i)=\{(0,2); i=4k+1,\quad f(v_i)=\{(2,2); i=4k+1$

$ne^{(0)}=6k+2, ne^{(1)}=6k+1$

Hence $|n_f(0)-n_f(1)|\leq 1$

Case 3: $n\equiv 2\ (mod4)$

Let $n=4k+2, k\geq 1$ for $1\leq i\leq 4k$ we labeled as in case(i), for remaining vertices we define

$$f(u_i)=\begin{cases}(1,0); i=4k+1\\ (0,2); i=4k+2\end{cases}$$,

$$f(v_i)=\begin{cases}(0,1); i=4k+1\\ (2,0); i=4k+2\end{cases}$$

$ne^{(0)}=6k+3, ne^{(1)}=6k+3$

Hence $|n_f(0)-n_f(1)|\leq 1$

Case 4: $n\equiv 3\ (mod4)$

Let $n=4k+3, k\geq 1$ for $1\leq i\leq 4k$ we labeled as in case(i),for remaining vertices we define

$$f(\mathrm{u_i}) = \begin{cases} (1,1); i = 4k+1 \\ (0,2); i = 4k+2 \\ (0,2); i \equiv 4k+3 \end{cases}$$,

$$f(\mathrm{v_i}) = \begin{cases} (0,1); i = 4k+1 \\ (2,2); i = 4k+2 \\ (2,2); i \equiv 4k+3 \end{cases}$$

$ne^{(0)} = 6k+5, ne^{(1)} = 6k+4$

Hence $|n_f(0) - n_f(1)| \leq 1$

Hence all **Splitting path $S^1(C_n)$, n>1 is edge matrix cordial labeling**

From the below table we can easily prove that all path $P_n, n > 1$ is edge matrix cordial labeling

Nature of n	$ne^{(0)}$	$ne^{(1)}$
4k	$6k$	$6k$
4k+1	$6k+2$	$6k$+1
4k+2	$6k+3$	$6k+3$
4k+3	$6k+5$	$6k+4$

Refrences

[1] J. Gross and J. Yellen, "Graph Theory and Its Applications," CRC Press, Boca Raton, 1999.

[2] L. W. Beineke and S. M. Hegde, "Strongly Multiplicative Graphs," Discussiones Mathematicae Graph Theory, Vol. 21, 2001, pp. 63-75.

[3] J. A. Gallian, "A Dynamic Survey of Graph Labeling," The Electronic Journal of Combinatorics, Vol. 1 DS6. http://www.combinatorics.org/Surveys/ds6.pdf

[4] I. Cahit, "Cordial Graphs: A Weaker Version of Graceful and Harmonious Graphs," Ars Combinatoria, Vol. 23, 1987, pp. 201-207.

[5] M. Sundaram, R. Ponraj and S. Somasundram, "Prime Cordial Labeling of Graphs," Journal of the Indian Academy of Mathematics, Vol. 27, No. 2, 2005 , pp. 373- 390.

[6] S. K. Vaidya and P. L. Vihol, "Prime Cordial Labeling for Some Graphs," Modern Applied Science, Vol. 4, No. 8, 2010, pp. 119-126.

[7] S. K. Vaidya and P. L. Vihol, "Prime Cordial Labeling for Some Cycle Related Graphs," International Journal of Open Problems in Computer Science and Mathematics, Vol. 3, No. 5, 2010, pp. 223-232.

[8] S. K. Vaidya and N. H. Shah, "Some New Families of Prime Cordial Graphs," Journal of Mathematics Research, Vol. 3, No. 4, 2011, pp. 21-30. doi:10.5539/jmr.v3n4p21