Advances in Nonlinear Variational Inequalities
ISSN: 1092-910X
Vol 28 No. 5s (2025)

Domination of Polynomial with Application

Alyaa A. Alwan[1,a)] and Alaa A. Najim[2,b)]
[1]College of .Sciences , University of Basrah , Basrah, Iraq
[2]Department of Mathematics , College of .Sciences University of Basrah , Basrah, Iraq
[a)] Corresponding author:[a)]alyaaalwan@utq.edu.iq
[b)]alaa.najim@uobasrah.edu.iq

Article History:

Received: *06-11-2024*

Revised: *05-12-2024*

Accepted: *15-12-2024*

Abstract:

In this paper, .We .initiate the study of domination. polynomial , consider G=(V,E) be a simple, finite, and directed graph without. isolated. vertex .We present a study of the Iraqi national power grid 400kv. We extracted the poly dominance as well as the reliability. dominance by dividing it in to .several regions. The MATLAB program was used to calculate the shortest path for the network.

Introduction: The study of domination polynomial and domination reliability are very important concepts ..The domination polynomial of a graphs is a graph invariant that encodes information about the number of dominating. Set of various sizes in the graphs [29].The domination. Polynomial has been studies extensively to have various applications, such as in the analysis of communication network.[30]. Domination reliability is a measure .of the. reliability .of a graph or network, it is defined as the probability. that a randomly selected subset of vertices in the graph forms a dominating sets . [31],[32] .

Objectives: the reliability of the Iraqi super grid will be calculated .Calculation the reliability of figure 1 is NP-Hard, as the electrical conductors interfering with each other. Therefore, we relied on the reduction method by definition " $DRel(G,p) = (1-p)^{|V|} \sum_{\substack{W \subset V \\ N[W]=V}} (\frac{p}{1-p})^{|W|}$" to calculate the reliability of figure 1 ,where the parallel law is used for reduction and the series law for merging components with each other then use the law of reliability of parallele-seriese to compute the reliability of figure 1.

Iraqi electrical power system is divided in to six subsystems R1 is Iraqi North Zone reliability index, R2 is Dyala- Anbar Zone reliability index, R3 is Baghdad North Zone reliability index, R4 is Iraqi Middle Zone reliability index and R6 is Iraqi South Zone reliability index

Methods: we want to look at the domination problem from the reliability point of view. We are interested in the probability that in a graph with random failing vertices or edges, a dominating set (with some properties) exists. In this section we only show some possible direction for further research . assume that the vertices of the graph are dominating with a given probability p and the edges are perfectly reliable.

Let $G = (V, E)$ be a graph whose vertices fail randomly and independently with a given probability q_v for all $v \in V$. A failure in the context of

domination means, that the vertex is not in the dominating set. In such a graph we are interested in the reliability that a dominating set exists. If we assume that $q = q_v$, for all $v \in V$, and $p = 1 - q$, then we can define the domination reliability polynomial as

$$DRel(G,p) = (1-p)^{|V|} \sum_{\substack{W \subset V \\ N[W]=V}} (\frac{p}{1-p})^{|W|}.$$

Remark: The domination polynomial and the domination reliability polynomial are equivalent.

Results: We study this work, as domination reliability is may reveal some new insights in to system reliability.

Conclusions: In this paper, .we introduced the Iraqi 400kv grid and .compute dominating polynomial of .this network also, determined the dominating reliability of Iraqi super grid , the computation of this network is NP- hard, that. divides the network to find the minimal path of Iraqi 400kv grid to compute the dominating .reliability .and dominating polynomial.We study this work, as domination reliability is may reveal some new .insights in .to system reliability.

Keywords: domination polynomial, the domination reliability**,** the domination reliability polynomial, Iraqi 400 KV super grid.

1. Introduction

The study of domination polynomial and domination reliability are very important concepts. The domination polynomial of a graph is a graph invariant that encodes information about the number of dominating sets of various sizes in the graph [29].

The domination polynomial has been studies extensively to have various applications, such as in the analysis of communication network.[30]

Domination reliability is a measure of the reliability of a graph or network, it is defined as the probability that a randomly selected subset of vertices in the graph forms a dominating set. [31],[32] .

In this paper we will compute the domination polynomial and domination reliability of Iraqi 400kv grid. The network of fig (1) is classified into SIX zones which represent the close practical operating condition of the network. To find the domination indices of the network the following procedure is followed:

For each zone & to apply the domination procedure calculations, the minimal paths are found .Hence finding the domination index (D)

Notation:

- D1 …. Iraqi North zone domination index
- D2 …. Dyala -Anbar zone domination index
- D3 …. Baghdad North zone domination index

Advances in Nonlinear Variational Inequalities
ISSN: 1092-910X
Vol 28 No. 5s (2025)

- D4 …. Baghdad South zone domination index
- D5 …. Iraqi Middle zone domination index
- D6 …. Iraqi South zone domination index

The adjacent matrices of the 400KV Iraqi super grid of Iraqi north region, Dyala and Anbar region, Baghdad north region, Baghdad south region, Iraqi middle region, and Iraqi south region are shown in tables above, which is input data to the first computer program, the output results are also shown in tables above, from these tables it is noted that there are Minimal paths sets.

Objectives

Domination and graph polynomials are each areas of graph theory with extensive research. A 1991 bibliography on domination in graphs [17] by Hedetniemi and Laskar traced domination back to the graph theory texts of K¨onig (1950), Berge (1958) and Ore (1962). Graph polynomials have also been of interest since 1912 when Birkhofffirst defined the chromatic polynomial [15] in an attempt to prove the Four Colour Conjecture. Although domination and graph polynomials have been areas of interest for quite some time, the domination polynomial was only introduced by Arocha and Llano in their 2000 paper [14]. In fact no other papers were published until 2008 when a seemingly independent work [8] was published by Alikhani and Peng. Results that have been of interest for the domination polynomial include computing the domination polynomial for families and products of graphs, finding recurrence relations, locating the roots, and finding the domination equivalence classes of families of graphs. The reader is directed to Alikhani's 2009 Ph.D. thesis [4] which is the culmination of six fundamental papers [2], [5], [8],[10],[12] covering each area of domination polynomials studied.

3. Methods

In this paper we will compute the domination reliability and domination polynomial. of Iraqi 400kv grid. By divided it in .to two parts, as .follows:

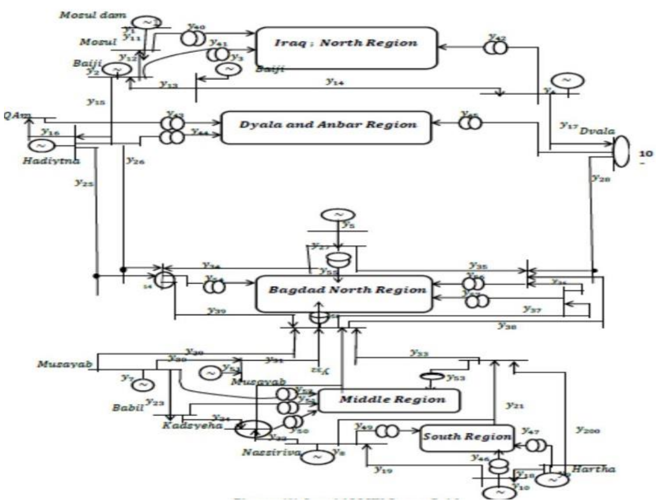

FIGURE 1 Iraqi 400 KV super grid

ISSN: 1092-910X
Vol 28 No. 5s (2025)

we have converted the electrical network into a graph to make it easier to study, then divided it into six regions to calculate the shortest path for each region, then calculated the polynomial for each path, then calculated the polynomial dominance for each path, then calculated the polynomial dominance reliability for each path and its overall reliability.

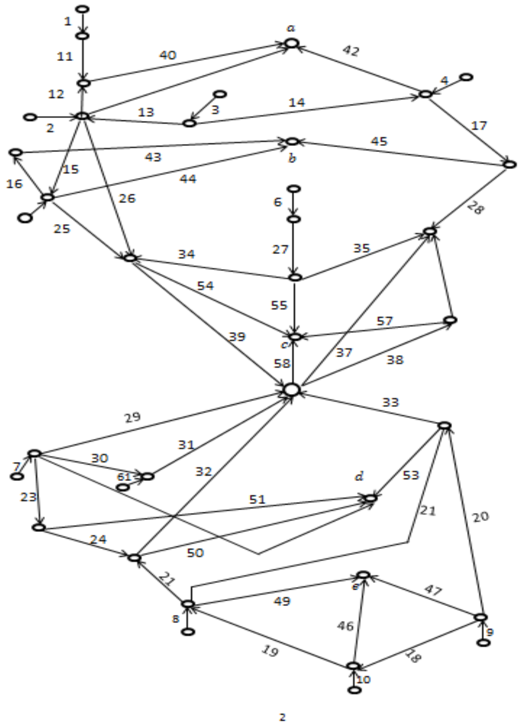

FIGURE 2 graph of analysisof the Iraqi 400 KV super grid

Input Representation

Where

No.	Region
1	Mosul dam
2	Mosul
3	Iraq north regin
4	Baiji
5	Baiji
6	Kirkok
7	Qam
8	Hadiytha
9	Dyala and Anbar region
10	Dyala
11	Qudis
12	Baghdad northregion
13	Baghdad south
14	Baghdad west

15	Middle baiji
16	Baghdad east
17	Baghdad north
18	Al ameareea

No.	**Region**
1	Baghdad south
2	Kut
3	Middle region
4	Musayab
5	Musayab
6	Babil
7	Kadsyeha
8	Nassiriya
9	South region
10	Hartha
11	Khor al zuber

Let's denote the domination reliability polynomial of a path graph with n vertices as $DRP(P_n, x, y)$, where x and y are variables For a path graph P_n, the domination reliability polynomial can be defined as

$$DRP(P_1, x, y) = x + y$$

$$DRP(P_2, x, y) = x^2 + 2xy + y^2$$

$$DRP(P_3, x, y) = x^3 + 3x^2y^2 + y^3 + xy$$

$$DRP(P_n, x, y) = (x + y) * DRP(P_{n-1}, x, y) - y * DRP(P_{n-2}, x, y)$$

Using this recursive formula, you can compute the domination reliability polynomial for any path graph by substituting the appropriate values of n For example, if you want to compute the domination reliability polynomial for a path graph with 4 vertices P_4, you can use the formula as follows :

$$DRP(P_4, x, y) = (x + y) * DRP(P_3, x, y) - y * DRP(P_2, x, y)(x + y) * \left((x + y) * DRP(P_2, x, y) - y * DRP(P_1, x, y)\right) - y$$

The domination reliability polynomial for a path graph P_n is given by the following formula :

$$DR(P_n, p) = p + (n - 1)(1 - p)p$$

Where:

P is the probability that a vertex is included in the dominating sets .

The step – by - step explanation.

1-In a path graph P_n , the only way to form a dominating set is to either include the first vertex, or include every other vertex starting from the second vertex.

2-The probability of including the first vertex is simply p .

3-The probability of including every other vertex starting from the second vertex is $(1-p)p$, and there are n-1 such vertices.

4- Therefore , the domination reliability polynomial is the sum of these 2 terms: $p+(n-1)(1-p)p$.

" Iraq North Region "

$p_1 = y_2 y_{41} \rightarrow D(P_2, y) = y^2 + 2y$, $p_2 = y_4 y_{42} \rightarrow D(P_2, y) = y^2 + 2y$. $p_3 = y_3 y_{13} y_{41} \rightarrow D(P_3, y) = y^3 + 3y^2 + y$.
$p_4 = y_1 y_{11} y_{40} \rightarrow D(P_3, y) = y^3 + 3y^2 + y$,
$p_5 = y_2 y_{12} y_{40} \rightarrow D(P_3, y) = y^3 + 3y^2 + y$, $p_6 = y_3 y_{13} y_{12} y_{40} \rightarrow D(P_4, y) = y^4 + 4y^3 + 3y^2 + y$.

" Iraq South Region "$p_1 = y_8 y_{49} \rightarrow D(P_2, y) = y^2 + 2y$. $p_2 = y_{10} y_{19} y_{49} \rightarrow D(P_3, y) = y^3 + 3y^2 + y$. $p_3 = y_9 y_{18} y_{19} y_{49} \rightarrow D(P_4, y) = y^4 + 4y^3 + 3y^2 + y$. $p_4 = y_{10} y_{46} \rightarrow D(P_2, y) = y^2 + 2y$.

$p_5 = y_9 y_{18} y_{46} \rightarrow D(P_3, y) = y^3 + 3y^2 + y$. $p_6 = y_9 y_{47} \rightarrow D(P_2, y) = y^2 + 2y$.

" Baghdad South Region "$p_1 = y_7 y_{29} y_{37} y_{36} y_{56}$. $p_2 = y_7 y_{29} y_{38} y_{56}$. $p_3 = y_7 y_{29} y_{37} y_{57}$. $p_4 = y_7 y_{29} y_{58}$. $p_5 = y_8 y_{22} y_{32} y_{37} y_{36} y_{56}$. $p_6 = y_8 y_{22} y_{32} y_{38} y_{56}$. $p_7 = y_8 y_{22} y_{32} y_{37} y_{57}$. $p_8 = y_8 y_{22} y_{32} y_{58}$.
$p_9 = y_8 y_{21} y_{33} y_{37} y_{36} y_{56}$. $p_{10} = y_8 y_{21} y_{33} y_{38} y_{56}$. $p_{11} = y_8 y_{21} y_{33} y_{37} y_{56}$. $p_{12} = y_8 y_{21} y_{33} y_{58}$.

$p_{13} = y_{10} y_{19} y_{22} y_{32} y_{37} y_{36} y_{56}$. $p_{14} = y_{10} y_{19} y_{22} y_{32} y_{37} y_{57}$.$p_{15} = y_{10} y_{19} y_{22} y_{32} y_{37} y_{57}$.

$p_{16} = y_{10} y_{19} y_{22} y_{32} y_{58}$. $p_{17} = y_{10} y_{19} y_{21} y_{33} y_{37} y_{36} y_{56}$. $p_{18} = y_{10} y_{19} y_{21} y_{33} y_{38} y_{56}$. $p_{19} = y_{10} y_{19} y_{21} y_{33} y_{37} y_{57}$. $p_{20} = y_{10} y_{19} y_{21} y_{33} y_{58}$ $p_{21} = y_9 y_{20} y_{33} y_{37} y_{36} y_{56}$. $p_{22} = y_9 y_{20} y_{33} y_{38} y_{56}$.

$p_{23} = y_9 y_{20} y_{33} y_{37} y_{57}$. $p_{24} = y_9 y_{20} y_{33} y_{58}$.

" Baghdad South Region "$p_1 \rightarrow D(P_5, y) = y^5 + 5y^4 + 4y^3 + 3y^2 + y$, $p_2 \rightarrow D(P_4, y) = y^4 + 4y^3 + 3y^2 + y$. $p_3 \rightarrow D(P_4, y) = y^4 + 4y^3 + 3y^2 + y$. $p_4 \rightarrow D(P_3, y) = y^3 + 3y^2 + y$. $p_5 \rightarrow D(P_6, y) = y^6 + 6y^5 + 5y^4 + 4y^3 + 3y^2 + y$. $p_6 \rightarrow D(P_5, y) = y^5 + 5y^4 + 4y^3 + 3y^2 + y$.

$p_7 \rightarrow D(P_5, y) = y^5 + 5y^4 + 4y^3 + 3y^2 + y$. $p_8 \rightarrow D(P_4, y) = y^4 + 4y^3 + 3y^2 + y$

$p_9 \rightarrow D(P_6, y) = y^6 + 6y^5 + 5y^4 + 4y^3 + 3y^2 + y$.$p_{10} \rightarrow D(P_5, y) = y^5 + 5y^4 + 4y^3 + 3y^2 + y$.

$p_{11} \rightarrow D(\,P_5, y) = \; y^5 + 5y^4 + 4y^3 + 3y^2 + y.$
$p_{12} \rightarrow D(\,P_4, y) = \; y^4 + 4y^3 + 3y^2 + y$

$p_{13} \rightarrow D(\,P_7, y) = y^7 + 7y^6 + 6\,y^5 + 5y^4 + 4y^3 + 3y^2 + y$

$p_{14} \rightarrow D(\,P_6, y) = y^6 + 6\,y^5 + 5y^4 + 4y^3 + 3y^2 + y$

$p_{15} \rightarrow D(\,P_6, y) = y^6 + 6\,y^5 + 5y^4 + 4y^3 + 3y^2 + y$

$p_{16} \rightarrow D(\,P_5, y) = \; y^5 + 5y^4 + 4y^3 + 3y^2 + y$

$p_{17} \rightarrow D(\,P_7, y) = y^7 + 7y^6 + 6\,y^5 + 5y^4 + 4y^3 + 3y^2 + y$

$p_{18} \rightarrow D(\,P_6, y) = y^6 + 6\,y^5 + 5y^4 + 4y^3 + 3y^2 + y$

$p_{19} \rightarrow D(\,P_6, y) = y^6 + 6\,y^5 + 5y^4 + 4y^3 + 3y^2 + y$

$p_{20} \rightarrow D(\,P_5, y) = \; y^5 + 5y^4 + 4y^3 + 3y^2 + y$

$p_{21} \rightarrow D(\,P_6, y) = y^6 + 6\,y^5 + 5y^4 + 4y^3 + 3y^2 + y$

$p_{22} \rightarrow D(\,P_5, y) = \; y^5 + 5y^4 + 4y^3 + 3y^2 + y$

$p_{23} \rightarrow D(\,P_5, y) = \; y^5 + 5y^4 + 4y^3 + 3y^2 + y$

$p_{24} \rightarrow D(\,P_4, y) = \; y^4 + 4y^3 + 3y^2 + y$

" Baghdad North Region "$p_1 = \; y_1 y_{11}\, y_{40} \rightarrow D(P_3, y) = \; y^3 + 3y^2 + y,$ $p_2 = y_2 y_{12}\, y_{40} \rightarrow D(P_3, y) = \; y^3 + 3y^2 + y$, $p_3 = \; y_3\, y_{13} y_{12}\, y_{40} \rightarrow D(P_4, y) = y^4 \; + 4y^3 + 3y^2 \; + y$

$p_4 = \; y_2 y_{41} \rightarrow D(P_2, y) = \; y^2 + 2y$, $p_5 = \; y_3 y_{13}\, y_{41} \rightarrow D(P_3, y) = \; y^3 + 3y^2 + y,$ $p_6 = y_4 y_{42} \rightarrow D(P_2, y) = \; y^2 + 2y$.

" Dyala - Anbar Region "

$p_1 = \; y_5 y_{44} \rightarrow D(P_2, y) = \; y^2 + 2y$

" Iraq North Region ", $DR(P_1, p) = P + (2-1)(1-P)P = 2P - \; P^2$ **,** $DR(P_2, p) = P + (2-1)(1-P)P = 2P - \; P^2$, $DR(P_3, p) = P + (3-1)(1-P)P = P + 2P - 2P^2 = 3P - 2P^2$

$DR(P_4, p) = P + (3-1)(1-P)P = P + 2P - 2P^2 = 3P - 2P^2$, $DR(P_5, p) = P + (3-1)(1-P)P = P + 2P - 2P^2 = 3P - 2P^2,$ $DR(P_6, p) = P + (4-1)(1-P)P = P + 3(1-P)P = 3P - 2P^2$ $DR(P_i, p) = \; 2P - \; P^2 \; + 3P - 2P^2 + \; 3P - 2P^2 + 3P - 2P^2 = 12P - 8P^2$.

" Iraq South Region " , $DR(P_1, p) = P + (2-1)(1-P)P = 2P - \; P^2$.**,** $DR(P_2, p) = 3P - 2P^2$.

$DR(P_3, p) = 4P - 3P^2$. , $DR(P_4, p) = 2P - \; P^2$. , $DR(P_5, p) = \; 3P - 2P^2$., $DR(P_6, p) = 2P - \; P^2$.

$DR(P_i,p) = \quad 2P - P^2 + 3P - 2P^2 + 4P - 3P^2 + 2P - P^2 + 3P - 2P^2 + 2P - P^2 = 16P - 10\,P^2$

" Baghdad South Region " , $DR(P_1,p) = 5P - 4P^2$ **,** $DR(P_2,p) = 4P - 3P^2$ **,** $DR(P_3,p) = 4P - 3P^2$

$DR(P_4,p) = 3P - 2P^2$, $DR(P_5,p) = 6P - 5P^2$, $DR(P_6,p) = 5P - 4P^2$, $DR(P_7,p) = 5P - 4P^2$

$DR(P_8,p) = 4P - 3P^2$, $DR(P_9,p) = 6P - 5P^2$, $DR(P_{10},p) = 5P - 4P^2$, $DR(P_{11},p) = 5P - 4P^2$

$DR(P_{12},p) = 4P - 3P^2$, $DR(P_{13},p) = 7P - 6P^2$, $DR(P_{14},p) = 6P - 5P^2$, $DR(P_{15},p) = 6P - 5P^2$

$DR(P_{16},p) = 5P - 4P^2$, $DR(P_{17},p) = 7P - 6P^2$, $DR(P_{18},p) = 6P - 5P^2$, $DR(P_{19},p) = 6P - 5P^2$

$DR(P_{20},p)\ 5P - 4P^2$, $DR(P_{21},p) = 6P - 5P^2$, $DR(P_{22},p) = 5P - 4P^2$. , $DR(P_{23},p) = 5P - 4P^2$

$DR(P_{24},p) = 4P - 3P^2$, $DR(P_i,p) = 95P - 76P^2$

" Baghdad north Region "

$DR(P_1,p) = 3P - 2P^2$,$DR(P_2,p) = 3P - 2P^2$, $DR(P_3,p) = 4P - 3P^2$, $DR(P_4,p) = 2P - P^2$

$DR(P_5,p) = 3P - 2P^2$, $DR(P_6,p) = 2P - P^2$

$DR(P_i,p) = 3P - 2P^2 + 3P - 2P^2 + 4P - 3P^2 + 2P - P^2 + 3P - 2P^2 + 2P - P^2 = 17P - 11P^2$.

" Dyala - Anbar Region " $DR(P_1,p) = 2P - P^2$.

Now, $DR(P_I,p) = 12P - 8P^2 + 16P - 10\,P^2 + 2P - P^2 + 17P - 11P^2 + 95P - 76P^2 = 142P - 106P^2$.

Domination and graph polynomials are each areas of graph theory with extensive research. A 1991 bibliography on domination in graphs [17] by Hedetniemi and Laskar traced domination back to the graph theory texts of K¨onig (1950), Berge (1958) and Ore (1962). Graph polynomials have also been of interest since 1912 when Birkhofffirst defined the chromatic polynomial [15] in an attempt to prove the Four Colour Conjecture. Although domination and graph polynomials have been areas of interest for quite some time, the domination polynomial was only introduced by Arocha and Llano in their 2000 paper [14]. In fact no other papers were published until 2008 when a seemingly independent work [8] was published by Alikhani and Peng. Results that have been of interest for the domination polynomial include computing the domination polynomial for families and products of graphs, finding recurrence relations, locating the roots, and finding the domination equivalence classes of families of graphs. The reader is directed to Alikhani's 2009 Ph.D.

thesis [4] which is the culmination of six fundamental papers [2], [5], [8],[10],[12] covering each area of domination polynomials studied.

2. Results

The domination reliability polynomial of a graphs is a polynomial that encodes information about the number of dominating sets in the graph and their reliability. The domination reliability polynomial of a path graph can be computed using a recursive formula

Let's denote the domination reliability polynomial of a path graph with n vertices as $DRP(P_n, x, y)$, where x and y are variables For a path graph P_n, the domination reliability polynomial can be defined as

$$DRP(P_1, x, y) = x + y$$

$$DRP(P_2, x, y) = x^2 + 2xy + y^2$$

$$DRP(P_3, x, y) = x^3 + 3x^2y^2 + y^3 + xy$$

$$DRP(P_n, x, y) = (x + y) * DRP(P_{n-1}, x, y) - y * DRP(P_{n-2}, x, y)$$

Using this recursive formula, you can compute the domination reliability polynomial for any path graph by substituting the appropriate values of n For example, if you want to compute the domination reliability polynomial for a path graph with 4 vertices P_4, you can use the formula as follows :

$$\begin{aligned} DRP(P_4, x, y) &\\ &= (x + y) * DRP(P_3, x, y) - y * DRP(P_2, x, y)(x + y) \\ &* \left((x + y) * DRP(P_2, x, y) - y * DRP(P_1, x, y)\right) - y \end{aligned}$$

The domination reliability polynomial for a path graph P_n is given by the following formula :

$$DR(P_n, p) = p + (n - 1)(1 - p)p$$

Where:

P is the probability that a vertex is included in the dominating set.

The step-by-step explanation.

1-In a path graph P_n , the only way to form a dominating set is to either include the first vertex, or include every other vertex starting from the second vertex.

2-The probability of including the first vertex is simply p .

3-The probability of including every other vertex starting from the second vertex is $(1 - p)p$, and there are n-1 such vertices.

4-Therefore, the domination reliability polynomial is the sum of these two terms: $p + (n - 1)(1 - p)p$.

Discussion

In this paper, we introduced the Iraqi 400kv grid and compute dominating polynomial of this network also, determined the dominating reliability of Iraqi super grid , the computation of this network is NP- hard, that divides the network to find the minimal path of Iraqi 400kv grid to compute the dominating reliability and dominating polynomial.

We study this work, as domination reliability is may reveal some new insights in to system reliability.

Refrences

[1] Karaboga, N., & Gorkemli, B. (2011). Combinatorial optimization of domination reliability in wireless sensor networks. Expert Systems with Applications, 38(9), 10880-10887.

[2] Reilley, J. L., & Zachariasen, M. (2009). Domination reliability in social networks. In International Workshop on Algorithms and Models for the Web-Graph (pp. 139-150). Springer, Berlin, Heidelberg.

[3] Haynes, T. W., Hedetniemi, S. T., & Slater, P. J. (1998). Fundamentals of domination in graphs. CRC Press.

[4] Dai, F., & Wu, J. (2004). An extended localized algorithm for connected dominating set formation in ad hoc wireless networks. IEEE Transactions on Parallel and Distributed Systems, 15(10), 908-920.

[5] E. J. Cockayne and S. T. Hedetniemi," Towards a Theory of Domination in Graphs", Networks, 7 (1977) 247-261.

[6] [Karaboga, N., & Gorkemli, B. (2011). Combinatorial optimization of domination reliability in wireless sensor networks. Expert Systems with Applications, 38(9), 10880-10887 .

[7] Reilley, J. L., & Zachariasen, M. (2009). Domination reliability in social networks. In International Workshop on Algorithms and Models for the Web-Graph (pp. 139-150). Springer, Berlin, Heidelberg.

[8] Haynes, T. W., Hedetniemi, S. T., & Slater, P. J. (1998). Fundamentals of domination in graphs. CRC Press.

[9] Dai, F., & Wu, J. (2004). An extended localized algorithm for connected dominating set formation in ad hoc wireless networks. IEEE Transactions on Parallel and Distributed Systems, 15(10), 908-920.

[10] M. C. Golumbic," Algorithmic Graph Theory and Perfect Graphs", Academic Press, New York, (1980).

[11] F. Harary," Graph Theory", Addison-Wesley, Boston, (1969).

[12] K. Dohmen, & P. Tittmann, " Domination Reliability " 09648 Mittweida,Germany, November 1, 2018.

[13] T. W. Haynes, S. T. Hedetniemi and P. J. Slater," Fundamentals of Domination in Graphs", Marcel Dekker, INC., New York, (1998).

[14] Alikhani, S., Peng, Y.H., Introduction to domination polynomial of a graph, Ars Combin., 114(2014), 257–266.

[15] V. R. Kulli and S. C. Sigarkanti," Inverse Domination in Graphs", National Academy Science Letters, 14 (1991) 473-475.

[16] Anthony Meadows "Tutte Polynomials of Some Graphs" A thesis submitted to the Graduate Faculty of Elizabeth City State University 2020

[17] G. Agnarsson and R. Greenlaw, (2007). Graph theory: Modeling, applications &algorithms, Pearson Education, New Delhi.

[18] R. J. Wilson," Introduction to Graph Theory", Longman Group Ltd, Fourth Edition, (1998)

[19] T.Kotek,J.Preen,andP.Tittmann,Dominationpolynomialsofgraphproducts,J.Combin.Math.Combin.Comput. 2013

[20] S. Muthammai, M. Bhanumathi and P. Vidhya," Complementary Tree Domination Number of a Graph", International Mathematical Forum, Bulgaria, 6 (26) (2011) 1273-1282.

[21] A. A. Omran," On Chess Combinatorics", Ph. D. Thesis, Mathematics Department, Faculty of Science, Ain Shams University, (2014).

[22] O. Ore," Theory of Graphs", American Mathematical Society Colloquium, Providence, R. I., (1962).

[23] M. S. Rahman," Basic Graph Theory", Springer, India, (2017).
[24] [24] T. Kotek, J. Preen, F. Simon, P. Tittmann, and M. Trinks, Recurrence relations and splitting formulas for the domination polynomial, Electron. J. Comb. 19 (2012), 1–27
[25] S. Alikhani and Y. H. Peng, Dominating sets and domination polynomials of paths, Int. J. Math. Math. Sci. 10 (2009), 1–10.
[26] S. Alikhani and Y. H. Peng, Dominating sets and domination polynomials of certain graphs. II., Opusc. Math. 30 (2010), 37–51.
[27] S. Alikhani and Y. H. Peng, Domination polynomials of cubic graphs of order 10, Turkish J. Math. 35 (2011), 355–366.
[28] S. Alikhani and Y. H. Peng, Introduction to domination polynomial of a graph, Ars Comb. 114 (2014), 257–266.
[29] Goddavd, W., Henning , M. A. (2013), independent domination in graphs: A survey and recent results, Discrete Mathematics, 313(7), 839-854.
[30] Harary, F., Livingston , M. (1988). On the domination polynomial of a graph.
[31] Bauev, D. , Prakash, Shier, D., R., Trotter, W.T. (2007). Domination in graph.
[32] Kariv, O., Hakimi, S.L.(1979). An algorithmic approach to network reliability.
[33] M. A. Abdlhusein," Doubly Connected Bi-Domination in Graphs", Discrete Mathematics, Algorithms and Applications, 13 (2) (2021) 2150009.