

Moments of Ordered Statistics Approach for Discrimination

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Abstract: Test statistics based on moments of ordered statistics are proposed for discriminating between the null population, Inverse Rayleigh Distribution (IRD), and two successive alternative populations, Rayleigh Distribution (RD) and Inverse Half Logistic Distribution (IHL). Due to the intractability of exact sampling distributions, the percentiles of the proposed test statistics are computed using simulated sampling distributions. The powers of the test statistics are also computed and compared. A comparative study of the powers is presented for a given sample size, evaluating the effectiveness of the proposed test statistics.

Keywords: Inverse Rayleigh distribution, Rayleigh distribution, Inverse Half Logistic distribution, Order Statistics, Population Quantile, Power of the test.

1. Introduction

The cumulative distribution functions (CDFs) for the following distributions are
Inverse Rayleigh Distribution (IRD)

$$F(x) = e^{-\frac{1}{\lambda^2 x^2}}, x > 0, \lambda > 0 \quad (1.1)$$

Rayleigh Distribution (RD)

$$F(x) = 1 - e^{-\frac{x^2}{2\sigma^2}}, x > 0, \sigma > 0 \quad (1.2)$$

Inverse Half Logistic Distribution (IHL)

$$F(x) = \frac{2e^{-\frac{b}{x}}}{1 + e^{-\frac{b}{x}}}, x > 0, b > 0 \quad (1.3)$$

The probability density functions (PDFs) for the following distributions are:

Inverse Rayleigh Distribution (IRD)

$$f(x) = \frac{2}{\lambda^2 x^3} e^{-\frac{1}{\lambda^2 x^2}}, x > 0, \lambda > 0 \quad (1.4)$$

Rayleigh Distribution (RD)

$$f(x) = \frac{x}{\sigma^2} e^{-\frac{x^2}{2\sigma^2}}, \quad x > 0, \sigma > 0 \tag{1.5}$$

Inverse Half Logistic Distribution (IHLD)

$$f(x) = \frac{2be^{-\frac{b}{x}}}{x^2(1+e^{-\frac{b}{x}})^2}, \quad x > 0, b > 0 \tag{1.6}$$

The scale parameters of IRD, RD, and IHLD are λ , σ , and b , respectively.

The striking similarities between the frequency curves of Inverse Rayleigh Distribution (IRD), Rayleigh Distribution (RD), and Inverse Half Logistic Distribution (IHLD) motivate investigating their interchangeability. Given IRD's established presence in literature, we explore RD and IHLD as alternative models. In a hypothesis testing framework, RD and IHLD serve as alternative populations with IRD as the null population. Figures 1.1-1.3 display the standard probability models' graphs, while Figure 1.4 provides a comparative visualization.

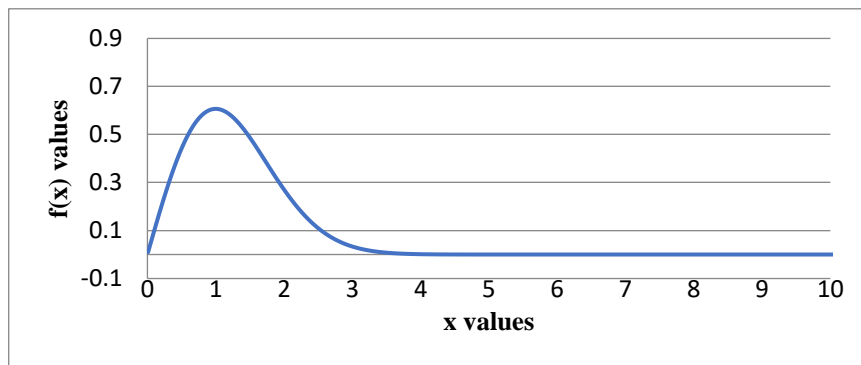


Figure 1.1. PDF graph of standard RD

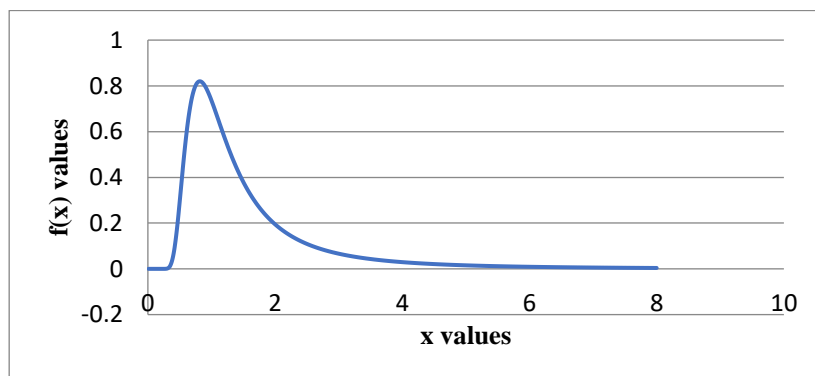


Figure 1.2. PDF graph of standard IRD

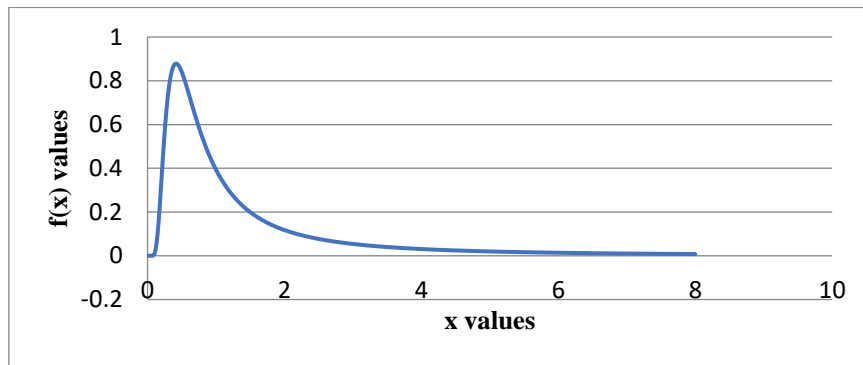


Figure 1.3. PDF graph of standard IHLD

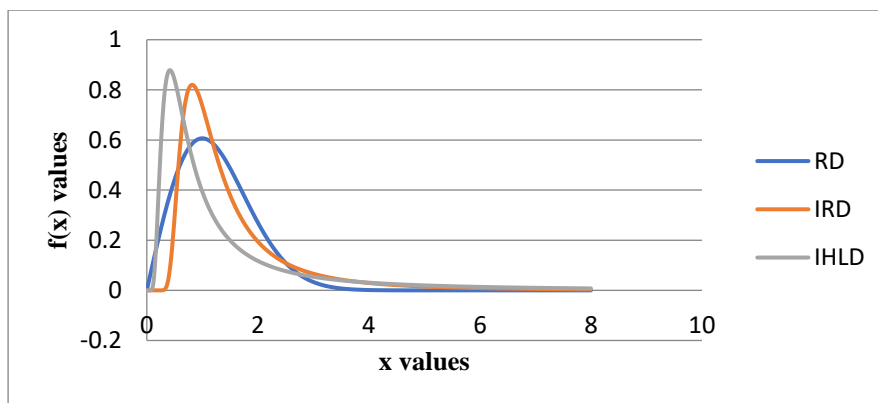


Figure 1.4. PDF graph of standard RD, IRD & IHLD

Lifetime data analysis frequently employs distributions, which can exhibit similar probability density functions or hazard functions for certain parameter ranges. However, one distribution's mathematical structure may be simpler to handle analytically than another's. This raises the question: Can a mathematically simpler model serve as an alternative to a structurally complicated one, given graphically similar probability density functions or hazard functions? To address this, a statistically admissible test procedure (with low error probabilities) is desirable, treating the complicated model as the null and the simpler model as the alternative. This study investigates the discrimination procedure between two distribution functions. We examine the asymptotic properties of the proposed criterion and find that the asymptotic distributions are independent of unknown parameters. These distributions are then used to determine the minimum sample size required to discriminate between the two distribution functions with a user-specified probability of correct selection.

It has been observed that point estimation methods for most probability models do not yield closed-form expressions for parameter estimators. This limitation motivates us to explore the possibility of using alternative probability models, such as those similar to Inverse Rayleigh Distribution (IRD), with a reasonable power of indistinguishability between the chosen distribution pairs. This motivation leads to the formulation of a statistical hypothesis testing problem: "Is a given sample of a specified size drawn from IRD or an alternative distribution?" In this context, IRD serves as the null population, while the alternative distribution is considered as a suitable substitute. This paper discusses procedures for identifying the given sample with IRD or the alternative distribution, minimizing risks and maximizing power.

A few notable researchers who have contributed to this area include: Gupta and Kundu (2003), Gupta and Kundu (2004), Kundu and Manglick (2004), Kundu and Raqab (2007), Dey and Kundu (2010), Dey and Kundu (2012), Rao and Kantam (2013), Rao *et al.* (2013), Rao *et al.* (2013), Rao and Kantam (2014), Kantam *et al.* (2014a), Kantam *et al.* (2014b), Prasad and Kantam (2015), Kantam and Ravikumar (2015), Kantam and Ravikumar (2016), Rao *et al.* (2016), Subhradevsen *et al.* (2020).

In this paper, we employ the Moments of Ordered Statistics approach to develop test criteria for identifying whether a given sample originates from the Inverse Rayleigh distribution or alternative models, specifically Rayleigh distribution and Inverse Half Logistic distribution. The choice of null and alternative populations is motivated by the striking similarities in the frequency curves of these distributions when plotted on the same scale, raising questions about their interchangeability. We investigate the discrimination between two pairs:

- (a) Inverse Rayleigh distribution (null) vs. Rayleigh distribution (alternative).
- (b) Inverse Rayleigh distribution (null) vs. Inverse Half Logistic distribution (alternative).

Furthermore, this study aims to explore whether Rayleigh distribution and Inverse Half Logistic distribution can be considered acceptable alternatives, or if it's feasible to adapt the simpler and admissible inferential procedures of Rayleigh distribution and Inverse Half Logistic distribution to Inverse Rayleigh distribution.

The test procedure based on moments of ordered statistics is described in Section 2. In Section 3, this approach is adapted to our chosen null and alternative populations, and critical values are computed. Section 4 presents the power of the test criteria for various sample sizes. A comparative study of the proposed test criteria for the chosen null and alternative populations is presented in Section 5.

2. Moments of Order Statistics Approach For Discrimination

Consider two continuous probability distributions, P_0 and P_1 , representing the null and alternative populations, respectively. Let $x_{(1)}, x_{(2)}, \dots, x_{(n)}$ denote an ordered random sample of size n drawn from the null population (Inverse Rayleigh Distribution, IRD) P_0 .

Consider the following hypothesis testing scenario:

Null hypothesis (H_0): The sample is drawn from population P_0 .

Alternative hypothesis (H_1): The sample is drawn from population P_1 .

Sultan (2007) introduced a test statistic, given by

$$T = \frac{\sum_{i=1}^n x_{(i)} \alpha_i}{\sqrt{\sum_{i=1}^n x_{(i)}^2 \sum_{i=1}^n \alpha_i^2}} \tag{2.1}$$

where α_i represents the expected value of the i^{th} standard order statistic in a sample of size n from the null population i.e., $\alpha_i = E(X_{(i)})$.

Assuming $x_{(1)}, x_{(2)}, \dots, x_{(n)}$ is a sample from the null population, Sultan's (2007) test statistic 'T' serves as a discriminator between the null and alternative populations, based on its critical values.

Consequently, the sampling distribution of 'T' and its percentiles are crucial for utilizing the test statistic 'T'. However, since 'T' involves non-linear functions of order statistics, its sampling distribution is generally intractable analytically for all populations. Therefore, empirical sampling distributions and percentiles of 'T' must be obtained through simulation. The percentiles of 'T' will serve as critical values for testing the null hypothesis. A large value of 'T' indicates a strong linear relationship between $x_{(i)}$ and $\alpha_{(i)}$, suggesting that the sample originates from the specified distribution. Conversely, a small value of 'T' implies that the sample comes from a different distribution. Sultan (2007) applied this approach using the generalized exponential distribution as the null population.

The statistic 'T', defined in (2.1), relies on moments of order statistics from samples of the null population, which may not be readily available for all probability models, including the Inverse Rayleigh distribution. In this paper, we derive the moments of ordered statistics, $E(X_i) = \alpha_i$, for the Inverse Rayleigh distribution, with the calculation procedure outlined in Section 3.

3. Applying The Test Procedure to the Inverse Rayleigh Distribution

Consider a complete ordered random sample, $x_{(1)}, x_{(2)}, \dots, x_{(n)}$, of size n from the Inverse Rayleigh distribution.

The null hypothesis is

H_0 : The sample comes from the Inverse Rayleigh distribution.

While the alternative hypotheses are

H_{11} : The sample comes from the Rayleigh distribution.

H_{12} : The sample comes from the Inverse Half Logistic distribution.

Extending the above approach, our proposed test statistic is

$$T = \frac{\sum_{i=1}^n x_{(i)} \alpha_i}{\sqrt{\sum_{i=1}^n x_{(i)}^2 \sum_{i=1}^n \alpha_i^2}} \tag{3.1}$$

The pdf of the i th order statistic $X_{r:n}, 1 \leq r \leq n$ as

$$f_{r:n}(x) = \frac{n!}{(r-1)!(n-r)!} [F(x)]^{r-1} [1-F(x)]^{n-r} f(x); -\infty < x < \infty \tag{3.2}$$

where $f(x)$ and $F(x)$ denote the pdf and cdf of the IRD.

The K^{th} moment of $X_{r:n}$ is given by

$$\alpha_{r:n}^{(k)} = E[X_{r:n}^k] = \int_0^{\infty} x^k f_{r:n}(x) dx \tag{3.3}$$

These moments are evaluated as follows:

In a sample of size 'n' from the Inverse Rayleigh Distribution (IRD), the expected value of the highest ordered statistic is: $\alpha_{n:n} = \sqrt{n\pi}$. This expression is used repeatedly for various 'n' (ranging from 1 to 25) in the recurrence relation proposed by Balakrishnan and Cohen (1991). The resulting values are presented in Table 1.

$$n\alpha_{i:(n-1)} = i\alpha_{i+1:n} + (n-i)\alpha_{i:n} \text{ in a sequential manner.}$$

4. Test Power Criteria

The power of the test statistic is evaluated against Rayleigh ($\sigma=1$) and Inverse Half Logistic ($b=1$) distributions as alternatives. We generated 10,000 random samples of sizes $n = 5, 10, 15, 20,$ and 25 from the Inverse Rayleigh distribution ($\lambda=1$). For each sample, the test statistic T was calculated. When calculating T for samples drawn from the Rayleigh distribution ($\lambda=1$), the δ_i 's values in the T formula were retained from the null population (Inverse Rayleigh distribution). The proportion of T values exceeding specified percentiles (90th, 95th, and 99th) out of 10,000 samples were recorded, representing the power of the test statistic T at respective significance levels. Through 10,000 Monte-Carlo simulations, we derived the percentiles of the empirical sampling distribution of T for $n = 5 (5) 25$, presented in Table 2.

The power of a test is the probability of correctly rejecting a false null hypothesis, contingent upon the alternative distribution.

To evaluate the power of the test statistic T, we follow these steps:

1. Generate a sample of specified size from each alternative population.
2. Calculate T using the α_i values from the Inverse Rayleigh distribution.
3. Compare the calculated T with the critical value from Table 2.2.
4. Repeat steps 1-3, 10,000 times.

The proportion of times the null hypothesis is rejected out of 10,000 simulations is taken as a measure of the test's power.

i.e., power of the test = $\frac{\text{number of rejections of } H_0}{10,000}$

The power calculations were conducted for the aforementioned alternative distributions using various sample sizes and significance levels. The results for complete samples are summarized in Table 3.

Table 1. Ordered Statistics Moments for the Standard IRD

n	i	α_i	n	i	α_i	n	i	α_i
1	1:1	1.77245	9	1:9	0.63668	12	7:12	1.32443
2	1:2	1.03828		2:9	0.78557		8:12	1.51896
	2:2	2.50663		3:9	0.92390		9:12	1.78608
3	1:3	0.86746		4:9	1.07539		10:12	2.19951
	2:3	1.37992		5:9	1.25851		11:12	3.00321
	3:3	3.06998		6:9	1.50230		12:12	6.13996

4	1:4	0.78506			7:9	1.87196			1:13	0.59210
	2:4	1.11465			8:9	2.58042			2:13	0.70789
	3:4	1.64520			9:9	5.31736			3:13	0.80517
	4:4	3.54491			10	1:10			0.62289	4:13
5	1:5	0.73458	2:10	0.76078		5:13	1.00084			
	2:5	0.98699	3:10	0.88472		6:13	1.11199			
	3:5	1.30615	4:10	1.01533		7:13	1.24057			
	4:5	1.87123	5:10	1.16548		8:13	1.39630			
	5:5	3.96333	6:10	1.35153		9:13	1.59562			
6	1:6	0.69964	7:10	1.60281		10:13	1.87073			
	2:6	0.90925	8:10	1.98730		11:13	2.29814			
	3:6	1.14246	9:10	2.72869		12:13	3.13140			
	4:6	1.46984	10:10	5.60499		13:13	6.39067			
	5:6	2.07193	11	1:11	0.61116	14	1:14	0.58419		
	6:6	4.34161		2:11	0.74024		2:14	0.69483		
7	1:7	0.67361		3:11	0.85321		3:14	0.78628		
	2:7	0.85583		4:11	0.96876		4:14	0.87442		
	3:7	1.04279		5:11	1.09682		5:14	0.96557		
	4:7	1.27536		6:11	1.24788		6:14	1.06433		
	5:7	1.61569		7:11	1.43791		7:14	1.17554		
	6:7	2.25442		8:11	1.69704		8:14	1.30561		
	7:7	4.68947		9:11	2.09615		9:14	1.46432		
8	1:8	0.65323		10:11	2.86926		10:14	1.66857		
	2:8	0.81631	11:11	5.87856	11:14	1.95160				
	3:8	0.97440	12	1:12	0.60100	12:14	2.39265			
	4:8	1.15678		2:12	0.72286	13:14	3.25453			
	5:8	1.39395		3:12	0.82716	14:14	6.63192			
	6:8	1.74874		4:12	0.93135	15	1:15	0.57712		
	7:8	2.42298		5:12	1.04359		2:15	0.68329		
	8:8	5.01326		6:12	1.17134		3:15	0.76983		

Contd...

n	i	α_i	n	i	α_i	n	i	α_i
15	4:15	0.85208	17	11:17	1.48196	19	14:19	1.76325
	5:15	0.93585		12:17	1.65054		15:19	1.99220
	6:15	1.02501		13:17	1.86970		16:19	2.31306
	7:15	1.12330		14:17	2.17583		17:19	2.81760
	8:15	1.23524		15:17	2.65586		18:19	3.81076
	9:15	1.36718		16:17	3.59863		19:19	7.72595
	10:15	1.52909		17:17	7.30801		20	1:20
	11:15	1.73831		18	1:18	0.55959		2:20
	12:15	2.02916	2:18		0.65535	3:20		0.71084
	13:15	2.48352	3:18		0.73085	4:20		0.77441

16	14:15	3.37315	19	4:18	0.80035	21	5:20	0.83596
	15:15	6.86468		5:18	0.86876		6:20	0.89795
	1:16	0.57072		6:18	0.93886		7:20	0.96208
	2:16	0.67300		7:18	1.01282		8:20	1.02989
	3:16	0.75534		8:18	1.09278		9:20	1.10297
	4:16	0.83265		9:18	1.18117		10:20	1.18315
	5:16	0.91037		10:18	1.28118		11:20	1.27280
	6:16	0.99189		11:18	1.39721		12:20	1.37509
	7:16	1.08021		12:18	1.53589		13:20	1.49457
	8:16	1.17870		13:18	1.70787		14:20	1.63811
	9:16	1.29178		14:18	1.93195		15:20	1.81689
	10:16	1.42583		15:18	2.24551		16:20	2.05063
	11:16	1.59104		16:18	2.73793		17:20	2.37867
	12:16	1.80524		17:18	3.70622		18:20	2.89505
	13:16	2.10380		18:18	7.51988		19:20	3.91251
	14:16	2.57115		1:19	0.55470		20:20	7.92665
15:16	3.48772	2:19		0.64770	1:21		0.54597	
16:16	7.08982	3:19		0.72038	2:21		0.63421	
17	1:17	0.56491	4:19	0.78672	3:21		0.70212	
	2:17	0.66374	5:19	0.85146	4:21		0.76321	
	3:17	0.74243	6:19	0.91719	5:21	0.82198		
	4:17	0.81555	7:19	0.98582	6:21	0.88072		
	5:17	0.88823	8:19	1.05912	7:21	0.94102		
	6:17	0.96351	9:19	1.13905	8:21	1.00421		
	7:17	1.04392	10:19	1.22798	9:21	1.07163		
	8:17	1.13206	11:19	1.32906	10:21	1.14475		
	9:17	1.23118	12:19	1.44678	11:21	1.22539		
	10:17	1.34564	13:19	1.58787	12:21	1.31590		

Contd...

n	i	α_i	n	i	α_i	n	i	α_i
21	13:21	1.41949	23	8:23	0.96103	24	23:24	4.29541
	14:21	1.54077		9:23	1.01966		24:24	8.68322
	15:21	1.68678		10:23	1.08209	25	1:25	0.53171
	16:21	1.86893		11:23	1.14949		2:25	0.61261
	17:21	2.10742		12:23	1.22326		3:25	0.67336
	18:21	2.44249		13:23	1.30520		4:25	0.72683
	19:21	2.97048		14:23	1.39769		5:25	0.77716
	20:21	4.01167		15:23	1.50403		6:25	0.82635
	21:21	8.12240		16:23	1.62902		7:25	0.87564
22	1:22	0.54205		17:23	1.77996		8:25	0.92595
	2:22	0.62823	18:23	1.96878	9:25		0.97810	
	3:22	0.69409	19:23	2.21655	10:25	1.03283		
	4:22	0.75298	20:23	2.56533	11:25	1.09099		

	5:22	0.80927	24	21:23	3.11583		12:25	1.15350	
	6:22	0.86518		22:23	4.20296		13:25	1.22147	
	7:22	0.92216		23:23	8.50039		14:25	1.29633	
	8:22	0.98142		1:24	0.53495		15:25	1.37987	
	9:22	1.04409		2:24	0.61747		16:25	1.47454	
	10:22	1.11140		3:24	0.67977		17:25	1.58376	
	11:22	1.18477		4:24	0.73488		18:25	1.71250	
	12:22	1.26601		5:24	0.78700		19:25	1.86834	
	13:22	1.35748		6:24	0.83818		20:25	2.06368	
	14:22	1.46242		7:24	0.88973		21:25	2.32047	
	15:22	1.58554		8:24	0.94264		22:25	2.68250	
	16:22	1.73402		9:24	0.99780		23:25	3.25467	
	17:22	1.91952		10:24	1.05610		24:25	4.38591	
	18:22	2.16268		11:24	1.11849		25:25	8.86227	
	19:22	2.50467		12:24	1.18613				
	20:22	3.04403		13:24	1.26040				
	21:22	4.10843		14:24	1.34311				
	22:22	8.31355		15:24	1.43667				
	23	1:23		0.53839	16:24		1.54444		
		2:23		0.62266	17:24		1.67130		
		3:23		0.68666	18:24		1.82470		
		4:23		0.74357	19:24		2.01680		
5:23		0.79766	20:24	2.26911					
6:23		0.85106	21:24	2.62458					
7:23		0.90516	22:24	3.18601					

Table 3. Powers of ‘T’ Statistic: IRD

Level of Significance	0.10		0.05		0.01	
	RD	IHLD	RD	IHLD	RD	IHLD
Alternative Population						
Sample Size						
5	0.04680	0.06450	0.02180	0.02880	0.00510	0.00620
10	0.00570	0.04490	0.00240	0.01980	0.00070	0.00360
15	0.00030	0.02970	0.00010	0.01150	0.00010	0.00140
20	0.00000	0.02730	0.00000	0.00970	0.00000	0.00120
25	0.00000	0.01940	0.00000	0.00810	0.00000	0.00150

Table 2. Percentiles of T: IRD

P n	0.998 65	0.995	0.99	0.975	0.95	0.90	0.10	0.05	0.025	0.01	0.005	0.001 35
5	0.999 64	0.999 21	0.998 79	0.998 02	0.996 90	0.994 80	0.920 07	0.905 71	0.893 62	0.881 35	0.873 15	0.858 36
10	0.999 07	0.998 61	0.998 02	0.997 15	0.995 96	0.993 65	0.913 19	0.896 35	0.881 61	0.860 36	0.840 03	0.811 24
15	0.998 78	0.998 32	0.997 88	0.996 82	0.995 48	0.992 92	0.910 75	0.893 86	0.878 68	0.854 55	0.831 92	0.802 05
20	0.999 02	0.998 35	0.997 91	0.996 88	0.995 54	0.992 76	0.910 01	0.891 44	0.875 45	0.845 77	0.819 82	0.774 65
25	0.998 80	0.998 11	0.997 58	0.996 49	0.995 12	0.992 59	0.909 35	0.891 39	0.871 85	0.836 83	0.800 04	0.760 21

5. Discrimination Analysis: Key Findings

This paper proposes a test criterion based on moments of ordered statistics to distinguish between populations. We test the null hypothesis that a sample originates from an Inverse Rayleigh distribution against two alternative populations: Rayleigh and Inverse Half Logistic distributions.

Null and Alternative Hypotheses:

Two cases are considered:

Inverse Rayleigh Vs Rayleigh

H_0 : The sample comes from the Inverse Rayleigh distribution

H_1 : The sample comes from the Rayleigh distribution.

2. Inverse Rayleigh Vs Inverse Half Logistic

H_0 : The sample comes from the Inverse Rayleigh distribution.

H_1 : The sample comes from the Inverse Half Logistic distribution.

Methodology:

Moments of ordered statistics are evaluated for sample sizes $n = 5(5)25$ for the null population (Table 2). Test powers for both alternatives are presented in Table 3.

Results and Discussion:

- A critical examination of powers reveals:
- For Inverse Half Logistic, the significance level exceeds the power.
- Powers decrease as sample size increases ($n = 5(5)25$) for fixed $\alpha = 1\%$, 5% , 10% (Table 3).
- Inverse Half Logistic powers are generally lower than Rayleigh powers, except for $n = 25$ at $\alpha = 0.05$.

Conclusion:

The test procedure through population quantiles yields an inadmissible test, as the minimum power requirement ($1 - \beta \geq \alpha$) is not satisfied. This indicates bias. Despite similar frequency curves, one distribution cannot replace the other as an alternative for the chosen models.

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