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Crossing İntuitionistic KUS-İdeals on KUS-Algebras as an Extension of Bipolar Fuzzy Sets

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Abstract:

The concept crossing intuitionistic structures set is combination of intuitionistic fuzzy set and ℵfunction.In this paper,the concept crossing intuitionistic ideals are introduced and several properties are investigated .Also,the relations between crossing intuitionistic KUSideals and crossing intuitionistic ideals are given .The image and the preimage of crossing intuitionistic KUS ideals under homomorphism of KUS algebras are defined and how the image and the preimage of crossing intuitionistic KUSideals under homomorphism of KUS algebras become crossing intuitionistic KUS ideals are studied.

Keywords: KUS -algebra,fuzzy KUS ideals,crossing intuitionistic, KUS ideals,the pre imageof crossing intuitionistic KUS ideals in KUS algebras.

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1.Introduction

Many mathematicians ([3],[4])have studied of BCKalgebras as a generalization of the concept of set theoretic difference and propositional calculus and alot of properties are investigated.The fuzzy set initiated by L.A.Zadeh [20].In ([3],[4],[5])Since then this concept has been applied in BCI algebras and several theorems and properties are investigated .S.M.Mostafa and et al [14,16]introduced KUSideals in KUS algebras and introduced the notions fuzzy KUS subalgebras ,fuzzy KUS ideals of KUS algebras and studied relationship among them .

2. Preliminaries

Now we give some definitions and preliminary results needed in the later sections .

Definition 2.1.([16]). Let $(\varphi,*,0)$be an algebra of type $(2,0)$with a single binary operation $(*)$. X is called an KUS − algebra if it satisfies the following identities:
for any $\acute{s}, \hat{s}, \tau \in \varphi$.

(kus1) $(\tau * \hat{s}) * (\tau * \acute{s}) = \hat{s} * x$,

(kus2) : $0 * \acute{s} = \acute{s}$,

(kus3) : $\acute{s} * \acute{s} = 0$,

(kus4) : $\acute{s} * (\hat{s} * \tau) = \hat{s} * (\acute{s} * \tau)$.

In X we can define a binary relation $(\leq)$ by : $\acute{s} \leq \hat{s}$ if and only if $\hat{s} * \acute{s} = 0$.
In what follows, let $(\varphi;*,0)$denote an KUSalgebra unless otherwise specified. For brevity we also called φ a KUS − algebra.

Lemma 2.2. ([16]). In any KU − algebra $(\varphi;*,0)$, the following properties holds: $\forall\ \acute{s}, \hat{s}, \tau \in \varphi$;

$\acute{s} * \hat{s} = 0\ \&\ \hat{s} * \acute{s} = 0 \ \rightarrow\ \acute{s} = \hat{s}$,

$\hat{s} * [(\hat{s} * \tau) * \tau] = 0,$
$(0 * \acute{s}) * (\hat{s} * \acute{s}) = \hat{s} * 0$,
$\acute{s} = 0 * (0 * \acute{s})$,
$(\acute{s} * \hat{s}) * 0 = \hat{s} * \acute{s}$,
$\acute{s} \leq \hat{s}$ implies that$\hat{s} * \tau \leq \acute{s} * \tau$,
$\acute{s} \leq \hat{s}$ implies that $\tau * \acute{s} \leq \tau * \hat{s}$,
$\acute{s} \leq \hat{s}$ and $\hat{s} \leq \tau$ imply $\acute{s} \leq \tau$,
$\acute{s} * \hat{s} \leq \tau$ implies that $\tau * \hat{s} \leq \acute{s}$.

Example 2.3.

1) Let $\varphi = \{0, 1, 2, 3\}$ in which $(*)$ be defined by the following Table (1)

Table (1)

*	0	1	2	2
0	0	1	2	3
3	1	0		2
2	2	3	0	1
3	3	2	1	0

Then $(\varphi; * , 0)$ is a KUS − algebra .

2) Let $\varphi = \{0, 1, 2, 3, 4\}$ in which $(*)$ be defined by the following table (2) :

Table (2)

*	0	1	2	3	4
0	0	1	2	3	4
1	4	0	1	2	3
2	3	4	0	1	2
3	2	3	4	0	1
4	1	2	3	4	0

By routine calculations,It is easy to show that$(\varphi; * , 0)$is a KUS algebra .

Definition 2. 4([16])**.** Let φbe a KUS-algebra and let ϑ be a nonempty of φ. ϑ is called a KUS sub algebra of X if $\acute{s} * \hat{s} \in \vartheta$ whenever $\acute{s} \in \vartheta$ and $\hat{s} \in \vartheta$.

Definition 2. 5 ([16]). A nonempty subset $\sigma$of a KUS-algebra φ is called a KUS-ideal of φ if it satisfies: for $\acute{s}, \hat{s}, \tau \in \varphi$,

($Ikus_1$) $(0 \in \sigma)$,
($Ikus_2$) $(\tau * \hat{s}) \in \sigma$ and $(\hat{s} * \acute{s}) \in \sigma$ imply $(\tau * \acute{s}) \in \sigma$.

Example 2. 6 . Let $\varphi = \{0, a, b, c\}$ in which $(*)$ is defined by the following

Table (3)

*	0	a	b	c
0	0	a	b	c
a	a	0	c	b

b	b	c	0	a
c	c	b	a	0

Then $(\varphi;*,0)$ is KUS − algebra . It is easy to show that $\sigma_1 = \{0, a\}$, $\sigma_2 = \{0, b\}$, $I_3 = \{0, c\}$, and $\sigma_4 = \{0, a, b, c\}$ are KUS − ideals of φ .
Not that: The example have been calculated by using the programs

Proposition 2. 7([16])EveryKUS ideal of KUS algebra φ is a KUs subalgebra.

Proposition 2.8 ([16 **]).** Let $\{I_i \mid i\in\Lambda\}$be a family of KUS ideals of KUS

algebra φ Then $\bigcap_{i\in V} I_i$ the intersection of any set of KUS ideals of KUS

algebra X is also KUS − ideal .

Definition 2. 9 ([16]). Let (Ë ;∗ ,0) and (È;∗ `, 0`) be nonempty sets. A mapping φ: (Ë ;∗ ,0) → (È;∗`, 0`) be called a homomorphism if it satisfying $\varphi(ś * ŝ) = \varphi(ś) *` \varphi(ŝ)$, for all ś, ŝ ∈ φ. The set {x∈ φ/φ(ś) = 0`} is called the Kernel of φ denoted by Kerφ.

Theorem 2. 10 ([16]). Let φ: Ë → Èbe a homomorphism of a KUS-algebra È into a KUS-algebra È, then :
$\varphi(0) = 0'$.
φ is injective ↔ Kerφ = {0}.
ś ≤ ŝ implieφ(ś) ≤ φ(ŝ).

Theorem 2. 11 ([16]). Let φ: (Ë ;∗ ,0) → (È;∗ `, 0`) be a homomorphism of a KUS-algebra Ë into a KUS-algebraÈ, then :
(F1) If Q is a KUS sub algebra of Ë, then φ (S)is a KUS subalgebra of È.
(F2) If σ is an KUS ideal of Ë, then φ(σ)is an KUSideal in È .
(F3) If N is a KUS sub algebra of È, thenφ^{-1}(B)is a KUS sub algebra of Ë .
(F4) If J is an KUS ideal in È, then φ^{-1}(J)is an KUSideal in Ë .
(F5) Kerφ is KUS ideal of Ë.
(F6) Im(φ)is a KUS sub algebra of È.

Definition 2.12([20]).Let (Ë;∗ ,0) be a nonempty set, a fuzzy subset ρ in Ë is a function ρ : φ → [0,1].

Definition 2.13.[14] Let (ϑ;∗ ,0) be a KUSalgebra , a fuzzy subset ρ in ϑ is called a fuzzy KUS sub algebra of ϑ if for all ś , ŝ∈ϑ,
ρ (ś ∗ ŝ) ≥ min ρ(ś), ρ (ŝ)} .

Definition 2.14.[14] Let (ϑ;∗ ,0) bea KUS algebra , a fuzzy subset σ in ϑ is called a fuzzy KUS ideal of ϑ if it satisfies the following conditions:
for all ś , ŝ, τ ∈ϑ,
(Fkus1) σ (0) ≥ σ (ś) ,
(Fkus2) σ (τ ∗ ś) ≥ min {σ (τ ∗ ŝ), μ (ŝ ∗ ś)} .

Example 2.15.(I) Let φ = {0, 1, 2, 3} in which (∗) is defined by the following Table (1)Then (φ ; ∗ , 0)is KUS − algebra . Define a fuzzy subset
ϑ : φ → [0,1] by

$$\varphi(ś) = \begin{cases} 0.8 & \text{if } ś \in [0,1] \\ 0.2 & \text{otherwise} \end{cases}$$

$\sigma_1 = \{0, 1\}$is an KUS ideal of φ. Routine calculation gives that ϑ is a fuzzy KUS ideal of KUS algebras φ.

(II) Consider $\varphi = \{0, a, b, c, d\}$ with$(*)$ defined by the Table (3)
Then(φ ;$*$,0) is a KUS-algebra. Define a fuzzy subset $\vartheta: \varphi \rightarrow [0,1]$such that
$V(0) = t_1$,$\vartheta(a) = \vartheta(b) = \vartheta(c) = \vartheta(d) = t_2$,where $t_1, t_2 \in [0,1]$ and $t_1 > t_2$.
Routine calculation gives that ϑ is a fuzzy KUS-ideal of KUS- algebra φ .

Definition 2.16 .[1 , 2] (An Intuitionistic fuzzy set (briefly I F S) A in a nonempty set φ is an object having the form $A = \{\acute{s}, \beta_A, \gamma(\acute{s}))|\acute{s} \in \varphi\}$, where the function
$\beta_A: \varphi \rightarrow [0,1]$and $\gamma_A: \varphi \rightarrow [0,1]$denote the degree of membership and degree of non membership, respectively and $0 \leq \beta_A(\acute{s}) + \gamma_A(\acute{s}) \leq 1$ for all $x \in \varphi$. An intuitionistic fuzzy set $A = \{(\acute{s}, \beta_A(\acute{s}), \gamma_A(\acute{s}))|\acute{s} \in \varphi\}$ in φ can be identified to an order pair(β_A, γ_A) in $I^X \times I^X$. We shall use the symbol $A = (\beta_A, \gamma_A)$.

Definition 2.17. [6] Let φ be a non $-$ empty set. A function from $\varphi \rightarrow [-1,0]$is called a negative valued function ($\omega -$ function)from φ to $[-1,0]$
Denote by $\omega(\varphi, [-1, 0])$the collection of functions from a set φ to $[-1, 0]$.
We say that, an element of $\omega(\varphi, [-1, 0])$is a negativevalued function from φ to $[-1, 0]$ (briefly , ω function on φ. By an ω structure we mean an ordered pair $(\varphi, V_A^{\aleph})$, where $V_A^{\aleph}$ is an $\omega -$ function on φ.
Let the set of all ω function on φ denoted by $\aleph\omega(\varphi)$.

Definition 2.18 ([**6**]). Let φ be a nonempty set and let $A, B \in \aleph\omega(\varphi)$.
(i)We say that the $\omega -$ function $A = (\hat{s}, \rho_A^{\aleph})$on φ is subset of the ω function $B = (\hat{s}, \rho_B^{\aleph})$, denoted by $A \subset B$, if $\forall \hat{s} \in \varphi$, $\rho_A^{\aleph}(\hat{s}) \geq \rho_B^{\aleph}(\hat{s})$.
(ii)The complement of $\omega -$ function $A = (x, \rho^{\aleph})$, denoted by $A^c = (\hat{s}, (\rho^{\aleph})^c)$ is a $\omega -$ function in φ defined as: for each $y \in \varphi$
$A^c(\hat{s}) = (\hat{s}, (\mu^{\aleph})^c(\hat{s})) = \hat{s}$ i. e., $(\rho^{\aleph}(\hat{s})))^c = -1 - (\rho^{\aleph})(\hat{s})$
(iii)The intersection of two $\omega -$ functions $A = (\hat{s}, \rho_A^{\aleph})$ and $B = (\hat{s}, \rho_B^{\aleph})$, denoted by $A \cap B$, is a $\omega -$ function in φ defined as: for each $\hat{s} \in \varphi$,
$(A \cap B)(\hat{s}) := \{(\rho_A^{\aleph}(\hat{s}) \vee \rho_B^{\aleph}(\hat{s}))\}$
(iv)The union of two $\omega -$ functions $A = (\hat{s}, \rho_A^{\aleph})$ and $B = (\hat{s}, \rho_B^{\aleph})$, denoted by $A \cup B$, is a$\omega -$ function in φ defined as: for each $y \in \varphi$,
$(A \cup B)(\hat{s}) := \{((\rho(\hat{s}) \wedge \rho_B^{\aleph}(\hat{s}))\}$

3. Crossing intuitionistic kUS -ideal

Definition 3.1. Let φ be a nonempty set. By a crossing intuitionistic set in φ we mean a structure $A = \{\varphi, \mu_A(\acute{s}), \lambda_A(\acute{s}), V_A^{\aleph})|\acute{s} \in \varphi\}$, where the function $\mu_A: \varphi \rightarrow [0,1]$ and $\lambda_A: \varphi \rightarrow [0,1]$denote the degree of membership and degree of non membership, respectively such that$0 \leq \rho_A(\acute{s}) + \mu_A(\acute{s}) \leq 1$ and $V_A^{\aleph}: \varphi \rightarrow [0,1]$ is $\omega -$ function on φ, where $-1 \leq V_A^{\aleph}(\acute{s}) \leq 0$ for all $\acute{s} \in \varphi$.
Let the set of all crossing intuitionistic set on φ be denoted by $CIN(\varphi)$.

Example3.2
Define $C_A = \{(\acute{s} \in X|\mu_A(\acute{s}), \lambda_A(\acute{s}), V_A^{\aleph})\}$, as follows:
$C_A(0) = \{0.9, 0.1, -0.5\}$
$C_A(a) = \{0.7, 0.2, -0.4\}$
$C_A(b) = \{0.5, 0.3, -0,2\}$

It is easy to check that $C_A = \{(ś \in \varphi | \mu_A(ś), \lambda_A(ś), V_A^{\aleph})\}$, is crossing intuitionistic set.

Definition3.3. Let $C_A = \{(ś \in \varphi | \mu_A(ś), \lambda_A(ś), V_A^{\aleph})\}$, and $C_B = \{(ś \in \varphi | \mu_B(ś), \lambda_B(ś), V_B^{\aleph})\}$ be two crossing intuitionistic sets of φ, then we say:

1. $C_A \subseteq C_B \leftrightarrow \mu_A \leq \mu_B$, $\lambda_A \geq \lambda_B$ and $V_A^{\aleph} \geq V_B^{\aleph}$

Example3.4.

$C_A = \{0.4,5,-0.2\}, C_B = \{0.7,3,-0.7\}$then $C_A \subseteq C_B$

2. $C_A = C_B \leftrightarrow \mu_A(ś) = \mu_B(ś), \lambda_A(ś) = \lambda_B(ś), V_A^{\aleph}(ś) \geq V_B^{\aleph}(ś)$

3. $[C]^c \leftrightarrow (C_A)^c = \{(ś \in \varphi | \left(\mu_A(ś)\right)^c, (\lambda_A(ś))^c, \left(V_A^{\aleph}(ś)\right)^c\}$,
where$(\mu_A)^c = 1 - \mu_A$, $(\lambda_A)^c = 1 - \lambda_A$ and $\left(V_A^{\aleph}\right)^c = -1 - V_A^{\aleph}$

Example3.5.

Let $C_A = \left\{\left(ŝ, \mu_A(ŝ), \lambda_A(ŝ), V_A^{\aleph}(ŝ)\right) \middle| ŝ \in \varphi\right\} = \{0.4,0.5,-0.2\}$

And $C_B = \left\{\left(ŝ, \mu_B(ŝ), \lambda_B(ŝ), V_B^{\aleph}(ŝ)\right) \middle| ŝ \in \varphi\right\} = \{0.7,0.3,-0.7\}$ are two crossing intuitionistic set of φ then

$C_A \cap C_B = \{ŝ, \min\{\mu_A(ŝ), \mu_B(ŝ)\}, \max\{\lambda_A(ŝ), \lambda_B(ŝ)\}, \max\{V_B^{\aleph}(ŝ), V_B^{\aleph}(ŝ)\}\} = \{0.4,0.5,-0.2\}$

$C_A \cup C_B = \{ŝ, \max\{\mu_A(ŝ), \mu_B(ŝ)\}, \min\{\lambda_A(ŝ), \lambda_B(ŝ)\}, \min\{V_B^{\aleph}(ŝ), V_B^{\aleph}(ŝ)\}\} = \{0.7,0.3,-0.7\}$

It is easy to prove the following

Result 3.7 Let $\Theta, \Phi, O \in CIN(\varphi)$ be three crossing intuitionistic sets of φ, where $\Theta = \{(ś, \mu_A(ś), \lambda_A(ś), V_A^{\aleph}) | ś \in \varphi\}$, $\Phi = \{(ś, \mu_B(ś), \lambda_B(ś), V_B^{\aleph}) | ś \in \varphi\}$and
$O = \{(ś, \mu_C(ś), \lambda_C(śx), V_C^{\aleph}) | ś \in \varphi\}$

Then

(1) (Idempotent laws):$(\Theta \cap \Theta) = \Theta, (\Theta \cup \Theta) = \Theta$
(2) (Commutative laws): $(\Theta \cap \Phi) = \Phi \cap \Theta, (\Theta \cup \Phi) = B \cup \Theta$,
(3) (Associative laws): $(\Theta \cap \Phi) \cap O = \Theta \cap (\Phi \cap O)$,
$(\Theta \cup \Phi) \cup O = \Theta \cup (\Phi \cup O)$
(4) (Distributive laws):$\Theta \cup (\Phi \cap O) = (\Theta \cup \Phi) \cap (\Theta \cup O)$,
$\Theta \cap (\Phi \cup O) = (\Theta \cap \Phi) \cup (\Theta \cap O)$
(5) (Absorption laws): $\Theta \cup (\Theta \cap \Phi) = \Theta, \Theta \cap (\Theta \cup \Phi) = \Theta$
(6) (DeMorgan's laws):$(\Theta \cap \Phi)^c = \Theta^c \cup \Phi^c, (\Theta \cup \Phi)^c = \Theta^C \cap \Phi^C$
(7) $(\Theta^c)^c = \Theta$
(8) $(\Theta \cap \Phi) \subset \Theta, (\Theta \cap \Phi) \subset \Phi$
(9) $\Theta \subset (\Theta \cup \Phi), \Phi\Phi \subset (\Theta \cup \Phi)$
(10) if $\Theta \subset \Phi$ and $\Phi \subset O$, then $\Theta \subset O$
(11) if $\Theta \subset \Phi$, then $\Theta \cap O \subset \Phi \cap O, \Theta \cup O \subset \Phi \cup O$

From now on φ is a KUS − algebra, unless otherwise is stated.

Defintion3.8. A set $đ = \{(ś, \mu_A(ś), \lambda_A(ś), V_A^{\aleph}) | ś \in \varphi\}$, in Xis called a crossing intuitionistic ideal of φ if it satisfies the following condition: $\forall ś, ŝ \in \varphi$

(b1) $\mu(0) \geq \mu(ś)$ and $\lambda(0) \leq \lambda(ś), V_A^{\aleph}(0) \leq V_A^{\aleph}(ś)$
(b2) $\mu(ś) \geq \min\{\mu(ś * ŝ), \mu(ŝ)\}, \lambda(ś) \leq \max\{\lambda(ś * ŝ), \lambda(ŝ)\}$,
(b3) $V^{\aleph}(0) \leq V^{\aleph}(ś), V^{\aleph}(ś) \leq \max\{V^{\aleph}(ś * ŝ), V^{\aleph}(ŝ)\}$,where the function

$\mu_A: \varphi \to [0,1]$ and $\lambda_A: \varphi \to [0,1]$denote the degree of membership and degree of non membership, respectively ,$0 \leq \mu_A(\acute{s}) + \lambda_A(\acute{s}) \leq 1$ and $V_A^{\aleph}: \varphi \to [-1,0]$ is ℵ-function on φ, such that $-1 \leq V_A^{\aleph}(\acute{s}) \leq 0$,$\forall \acute{s} \in \varphi$.

Defintion3.9. A crossing set $đ = \{(\acute{s}, \mu_A(\acute{s}), \lambda_A(\acute{s}), V_A^{\aleph}) | \acute{s} \in \varphi\}$in φis called
a crossing intuitionistic sub algebras of φ if it satisfies the following
condition: $\forall \acute{s}, \hat{s}, \tau \in \varphi$.
(Sb1) $\mu(\acute{s} * \hat{s}) \geq \min\{\mu(\acute{s}), \mu(\hat{s})\}$, $\lambda(\acute{s} * \hat{s}) \leq \max\{\lambda(\acute{s}), \lambda(\hat{s})\}$.
(Sb2) $V^{\aleph}(\acute{s} * \hat{s}) \leq \max\{V^{\aleph}(\acute{s}), V^{\aleph}(\hat{s})\}$, where the function $\mu_A: \varphi \to [0,1]$,
$\lambda_A: \varphi \to [0,1]$denote the degree of membership and degreeof non
membership, respectively , such that $0 \leq \mu_A(\acute{s}) + \lambda_A(\acute{s}) \leq 1$ and $V_A^{\aleph}: X \to [0,1]$ is ω function on φ, such that $-1 \leq V_A^{\aleph}(\acute{s}) \leq 0$ for all $\acute{s} \in \varphi$.

Lemma 3.10. If $A = \{(\acute{s}, \mu_A(\acute{s}), \lambda_A(\acute{s}), V_A^{\aleph}) | \acute{s} \in \varphi\}$ is crossing intuitionistic KUS-sub-algebra of φ, then $\mu(0) \geq \mu(\acute{s})$, $\lambda(0) \leq \lambda(\acute{s})$, and $V^{\aleph}(0) \leq V^{\aleph}(\acute{s})$
Proof. Put $\acute{s} = \hat{s}$ in the defintion 3.9,then we have
$\mu(\acute{s} * \acute{s}) = \mu(0) \geq \min\{\mu(\acute{s}), \mu(\acute{s})\} = \mu(\acute{s})$,
$\lambda(\acute{s} * \acute{s}) = \lambda(0) \leq \max\{\lambda(\acute{s}), \lambda(\acute{s})\} = \lambda(\acute{s})$.
Similar we have $V^{\aleph}(\acute{s} * \acute{s}) = V^{\aleph}(0) \leq \max\{V^{\aleph}(\acute{s}x)V^{\aleph}(\acute{s})\} = V^{\aleph}(\acute{s})$

Defintion3.11. set $đ = \{(\acute{s}, \mu_A(\acute{s}), \lambda_A(\acute{s}), V_A^{\aleph}) | \acute{s} \in X\}$in φ is called crossing intuitionistic KUS-ideal of φ if it satisfies the following condition: for all $\acute{s}, \hat{s}, \tau \in \varphi$
(b_1) $\mu(0) \geq \mu(\acute{s}), \mu(\tau * \acute{s}) \geq \min\{\mu(\tau * \hat{s}), \mu(\acute{s} * \hat{s})\}$
$\lambda(\tau * \acute{s}) \leq \max\{\lambda(\tau * \hat{s}), \lambda(\acute{s} * \hat{s})\}$
(b_2) $\lambda(0) \leq \lambda(\acute{s}), \lambda(\tau * \acute{s}) \leq \max\{\lambda(\tau * \hat{s}), \lambda(\acute{s} * \hat{s})\}$,
(b_2) $V_A^{\aleph}(0) \leq V_A^{\aleph}(\acute{s}), V_A^{\aleph}(\tau * \acute{s}) \leq \max\{V_A^{\aleph}(\tau * \hat{s}), V_A^{\aleph}(\acute{s} * \hat{s})\}$

Example 3.12 Let $X = \{0, 1, 2, 3\}$ be a set with a binary operation ∗ define by the following Table (1)

*	0	1	2	3
0	0	1	2	3
1	1	0	3	2
2	2	3	0	1
3	3	2	1	0

Table (4)

	0	1	2	3
$V_A^{\aleph}$	$\mathbf{-0.7}$	$\mathbf{-0.7}$	$\mathbf{-0.6}$	$\mathbf{-0.4}$
μ	$\mathbf{0.6}$	$\mathbf{0.5}$	$\mathbf{0.3}$	$\mathbf{0.3}$
λ	$\mathbf{0.1}$	$\mathbf{0.2}$	$\mathbf{0.3}$	$\mathbf{0.4}$

By routine calculations ,we can prove , that $đ = \{(\acute{s}, \mu_A(\acute{s}), \lambda_A(\acute{s}), V_A^{\aleph}(\acute{s})) | \acute{s} \in \varphi\}$ is crossing intuitionistic KUS-ideal of φ.

Proposition 3.13. Every a crossing intuitionistic KUS - ideal of φis a crossing intuitionistic ideal of φ .
Proof. clear.

Proposition 3.14. If $B = \{(\acute{s}, \mu_A(\acute{s}), \lambda_A(\acute{s}), V_A^{\aleph}(\acute{s})) | \acute{s} \in \varphi\}$ is crossing intuitionistic KUS - ideal of φand $\hat{s} \leq \acute{s}$, then $\mu(\acute{s}) \geq \mu(\hat{s}), \lambda(\acute{s}) \leq \lambda(\hat{s})$ and $V_A^{\aleph}(\acute{s}) \leq V_A^{\aleph}(b)$

Proof. If $\hat{s} \leq \acute{s}$, then $\acute{s} * \hat{s} = 0$,since $B = \{(\acute{s}, \mu_A(\acute{s}), \lambda_A(\acute{s}), V_A^{\aleph})|\acute{s} \in \varphi\}$is crossing intuitionistic KUS - ideal of φ, put $z = 0$ in defintion3.11, then we get
$\mu(0 * \acute{s}) = \mu(\acute{s}) \geq \min\{\mu\hat{s}, \mu(\acute{s} * \hat{s})\} = \min\{ \mu(\hat{s}), \mu(0)\} = \mu(\hat{s})$,
$\lambda(0 * \acute{s}) = \lambda(\acute{s}) \leq \max\{\lambda(0 * \hat{s}), \lambda(\acute{s} * \hat{s})\} = \max\{\lambda(\hat{s}), \lambda(0)\} = \lambda(\hat{s})$,finally
$V_A^{\aleph}(0 * \acute{s}) = V_A^{\aleph}(\acute{s}) \leq \max\{V_A^{\aleph}(0 * \hat{s}), V_A^{\aleph}(\acute{s} * \hat{s})\} = \max\{V_A^{\aleph}(\hat{s}), V_A^{\aleph}(0)\} = V_A^{\aleph}(\hat{s})$.

Theorem 3.15 Every crossing intuitionistic KUS- ideal of φ is a crossing intuitionistic sub-algebra of φ.
Proof . Let $E = \{(\acute{s}, \mu_A(\acute{s}), \lambda_A(\acute{s}), V_A^{\aleph}(\acute{s}))|\acute{s} \in \varphi\}$be crossing intuitionistic KUS- ideal of φ. Put $\tau = 0$ in Defintion3.11 , we get
Then, $\mu(0 * \acute{s}) \geq \min\{\mu(0 * \hat{s}), \mu(\acute{s} * \hat{s})\}$, i.e $\mu\acute{s} \geq \min\{\mu(\hat{s}), \mu(\acute{s} * \hat{s})\}$, Similar we have $\lambda(\acute{s}) \leq \max\{\lambda(\hat{s}), \lambda(\acute{s} * \hat{s})\}$and $V_A^{\aleph}(\acute{s}) \leq \max\{V_A^{\aleph}(\hat{s}), V_A^{\aleph}(\acute{s} * \hat{s})\}$
Hence $E = \{(\acute{s}, \mu_A(\acute{s}), \lambda_A(\acute{s}), V_A^{\aleph})|\acute{s} \in \varphi\}$ is crossing intuitionistic sub-algebra of φ.

Theorem 3.16. A crossing intuitionistic set $E = \{(\acute{s}, \mu_A(\acute{s}), \lambda_A(\acute{s}), V_A^{\aleph})|\acute{s} \in \varphi\}$ is crossing intuitionistic KUS- ideal of KU- algebra φ if and only if
$B^{t,u,s} = \{\acute{s} \in \varphi: \mu(\acute{s}) \geq t, \lambda(\acute{s}) \leq u, V^{\aleph}(\acute{s}) \leq s\} \neq \emptyset$ is a KUS- ideal of φ.
Proof: Suppose that $E = \{(\acute{s}, \mu_A(\acute{s}), \lambda_A(\acute{s}), V_A^{\aleph})|\acute{s} \in \varphi\}$ crossing intuitionistic crossing intuitionistic KUS- ideal of KUS – algebra and
$B^{t,u,s} = \{\acute{s} \in \varphi: \mu(\acute{s}) \geq t, \lambda(\acute{s}) \leq u, V^{\aleph}(\acute{s}) \leq s\} \neq \emptyset$for any $t, u \in [0,1], s \in [-1,0]$. there exists $\acute{s} \in B^{t,u,s}$ so that $\mu(\acute{s}) \geq t, \lambda(\acute{s}) \leq u, V^{\aleph}(\acute{s}) \leq s$. It follows from Defintion3.5 that $\mu(0) \geq \mu(\acute{s}) \geq t, \lambda(0) \leq \lambda(\acute{s}) \leq u, V^{\aleph}(0) \leq V^{\aleph}(\acute{s}) \leq s$ so that $0 \in B^{t,u,s}$. Let $\acute{s}, \hat{s}, \tau \in \varphi$ be such that $\tau * \hat{s} \in B^{t,u,s}$ and $\acute{s} * \hat{s} \in B^{t,u,s}$. Using Defintion3.11 we know that
$\mu(\tau * \acute{s}) \geq \min\{\mu(\tau * \hat{s}), \mu(\acute{s} * \hat{s})\} = \min\{t, t\} = t$,
$\lambda(\tau * \acute{s}) \leq \max\{\lambda(\tau * \hat{s}), \lambda(\acute{s} * \hat{s})\} = \max\{u, u\} = u$ and
$V^{\aleph}(\tau * \acute{s}) \leq \max\{V^{\aleph}(\tau * \hat{s}), V^{\aleph}(\acute{s} * \hat{s})) = \max\{s, s\} = s$,Thus $\tau * \tau \in B^{t,u,s}$. Hence $B^{t,u,s}$ is a KUS- ideal of φ.
Conversely, suppose that $B^{t,u,s} \neq \emptyset$ is a KUS- ideal of X,for every $t, u \in [0,1], s \in [-1,0]$ and any $a \in \varphi$, let $\mu(\acute{s}) = t, \lambda(\acute{s}) = u, V^{\aleph}(\acute{s}) = s$. Then $a \in B^{t,u,s}$, Since$0 \in B^{t,u,s}$, it follows that $\mu(0) \geq \mu(\acute{s}) = t, \lambda(0) \leq \lambda(\acute{s}) = u, V^{\aleph}(0) \leq V^{\aleph}\acute{s} = s$ for all $\acute{s} \in \varphi$. Now, we need to show that $\mu(\acute{s}), \lambda(\acute{s}), V^{\aleph}(\acute{s})$ satisfies Defintion3.11 If not, then there exist $\acute{s}, \hat{s}, \tau \in \varphi$ such that
$\mu(\tau * \acute{s}) \leq \min\{\mu(\tau * \hat{s}), \mu(\acute{s} * \hat{s})\}$
Taking $t_0 = \frac{1}{2}[\mu(\tau * \acute{s}) + \min\{\mu(\tau * \hat{s}), \mu(\acute{s} * \hat{s})\}]$then we have
$\mu(\tau * \acute{s}) < t_0 < \min\{\mu(\tau * \hat{s}), \mu(\acute{s} * \hat{s})\}$,
then there exist $\acute{s}, \hat{s}, \tau \in \varphi$such that
$\lambda(\tau * \acute{s}) \geq \max\{\lambda(\tau * \hat{s}), \lambda(\acute{s} * \hat{s})\}$
Taking $\varsigma_0 = \frac{1}{2}[\lambda(\tau * \acute{s}) + \max\{\lambda(\tau * \hat{s}), \lambda(\acute{s} * \hat{s})\}]$ then we have
$\lambda(\tau * \acute{s}) > \varsigma_0 > \max\{\lambda(\tau * \hat{s}), \lambda(\acute{s} * \hat{s})\}$ and
$V^{\aleph}(\tau * \acute{s}) \geq \max\{V^{\aleph}(\tau * b), V^{\aleph}(\acute{s} * \hat{s})\}$
. Taking $t_0 = \frac{1}{2}[V^{\aleph}(\tau * \acute{s}) + \max\{V^{\aleph}(\tau * \hat{s}), V^{\aleph}(\acute{s} * \hat{s})\}]$ then we have
$V^{\aleph}(\tau * \acute{s}) > \varsigma_0 > \max\{V^{\aleph}(\tau * \hat{s}), V^{\aleph}(\acute{s} * \hat{s})\}$
Hence in all cases $\tau * \hat{s} \in B^{t,u,s}$and $\acute{s} * \hat{s} \in B^{t,u,s}$, but $\tau * \acute{s} \notin B^{t,u,s}$ which means that $B^{t,u,s}$is not a KUS- ideal of X, this is contradiction. Therefore $E = \{(\acute{s}, \mu_A(\acute{s}), \lambda_A(\acute{s}), V_A^{\aleph})|\acute{s} \in \varphi\}$ is crossing intuitionistic crossing intuitionistic KUS- ideal of KUS – algebra φ . The proof is complete.

4-Image (Pre-image) crossing intuitionistic KUS- ideal

Definition 4.1 Let $(\ddot{E}, *, 0)$ and $(\grave{E}, *^{`}, 0^{`})$ be BCI-algebras. A mapping $\varphi: \ddot{E} \to \grave{E}$ is said to be a homomorphism if $\varphi(\acute{s} * \hat{s}) = \varphi(\acute{s}) *^{`} \varphi(\hat{s})$ for all $\acute{s}, \hat{s} \in \varphi$ Note that if $\varphi: \ddot{E} \to \grave{E}$ is a homomorphism of KUS-algebras, then $\varphi(0) = 0^{`}$.

Let φ: Ë → È be a homomorphism of KUS-algebras for any crossing intuitionistic set $A = \{(\acute{s}, \mu_A(\acute{s}), \lambda_A(\acute{s}), V_A^{\aleph}(\acute{s})) | \acute{s} \in \varphi\}$ in È, we define new crossing intuitionistic fuzzy set $A^{\varphi} = \{(\acute{s}, \mu_A^{\varphi}(\acute{s}), \lambda_A^{\varphi}(\acute{s}), (V_A^{\aleph}(\acute{s})_A^{\varphi}) | \acute{s} \in \varphi\}$ in φ by
$\mu_A^{\varphi}(\acute{s}) := \mu(\varphi(\acute{s})), \lambda_A^{\varphi}(\acute{s}) \coloneqq \lambda(\varphi(\acute{s}))$ and $V_A^{\aleph}(\acute{s})_A^{\varphi} = V_A^{\aleph}(\varphi(\acute{s})$ for all a∈ φ

Theorem 4.2 Let φ: Ë → È be a homomorphism of KUS- algebras. If $E = \{(A, \mu_A(\acute{s}), \lambda_A(\acute{s}), V_A^{\aleph}(\acute{s})) | \acute{s} \in \varphi\}$ be a crossing intuitionistic KUS- ideal of È, then $A^{\varphi} = \{(\acute{s}, \mu_A^{\varphi}(\acute{s}), \lambda_A^{\varphi}(\acute{s}), (V_A^{\aleph}(\acute{s})_A^{\varphi}) | \acute{s} \in \varphi\}$is crossing intuitionistic KUS- ideal of φ.
Proof. $\mu_A^{\varphi}(a) := \mu(\varphi(a)) \leq \mu(0) = \mu(\varphi(0)) = \mu_A^{\varphi}(0)$for all, $\acute{s}, \hat{s} \in \varphi$. And
$\mu_A^{f}(z * x) \coloneqq \mu(\varphi(\tau * \acute{s})) \geq \min\{\mu(\varphi(\tau) * \varphi(\hat{s})), \mu(\varphi(\acute{s}) * \varphi(\hat{s}))\}$
$\min\{\mu(\varphi(z * \hat{s})), \mu(\varphi(\acute{s} * \hat{s}))\} = \min\{\mu_A^{\varphi}(\tau * y), \mu_A^{\varphi}(\acute{s} * y)\}$, for all, $\acute{s}, \hat{s} \in \varphi$.And
$\lambda_A^{\varphi}(\tau * \acute{s}) \coloneqq \lambda(\varphi(\tau * \acute{s})) \leq \max\{\lambda(\varphi(\tau) * \varphi(\hat{s})), \lambda(\varphi(\acute{s}) * \varphi(\hat{s}))\}$
$= \max\{\lambda(\varphi(\tau * \hat{s})), \lambda(\varphi(\acute{s} * \hat{s}))\} = \max\{\lambda_A^{\varphi}(\tau * \hat{s}), \lambda_A^{\varphi}(\acute{s} * \hat{s})\}$ for all $\acute{s}, y \in$ Ë.And
$(V_A^{\aleph})_A^{\varphi}(\tau * \acute{s}) \coloneqq V_A^{\aleph}(\varphi(\tau * \acute{s})) \leq \max\{V_A^{\aleph}(\varphi(z) * \varphi(\hat{s})), V_A^{\aleph}(\varphi(\acute{s}) * \varphi(\hat{s}))\}\}$
$= \max\{V_A^{\aleph}(\varphi(\tau * \hat{s})), V_A^{\aleph}(\varphi(\acute{s} * \hat{s}))\}$
$= \max\{(V_A^{\aleph})_A^{\varphi}(\tau * (\hat{s} * \acute{s})), (V_A^{\aleph})_A^{\varphi}(\hat{s})\}$.Hence
$A^{\varphi} = \{(\acute{s}, \mu_A^{\varphi}(\acute{s}), \lambda_A^{\varphi}(\acute{s}), (V_A^{\aleph}(\acute{s}))_A^{\varphi}) | \acute{s} \in \varphi\}$is crossing intuitionistic KUS- ideal of φ.

Theorem 4.3 Let φ: Ë → È be an epimorphism of KUS- algebras .If
$A^{\varphi} = \{(\hat{s}, \mu_A^{\varphi}(\hat{s}), \lambda_A^{\varphi}(\hat{s}), (V_A^{\aleph})_A^{\varphi}) | \hat{s} \in \vartheta\}$is crossing intuitionistic KUS- ideal of Y, then $E = \{(\hat{s}, \mu_A(\hat{s}), \lambda_A(\hat{s}), V_A^{\aleph}(\hat{s})) | \hat{s} \in \vartheta\}$ is crossing intuitionistic KUS- ideal of È.
Proof. For any a ∈ È,there exists $\hat{s} \in \varphi$ such that $\varphi(y) = a$.Then
$\mu(a) = \mu(\varphi(\hat{s})) = \mu_A^{\varphi}(\hat{s}) \leq \mu_A^{\varphi}(0) = \mu(\varphi(0)) = \mu(0)$,
Let$a, b, c \in$ Ỳ. Then $\varphi(\hat{s}) = a, \varphi(\acute{s}) = b, \varphi(\tau) = c$, for some $\acute{s}, \hat{s}, \tau \in$ Ë. It follows that $\mu(c * a) = \mu(\varphi(\tau * \hat{s})) = \mu_A^{\varphi}(\tau * \hat{s}) \geq \min\{\mu_A^{\varphi}(\tau * \acute{s}), \mu_A^{\varphi}(\hat{s} * \acute{s}) =$
$\min\{\mu(\varphi(\tau * \acute{s})), \mu(\varphi(\hat{s} * \acute{s}))\} = \{\mu(\varphi(\tau) * \varphi(\acute{s})), \mu(\varphi(\hat{s}) * \varphi(\acute{s}))\} = \min\{\mu(c * b), \mu(a * b)\}$,
Now $\lambda(a) = \lambda(\varphi(\acute{s})) = \lambda_A^{\varphi}(x) \geq \lambda_A^{\varphi}(0) = \lambda(\varphi(0)) = \lambda(0)$,
Let $a, b, c \in$ Ỳ.Then $\varphi(\hat{s}) = a, \varphi(\acute{s}) = b, \varphi(\tau) = c$, for some $\acute{s}, \hat{s}, \tau \in$ Ë. It follows that $\lambda(c * a) = \lambda(\varphi(\tau * \hat{s})) = \lambda_A^{\varphi}(\tau * \hat{s}) \leq \max\{\lambda_A^{\varphi}(\tau * \acute{s}), \lambda_A^{\varphi}(y * \acute{s})\}$
$= \max\{\lambda(\varphi(\tau * \acute{s})), \lambda(\varphi(\hat{s} * \acute{s})\}$
$= \max\{\lambda(\varphi(\tau) * \varphi(\acute{s})), \lambda(\varphi(\hat{s}) * \varphi(\acute{s}))\} = \max\{\lambda(c * b), \lambda(a * b)\}$
Finally
$V_A^{\aleph}(a) = V_A^{\aleph}(\varphi(\hat{s})) = (V_A^{\aleph})_A^{\varphi}(\hat{s}) \geq (V_A^{\aleph})_A^{\varphi}(0) = V_A^{\aleph}(\varphi(0)) = V_A^{\aleph}(0)$,
Let $a, b, c \in$ Ë. Then $\varphi(\acute{s}) = a, \varphi(\hat{s}) = b, \varphi(\tau) = c$, for some $\acute{s}, \hat{s}, \tau \in \varphi$. It follows that
$V_A^{\aleph}(c * a) = V_A^{\aleph}(\varphi(z * x)) = (V_A^{\aleph})_A^{\varphi}(z * x) \leq \max\{(V_A^{\aleph})_A^{\varphi}(z * y)), (V_A^{\aleph})_A^{\varphi}(x * y) =$
$\max\{V_A^{\aleph}(\varphi(z * y)), V_A^{\aleph}(\varphi(x * y))\} = \max\{V_A^{\aleph}(\varphi(z) * \varphi(y)), V_A^{\aleph}(\varphi(x) * \varphi(y))\}$
$= \max\{V_A^{\aleph}(c * b), V_A^{\aleph}(a * b)\}$.This completes the proof.

Conclusion.

During this work,we present the crossing intuitionistic of KUSideal in KUS algebras as an extension of bipolar fuzzy sets.Also we discussed few results of crossing intuitionistic of KUS ideal in KUS algebras under homomorphism,the notion of image and the pre image of crossing intuitionistic of KUS ideal under homomorphism of KUS algebras are discussed.Moreover,We studied the image and the preimage of crossing intuitionistic of KUS ideal under homomorphism of KUS algebras become crossing intuitionistic fuzzy of KUS- ideal.

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Conflicts of Interest State any potential conflicts of interest here or "The authors declare no conflict of interest".

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