

Centered Triangular, Triangular, Square, Centered square, Hexagonal and Heptagonal Numbers

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Abstract:

Among the many relationships existing between various types of numbers, the arrangement of points representing numbers that forms various geometrical figures in a plane is both fascinating and important. Such numbers are called Polygonal numbers. The Centred Polygonal numbers are less well known family belonging to figurate numbers. In this paper we will consider centred triangular numbers, triangular numbers, centred square numbers, square numbers, hexagonal numbers and heptagonal numbers and explore their inter – relationship.

Keywords: Centred triangular number, Centred square number, Square, Hexagonal, Heptagonal numbers.

1. Introduction

The study of Figurate numbers, have long and rich history. They were supposed to be introduced by ancient Greek philosopher and mathematician Pythagoras and his followers Pythagoreans, as an attempt to connect Geometry and Arithmetic. In general, a Figurate number is a number that can be represented by regular and discrete geometric pattern of equally spaced points. In two dimensions, figurate numbers are called as polygonal numbers. Figurate numbers can also form other shapes such as centred polygons. The study of polygonal and centred polygonal numbers is so pleasing and would infuse great interest in the study of mathematics.

2. Definitions

We will first define certain class of important numbers.

2.1 Polygonal Numbers

General formula for n th polygonal number of order k is defined by

$p_{k,n} = \frac{n}{2}[(k-2)n - (k-4)]$ where n, k are natural numbers, and $k \geq 3$.

If $k = 3$, $p_{3,n} = \frac{n}{2}[n+1]$ (1) are Triangular numbers.

If $k = 4$, $p_{4,n} = n^2$ (2) are Square numbers.

If $k = 6$, $p_{6,n} = \frac{n}{2}[4n-2]$ (3) are Hexagonal numbers.

If $k = 7$, $p_{7,n} = \frac{n}{2}[5n-3]$ (4) are Heptagonal numbers.

2.2 Centred Polygonal Numbers

General formula for n th Centred polygonal number of order k is defined by

$$cp_{kn} = \frac{kn(n+1)}{2} + 1$$

where n, k are natural numbers, and $k \geq 3$.

If $k = 3$, then $cp_{3,n} = \frac{3n(n+1)}{2} + 1$ (5) are called centred triangular numbers.

If $k = 4$, then $cp_{4,n} = \frac{4n(n+1)}{2} + 1$ (6) are called centred square numbers.

We now prove a useful lemma.

2.3 Lemma

Consider the quadratic Diophantine equation $ax^2 - by^2 = c$ (7). Consider $D = ab$. Let (x_0, y_0) be a solution of equation (7) and let $u^2 - Dv^2 = 1$ (8).

Then $x = x_0u + b\beta v$, $y = \beta u + av$ (9) and $x = |-x_0u + b\beta v|$, $y = |\beta u - av|$ (10) where $\alpha = ax_0, \beta = y_0$ (11) are also solutions of equation (7).

Proof:

From the hypothesis, we have $ax_0^2 - by_0^2 = c$. Using (8),(9) and (11), we have

$$\begin{aligned} ax^2 - by^2 &= a(x_0u + by_0v)^2 - b(y_0u + ax_0v)^2 = u^2(ax_0^2 - by_0^2) - abv^2(ax_0^2 - by_0^2) \\ &= (u^2 - abv^2)(ax_0^2 - by_0^2) = 1(c) = c \end{aligned}$$

Hence the values of x and y from (9), forms solution to (7). Now using (10), we have

$$\begin{aligned} ax^2 - by^2 &= a(-x_0u + by_0v)^2 - b(y_0u - ax_0v)^2 = u^2(ax_0^2 - by_0^2) - abv^2(ax_0^2 - by_0^2) \\ &= (u^2 - abv^2)(ax_0^2 - by_0^2) = 1(c) = c \end{aligned}$$

Thus the values of x and y from (10), forms solution to (7).

Also we notice that if (x_0, y_0) is one of the solutions to (7), then we can generate infinitely many solutions of (7), using (9) and (10).

In the following section we prove some interesting results.

3.1 Theorem 1

There exists infinitely many centred triangular numbers which are also triangular numbers.

Proof: let CT_n be the n th centred triangular number and let T_m be m th triangular number.

$$\text{If } CT_n = T_m \text{ for some } m, n \text{ then we have } \frac{3n(n+1)}{2} + 1 = \frac{m(m+1)}{2} \quad (12)$$

$$3n^2 + 3n + 2 - m^2 - m = 0$$

$$n = \frac{-3 \pm \sqrt{3^2 - 4(3)(2 - m^2 - m)}}{2(3)}$$

$$(6n + 3)^2 = 12m^2 + 12m - 15$$

$$(3(2n + 1))^2 = 12m^2 + 12m - 15$$

$$3(2n + 1)^2 = (2m + 1)^2 - 6$$

$$(2m + 1)^2 - 3(2n + 1)^2 = 6 \quad (13)$$

If we now consider $X = 2m + 1$ and $Y = 2n + 1$ then (13) can be written as

$$X^2 - 3Y^2 = 6 \quad (14)$$

Comparing with (7), we get $a = 1; b = 3; D = ab = 3, c = 6$

One of the solutions for (14) is $(X_0, Y_0) = (9, 5)$

Now, consider $u^2 - 3v^2 = 1 \quad (15)$

The continued fraction for $\sqrt{3}$ is given by $\sqrt{3} = [1; \overline{1, 2}] \quad (16)$

The successive convergent obtained from continued fraction of $\sqrt{3}$ given by (16) are

$$\frac{1}{1}, \frac{2}{1}, \frac{5}{3}, \frac{7}{4}, \frac{19}{11}, \frac{26}{15}, \frac{71}{41}, \frac{97}{56}, \frac{265}{153}, \frac{362}{209}, \frac{985}{571}, \frac{1351}{780}, \frac{3691}{2131}, \frac{5042}{2911}, \frac{13775}{7953}, \frac{18817}{10864}, \frac{51409}{29681}, \dots (17)$$

We notice that the numerators and denominators of second, fourth, sixth, eighth, tenth, ... convergents from (17) forms solution to (15).

In particular, we observe that the pairs,

$$(u, v) = (2, 1); (7, 4); (26, 15); (97, 56); (362, 209); (1351, 780); (5042, 2911);$$

$(18817, 10864), \dots$ (18) forms solutions to (15).

By lemma (2.3), we obtain $\alpha = aX_0 = 9, \beta = Y_0 = 5$

$$X = X_0u + b\beta v = 9u + 15v \quad (19); \quad Y = \beta u + \alpha v = 5u + 9v \quad (20)$$

From equations (19) and (20) for every pair of (u, v) given by (18), we obtain solutions of equation (14). Such solutions are given by

$$(X, Y) = (33, 19); (123, 71); (459, 265); (1713, 989); (6393, 3691); (23859, 13775); (89043, 51409); (332313, 191861); \dots (21)$$

Since $m = \frac{X-1}{2}$ and $n = \frac{Y-1}{2}$, we have

$$(m, n) = (16, 9); (61, 35); (229, 132); (856, 494); (3196, 1845); (11929, 6887); (44521, 25704); (166156, 95930); \dots (22)$$

By lemma 2.3 we obtain

$$X = |-X_0u + b\beta v| = |-9u + 15v| \quad (23)$$

$$Y = |\beta u - \alpha v| = |5u - 9v| \quad (24)$$

From equations (23) and (24) for every pair of (u, v) given by (18) we obtain the solution of equation (14). Such solutions are given by

$$(X, Y) = (3, 1); (9, 5); (33, 19); (123, 71); (459, 265); (1713, 989); (6393, 3691);$$

(23859,13775); (89043,51409); (332313,191861); ... (25)

Since $m = \frac{x-1}{2}$, and $n = \frac{y-1}{2}$ we have

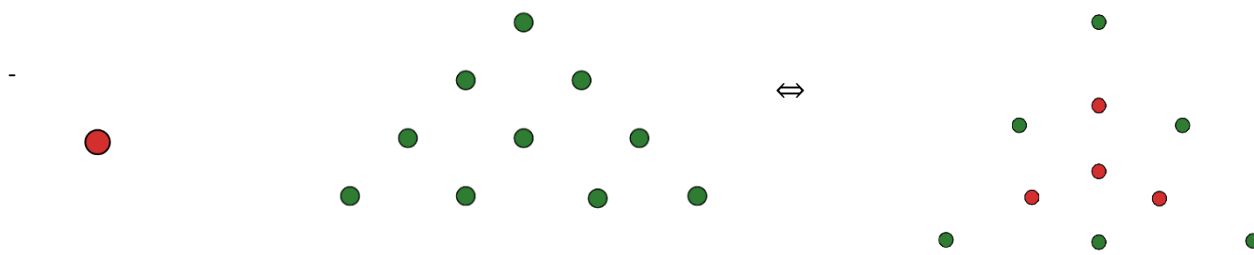
$(m, n) = (1,0); (4,2); (16,9); (61,35); (229,132); (856,494), (3196,1845); ... (26)$

Now considering positive values of (m, n) through (22) and (26), and neglecting the repeated values and substituting in (12), we notice that the numbers

1, 10, 136, 1891, 26335, 366796, 5108806, 71156485, 991081981, 13803991246, ... (27) are both Centred Triangular numbers as well as Triangular numbers.

Since there are infinitely many convergents that can be generated using (16), the number of solutions for (m, n) through (22) and (26) is infinite and so there exists infinitely many Centred Triangular and Triangular numbers, first few of which are listed in (27).

The first two triangular and centred triangular numbers are shown in the following pictures



$$T_1 = 1 = CT_0$$

$$T_4 = 10 \Leftrightarrow CT_2 = 10$$

3.2 Theorem 2

There exists infinitely many centred triangular numbers which are also square numbers.

Proof:

Let CT_n be the n th centred triangular number and S_m be the m th square number.

Let $CT_n = S_m$ for some m, n then according to (5) and (2), we get,

$$\frac{3n(n+1)}{2} + 1 = m^2 \quad (28)$$

$$3n^2 + 3n + 2(1 - m^2) = 0$$

$$n = \frac{-3 \pm \sqrt{3^2 - 4(3)(2(1 - m^2))}}{2(3)}$$

$$(6n + 3)^2 = 9 - 24(1 - m^2)$$

$$3(2n + 1)^2 = 8m^2 - 5$$

$$3(2n + 1)^2 - 2(2m)^2 = -5 \quad (29)$$

If we now consider $X = 2n + 1$ and $Y = 2m$ then (29) can be written as

$$3X^2 - 2Y^2 = -5 \quad (30)$$

Comparing with (7), we get $a = 3; b = 2; D = ab = 6$

One of the solutions for equation (30) is $(X_0, Y_0) = (1,2)$

Now, consider $u^2 - 6v^2 = 1$ (31)

Consider $(5 - 2\sqrt{6})(5 + 2\sqrt{6}) = 1$

$$5 - 2\sqrt{6} = \frac{1}{5 + 2\sqrt{6}} = \frac{1}{10 - (5 - 2\sqrt{6})}$$

$$5 - 2\sqrt{6} = \frac{1}{10 - \frac{1}{10 - \frac{1}{10 \dots}}}$$

$$2\sqrt{6} = 5 - \frac{1}{10 - \frac{1}{10 - \frac{1}{10 \dots}}}$$

The successive convergent of the continued fraction expansion of $2\sqrt{6}$ are given by

$$\frac{5}{1}, \frac{49}{10}, \frac{485}{99}, \frac{4801}{980}, \frac{47525}{9701}, \frac{470449}{96030}, \frac{4656961}{950599}, \dots$$

Multiplying each of the denominators of the convergents obtained above by 2, the solutions to (31) are given by

$$(u, v) = (5,2), (49,20), (485,198), (4801,1960), (47525, 19402), (470449,192060), \dots (32)$$

By lemma (2.3),we obtain

$$\alpha = aX_0 = 3, \beta = Y_0 = 2$$

$$X = X_0u + b\beta v = u + 4v \quad (33)$$

$$Y = \beta u + \alpha v = 2u + 3v \quad (34)$$

From equations (33) and (34) for every pair of (u, v) given by (32), we obtain the solutions of equation (30). Such solutions are given by

$$(X, Y) = (13,16); (129, 158); (1277,1564); (12641,15482); (125133,153256), \dots (35)$$

Since $m = \frac{Y}{2}$ and $n = \frac{X-1}{2}$ from (35) we have

$$(m, n) = (8,6), (79,64), (782,638), (7741,6320), (76628,62566), \dots (36)$$

By lemma (2.3) we obtain

$$X = |-X_0u + b\beta v| = |-u + 4v| \quad (37)$$

$$Y = |\beta u - \alpha v| = |2u - 3v| \quad (38)$$

From equations (37) and (38) for every pair of (u, v) given by (32), we obtain the solutions of equation (30). Such solutions are given by

$$(X, Y) = (3,4); (31,38); (307,376); (3039,3722), (30083,36844); \dots (39)$$

Since $m = \frac{Y}{2}$ and $n = \frac{X-1}{2}$,, from(39) we have

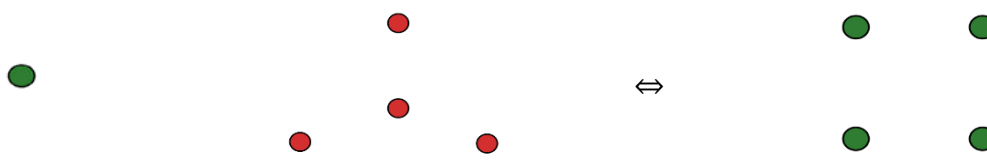
$$(m, n) = (2,1); (19,15); (188,153); (1861,1519); (18422,15041); \dots \quad (40)$$

Now considering positive values of (m, n) from (36) and (40) and substituting in (28), we notice that the numbers.

$$1, 4, 64, 361, 6241, 35344, 346321, 645234, 59923081, 339370084, 33254804881, 5871850384, \dots \quad (41)$$

are both Centered Triangular numbers as well as Square numbers. Since there are infinitely many solutions for (m, n) from (36) and (40), there exists infinitely many Centered Triangular numbers which are also Square numbers, first few of which are listed in (41).

The first two centered triangular numbers and square numbers are shown in the following pictures.



$$CT_0 = 1 = S_1$$

$$CT_1 = 4 \Leftrightarrow S_2 = 4$$

3.3 Theorem 3

There exists infinitely many centred triangular numbers which are also centred square numbers.

Proof:

Let CT_n be the n th Centred triangular number and let CS_m be the m th Centered square number. If $CT_n = CS_m$ for some n, m then we have,

$$\frac{3n(n+1)}{2} + 1 = \frac{4m(m+1)}{2} + 1 \quad (42)$$

$$3n^2 + 3n - 4m(m + 1) = 0$$

$$n = \frac{-3 \pm \sqrt{3^2 - 4(3)(-4m(m + 1))}}{2(3)}$$

$$(6n + 3)^2 = 9 + 48m^2 + 48m$$

$$3(2n + 1)^2 = 3 + 16m^2 + 16m$$

$$(4m + 2)^2 - 3(2n + 1)^2 = 1 \quad (43)$$

If we now consider $X = 4m + 2$ and $Y = 2n + 1$ then (43) can be written as

$$X^2 - 3Y^2 = 1 \quad (44)$$

The continued fraction for $\sqrt{3}$ is given by $\sqrt{3} = [1; \overline{1,2}]$ (45)

The successive convergent obtained from continued fraction of $\sqrt{3}$ given by (45) are

$$\frac{1}{1}, \frac{2}{1}, \frac{5}{3}, \frac{7}{4}, \frac{19}{11}, \frac{26}{15}, \frac{71}{41}, \frac{97}{56}, \frac{265}{153}, \frac{362}{209}, \frac{985}{571}, \frac{1351}{780}, \frac{3691}{2131}, \frac{5042}{2911}, \frac{13775}{7953}, \frac{18817}{10864}, \frac{51409}{29681}, \frac{70226}{40545}, \dots$$

$$\frac{262087}{151310}, \frac{191861}{413403}, \frac{978122}{564719}, \dots (46)$$

We observe that X is even, so considering the second, sixth, tenth, fourteen, eighteen, twentyfirst,... convergents from (46) forms solution to (44). In particular, if we consider numerators as X and denominators as Y then we obtain the pairs

$$(X, Y) = (2,1); (26,15); (362,209); (5042,2911); (70226,40545);$$

$$(978122,564719); \dots (47)$$

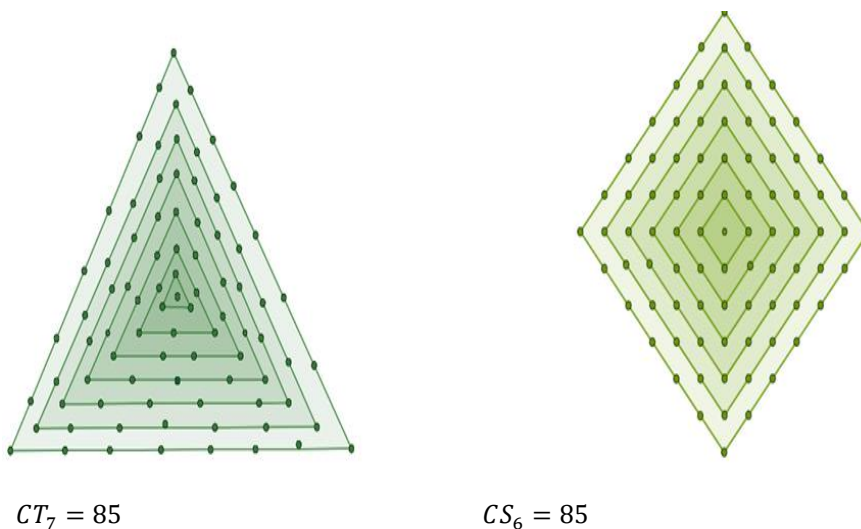
Since $m = \frac{X-2}{4}$ and $n = \frac{Y-1}{2}$, from (47) we have

$$(m, n) = (0,0); (6,7); (90,104); (1260,1455); (17556,20272); (244530,282359); \dots(48)$$

Now considering integer values of (m, n) from (48), and substituting in (42), we notice that the numbers 1, 85, 32760, 317721, 616461385, ... (49) are both Centered triangular numbers as well as Centered square numbers.

Since there are infinitely many convergents that can be generated using (45), the number of solution for (m, n) through (48) is infinite and so there exists infinitely many Centered triangular numbers and Centred square numbers, first few of which are listed in (49).

The second centerd triangular number and centred square number namely 85 is shown in the following picture.



3.4 Theorem 4

There exists infinitely many centred triangular numbers which are also hexagonal numbers.

Proof:

Let CT_n be the n th Centered triangular number and let H_m be the m th Hexagonal number.

Let $CT_n = H_m$ for some n, m . Then we have, $\frac{3n(n+1)}{2} + 1 = \frac{m(4m-2)}{2}$ (50)

$$3n^2 + 3n + 2 - 4m^2 + 2m = 0$$

$$n = \frac{-3 \pm \sqrt{3^2 - 4(3)(2 - 4m^2 + 2m)}}{2(3)}$$

$$(6n + 3)^2 = -15 + 48m^2 - 24m$$

$$3(2n + 1)^2 = -6 + (4m - 1)^2$$

$$(4m - 1)^2 - 3(2n + 1)^2 = 6 \quad (51)$$

If we now consider $X = 4m - 1$ and $Y = 2n + 1$ then (51) can be written as

$$X^2 - 3Y^2 = 6 \quad (52)$$

Comparing with (7), we get, $a = 1, b = 3, c = 6, D = ab = 3$

One of the solutions for equation (52) is $(X_0, Y_0) = (3, 1)$

Now, consider $u^2 - 3v^2 = 1 \quad (53)$

The continued fraction for $\sqrt{3}$ is given by $\sqrt{3} = [1; \overline{1, 2}] \quad (54)$

The successive convergent obtained from continued fraction of $\sqrt{3}$ is given by

$$\frac{1}{1}, \frac{2}{1}, \frac{5}{3}, \frac{7}{4}, \frac{19}{11}, \frac{26}{15}, \frac{71}{41}, \frac{97}{56}, \frac{265}{153}, \frac{362}{209}, \frac{985}{571}, \frac{1351}{780}, \frac{3691}{2131}, \frac{5042}{2911}, \frac{13775}{7953},$$

$$\frac{18817}{10864}, \frac{51409}{29681}, \frac{70226}{40545}, \frac{262087}{151310}, \frac{191861}{413403}, \frac{978122}{564719}, \frac{978122}{564719}, \dots \quad (55)$$

The second, sixth, tenth, fourteen, eighteen,...convergents from (55) forms solution to (53). In particular, we obtain the pairs.

$$(u, v) = (2, 1); (26, 15); (362, 209); (5042, 2911); (70226, 40545); \dots (56)$$

By lemma (2.3), we obtain

$$\alpha = aX_0 = 3, \quad \beta = Y_0 = 1,$$

$$X = X_0u + b\beta v = 3u + 3v \quad (57)$$

$$Y = \beta u + \alpha v = u + 3v \quad (58)$$

From equations (57) and (58) for every pair of (u, v) given by (56), we obtain the solutions of equation (52) which are given by

$$(X, Y) = (9, 5); (123, 71); (1713, 989); (23859, 13775); (332313, 191861); \dots (59)$$

Since $m = \frac{X+1}{4}$, $n = \frac{Y-1}{2}$, from the second, fourth,... values of (X, Y) in (59), we have

$$(m, n) = (31, 35); (5965, 6887); \dots (60)$$

We have omitted the values of (m, n) which are non-integers obtained from first, third, fifth,... pairs of (59)

By lemma (2.3), we obtain

$$X = |-X_0u + b\beta v| = |-3u + 3v| \quad (61)$$

$$Y = |\beta u - \alpha v| = |u - 3v| \quad (62)$$

From equation (61) and (62) for every pair of (u, v) given by (56), we obtain the solution of equation (52), which are given by

$$(X, Y) = (3,1); (33,19); (459,265); (6393,3691); (89043,51409); \dots(63)$$

Since $m = \frac{X+1}{4}$, $n = \frac{Y-1}{2}$, from the first, third, fifth,... values of (X, Y) in (63), we have

$$(m, n) = (1,0); (115,132); (22261,25704); \dots \quad (64)$$

We have omitted the values of (m, n) which are non-integers obtain from second, fourth,... pairs of (63)

Now considering values of (m, n) from (60), (64) and substituting in (50), we notice that the numbers 1, 1891, 26355, 71156485, 991081981, ... (65) are both Centered triangular numbers as well as Hexagonal numbers.

Since there are infinitely many convergents that can be generated using (54) the number of solutions for (m, n) from (60) and (64) through (60) and (64) are infinite and so there exists infinitely many Centered triangular numbers – Hexagonal numbers, first few of which are listed in (65). Further, since every hexagonal number is a triangular the list of numbers obtained in (65) is subset of the list of numbers obtained in (27) of Theorem 1.

3.5 Theorem 5

There exists infinitely many centered triangular numbers which are also Heptagonal numbers.

Proof:

Let CT_n be the n th Centerd triangular number and let HP_m be the m th Heptagonal number.

Let $CT_n = HP_m$ for some n, m then we have

$$\frac{3n(n+1)}{2} + 1 = \frac{5m^2 - 3m}{2} \quad (66)$$

$$5m^2 - 3m - 2 - 3n^2 - 3n = 0$$

$$m = \frac{3 \pm \sqrt{3^2 - 4(5)(-2 - 3n^2 + 3n)}}{2(5)}$$

$$(10m - 3)^2 = 15(2n + 1)^2 + 34$$

$$(10m - 3)^2 - 15(2n + 1)^2 = 34 \quad (67)$$

If we now consider $X = 10m - 3$ and $Y = 2n + 1$ then (67) can be written as

$$X^2 - 15Y^2 = 34 \quad (68)$$

Comparing with (7), we get, $a = 1, b = 15, D = ab = 15$. One of the solutions for equation (68) is $(X_0, Y_0) = (7, 1)$.

Now, consider $u^2 - 15v^2 = 1$ (69)

The continued fraction for $\sqrt{15}$ is given by

$$\sqrt{15} = 4 - \frac{1}{8 - \frac{1}{8 - \frac{1}{8 - \frac{1}{8 - \dots}}}} \quad (70)$$

The successive convergent obtained from continued fraction of $\sqrt{15}$ is given by

$$\frac{4}{1}, \frac{31}{8}, \frac{244}{63}, \frac{1921}{496}, \frac{15124}{3905}, \frac{119071}{30744}, \frac{937444}{242047}, \frac{7380481}{1905632}, \frac{58106404}{15003009}, \dots \quad (71)$$

We notice that the second, fourth, sixth,... convergents of (71) form solutions to (69). In particular, the solutions of (69) are given by the pairs

$$(u, v) = (31,8); (1921,496); (119071,30744); (7380481,1905632); \dots(72)$$

Now, $(x_0, y_0) = (7,1)$ is initial solution to $X^2 - 15Y^2 = 34$

$$\alpha = aX_0 = 7, \quad \beta = Y_0 = 1,$$

By lemma (2.3), we obtain

$$X = X_0u + b\beta v = 7u + 15v \quad (73)$$

$$Y = \beta u + \alpha v = u + 7v \quad (74)$$

From equations (73) and (74) for every pair of (u, v) given by (72), we obtain solutions of equation (68). Such solutions are given by

$$(X, Y) = (337,87); (20887,5393); (1294657,334279); \dots(75)$$

Since $m = \frac{X+3}{10}, n = \frac{Y-1}{2}$, from (75)

$$\text{We have } (m, n) = (34,43); (2089,2696); (129466,167139); \dots (76)$$

By lemma (2.3), we obtain

$$X = |-X_0u + b\beta v| = |-7u + 15v| \quad (77)$$

$$Y = |\beta u - \alpha v| = |u - 7v| \quad (78)$$

From equations (77) and (78) for every pair of (u, v) given by (72), we obtain solutions of equation (68) given by

$$(X, Y) = (97,25); (6007,1551); (372337,96137); \dots(79)$$

Since $m = \frac{X+3}{10}, n = \frac{Y-1}{2}$, from (79) we have

$$(m, n) = (10,12); (601,775); (37234,48068); \dots (80)$$

Now considering values of (m, n) from (76) and (80) and substituting in (66), we notice that the numbers,

$$235, 2839, 902101, 10906669, 3465871039, 41903418691, \dots (81)$$

are both Centered triangular numbers as well as Heptagonal numbers.

Since there are infinitely many convergents that can be generated using (70), the number of solutions for (m, n) through (76) and (80) are infinite and so there exists infinitely many Centred triangular numbers- Heptagonal numbers, first few of which are listed in (81).

4. Conclusion

Among several existing known ideas related to Polygonal numbers, this paper has produced few theorems which provide easier way of computing numbers which belong to two different categories. The study of centred polygonal numbers though is not as widely done as polygonal numbers are also significantly important. In this paper, we have introduced centred polygonal numbers and have established five interesting theorems related to centred triangular numbers.

In particular, we have shown that there exist infinitely many:

- (i) centred triangular numbers which are also triangular through theorem 1
- (ii) centred triangular numbers which are also square numbers through theorem 2
- (iii) centred triangular numbers which are also centred square numbers through theorem 3
- (iv) centred triangular numbers which are also hexagonal numbers through theorem 4
- (v) centred triangular numbers which are also heptagonal numbers through theorem 5

Thus the main purpose of this paper is to determine numbers which are simultaneously centred triangular numbers and other category of numbers. This is done through solving appropriate Pell's equation obtained in each case. Thus the enumeration of numbers belonging to two distinct families is done elaborately in this paper. One can follow similar methods to obtain many similar results.

References

- [1] R. Sivaraman, Curious Fact about Figurate Numbers, CLIO, Volume 6, Issue 6, April 2020, pp. 541 – 546.
- [2] R. Sivaraman, Generalized Narayana sequences and Figurate Numbers, Advances in Mathematics: Scientific Journal, Volume 9 (2020), no.10, pp. 7977 – 7984.
- [3] R. Sivaraman, Triangular – Pentagonal Numbers and Interesting Puzzle, International Journal of Engineering Research and Modern Education, Volume 7, Issue 1, 1 – 3, 2022.
- [4] R. Sivaraman, Square – Pentagonal Numbers, International Journal of Multidisciplinary Research and Modern Education, Volume 8, Issue 1,1 – 2, 2022.
- [5] R. Sivaraman, Octagonal – Pentagonal Numbers and an Interesting Puzzle, Bulletin of Mathematics and Statistics Research, Volume 10, Issue 1, January – March 2022, pp. 16 – 20.
- [6] R. Giridharan, R. Sivaraman, Octagonal – Triangular, Octagonal – Hexagonal Numbers and Related Puzzle, Stochastic Modeling & Applications, Vol. 26, No. 3, Special Issue Part – 8, 2022, 214 – 218.
- [7] Andreescu, T., D. Andrica, and I. Cucurezeanu, An introduction to Diophantine equations: A problem-based approach, Birkhäuser Verlag, New York, 2010.
- [8] Andrews, G. E. 1971, Number theory, W. B. Saunders Co., Philadelphia, Pa.-London- Toronto, Ont.
- [9] Isabella G. Bashmakova, Diophantus and Diophantine Equations, The Mathematical Association of America, 1998.
- [10] R. Sivaraman, Understanding Ramanujan Summation, International Journal of Advanced Science and Technology, Volume 29, No. 7, (2020), pp. 1472 – 1485.
- [11] R. Sivaraman, Remembering Ramanujan, Advances in Mathematics: Scientific Journal, Volume 9 (2020), no.1, pp. 489–506.
- [12] R. Sivaraman, Summing Through Integrals, Science Technology and Development, Volume IX, Issue IV, April 2020, pp. 267 – 272.
- [13] R. Sivaraman, Recognizing Ramanujan's House Number Puzzle, German International Journal of Modern Science, 22, November 2021, pp. 25 – 27.
- [14] R. Sivaraman, On Solving Brahmagupta's Puzzle, International Journal of Scientific Research and Modern Education, Volume 7, Issue 1, 2022.
- [15] Elena Deza, Michel Marie Deza, Figurate Numbers, World Scientific, Singapore, 2012, 46 – 47.
- [16] Morris Kline, Mathematical Thought from Ancient to Modern Times, Oxford University Press, New York, 1972, 136 – 137.