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Analysis of Resource Allocation Problem with Different Techniques: A Comparative Study

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Abstract:

Assigning n's entities to m's other entities in an invertible manner is a challenging task that arises in various circumstances. In combinatorial optimisation, assignment problems are a heavily researched area. These problems have many applications in different domains like production planning, VLSI design for telecommunications, economics, etc. The assignment problem plays a crucial role in assigning things to machines so as to reduce assignment costs. To achieve this purpose, this paper examines many ways for solving assignment problems, including Game theory, choice under uncertainty and Graph theory approaches. In order to demonstrate the efficacy of these models, the results were evaluated against one another. Throughout the study, modifications and special situations of the assignment problem and its applications are also examined. The simulation of the proposed WAP approaches is carried out using MATLAB software. The computational findings for a collection of randomly created problems of varying sizes demonstrate that the proposed methods are capable of locating solutions of high quality.

Keywords: undirected, directed, vertices, indegree, outdegree.

1. Introduction

When assigning the m-th worker to the n-th work, the assignment problem is often chosen in such a way that it will result in the least amount of expense and the highest amount of profit (in terms of time and distance). This kind of problem is referred to as the assignment problem, and its goal is to choose an allocation (which task should be given to which person on a one-to-one basis) in such a way that the overall cost of carrying out all of the jobs is reduced to the greatest extent possible. This article presents helpful principles from graph theory, game theory, and choice under uncertainty, all of which show how the assignment problem can be dealt with in the most effective manner possible. The majority of the industrial sectors in India make use of the routes that allow them to get to their destinations via the quickest route and in the least amount of time[1]. The Operation research makes use of these several feasible approaches in order to locate the best possible solution[2]. For the purpose of resolving the assignment problem issue, a unique game-theoretical technique is being utilized in the present work. While assignment challenges from diverse applications have a special emphasis on experimental study[3] and implementation of graph theory methods[4], this article focuses on theoretical research. The automatic production of drawing graphs has significant applications in major computer science technologies, such as data modelling, software development, automated signal conditioning, automated validation of circuits, computer network architecture, and potential input[5].

1.1 State of art

Because of growing rivalry on the international market, commercial businesses are compelled to enhance their operational processes by reducing costs and enhancing efficiency levels[5]. In this regard, it has become essential for decision-makers to identify the most effective techniques for utilizing their resources in order to achieve the highest levels of performance[6]. Resource Assignment Problem (RAP) refers to the difficulty of determining the optimal use of resources in industrial organizations. Among numerous types of resources, machine resources play a key part in the achievement of an industry sector if they are properly allocated to various services or systems with the goal of maximizing or minimizing definite performance and productivity-related objectives [2].

Cattrysse et al. (1994)[7] suggested a column generation fuzzy inference system in which the problem was stated as a set partitioning problem. In contrast, among the numerous metaheuristic approaches, Ant Colony Optimization (ACO) was the most prevalent solution. Demiral (2017)[8] created ACO for a collection of RAP generated at random. The study explored three RAP objectives (minimization of costs, maximization of sales, and maximizing of profits). A statistical analysis based on the mean, standard deviation, and variance was conducted to assist decision makers in selecting the aim that best suited their needs. In a similar vein, Suliman (2019)[9] compared the running time, number of iterations, and quality of solutions between ACO and the traditional Hungarian technique for solving the RAP with a size of 3x3. Chu and Beasley (1997)[10] also presented a genetic algorithm (GA) for solving the AP in addition to the ACO. Jia and Gong (2008)[11] utilized Multi-Objective Particle Swarm Optimization to solve the multi-objective RAP (MOPSO). In this study, various evolutionary algorithms , including Game theory, graph theory, and Decision under Certainty, are used to determine the optimal solution that minimizes overall cost. The evaluation of these proposed methods in RAP is performed using MATLAB.

1.2 Graph theory

Graph theory is indeed utilized in other biological fields. Graph can be employed to identify pharmacological targets by determining the function of a protein or gene. The principles of graph theory are also applicable to the study of DNA and RNA structures. It may be employed to examine the food chain of various animals in an ecological framework by creating arrow diagrams that depict the need of one animal on another for food. This diagram can be viewed as a graph in which the animals serve as vertices and are connected if they depend on one another for sustenance.

In chemistry, graph theory is utilized for mathematical modeling of chemical events. A graph is formed to reflect the natural model of a molecule, with vertices representing atoms and edges signifying bonds. computational biology also uses graph theory.

Graph theory plays an important role in data science. Concepts from graph theory are used to construct software algorithms. There are a number of algorithms derived from graph theory, including the shortest path approach in a network, the Finding minimum spanning tree, and the Determining graph planarity algorithms, Methods for discovering adjacency matrices, connectedness, cycles in a network, etc.

1.3 Game theory

Game theory can provide useful insights into the strategic behaviour of participants in diverse domains where stakeholder interactions typically result in suboptimal (from a system's perspective) results. Game theory can be applied in any field where multiple actors are involved in the decision-making process and the final outcome depends on the participants' strategic behaviour, their ability to collaborate, perception of risk, information availability, exposure to uncertainty, and other affective factors. This method exposes how the preferences of players, their potential moves, and their counter-moves play out in dynamic interplay producing a variety of outcomes.

Throughout the interactive decision-making process, game theory is able to anticipate or describe how individuals behave and pursue their own interests. Games are defined as mathematical frameworks consisting of a set of players, a collection of methods available to them (preferences or movements), and players' payoffs (utilities) for every possible combination of 'game' outcomes. Each player's decisions are primarily motivated by their prospective gain. In a normal game, players attempt to outwit each other by forecasting their moves. The game concludes as a result of the players' decisions.

1.4 Decision under uncertainity

According to C. Zamfir's (2008)[12] explanation of rational decision-making process, "a decision-making procedure is reasonable if it uses rational analysis of conceptually relevant knowledge to select the best conclusion."

The phases of decision-making refer to the actual selection of the solution, whereas the phases following decision-making refer to the implementation of the decision and subsequent evaluation of its long-term effectiveness.

There are four decision models based on the degree of decisional certainty: the definite decision in a rigidly predetermined world, the conclusive type stochastic decision, the decision under uncertainty, and the persistent and cybernetic model.

Alternative solution formulation is a significant source of uncertainty. If the decision-maker has only one option, the uncertainty relates to the potential of success or failure. When there are multiple potential options, uncertainty increases.

Reducible uncertainty refers to the condition of the decider, which is characterized by a high probability that certainty can be greatly decreased through the application of information. Consequently, the decider has the most efficient knowledge acquisition, with this information resulting in a more stable and structured cognitive representation.

1.5 Mathematical model

1.5.1 Graph theory

In this paper, a new graph theory is used to an assignment problem in the context of Operation Research. The employed terminology and theorems are provided first, followed by an explanation of the proposed graph approach. In a graph, a vertex v with zero in-degree is referred to as a vertices (source), whereas a vertex V with zero out-degree is referred to as a degree (sink). Then, we initiated the r x r matrix displaying the assignment of resources to activity in Table 1. It is possible to generate a graph from the supplied assignment table.

Table 1 Approach of Assignment Problem

Activity

	row/column	1	2	3	–	j	–	or	Available
Resource	1	h_{11}	h_{12}	h_{13}	–	h_{ij}	–	h_{1r}	1
	2	h_{21}	h_{22}	h_{23}	–	h_{2j}	–	h_{2r}	1
	3	h_{31}	h_{32}	h_{33}	–	h_{3j}	–	h_{3r}	1
	–	–	–	–	–	–	–	–	1 –
	i	h_{11}	h_{12}	h_{13}	–	h_{ij}	–	h_{im}	1
	r	h_{r1}	h_{r2}	h_{r3}	–	h_{rj}	–	h_{rr}	1
	required	1	1	1	–	1	–	1	–

The basic concepts used in graphs are: A is considered finite if both its set of vertices V and its set of edges E are finite. The in-degree of a vertex in a directed graph is the number of edges incident to the vertex, while the out-degree of a vertex is the count of edges incident from the vertex. A directed graph is one whose vertices can only be explored in one direction. An ordered pair (v_i, v_j) of the pair of vertices that an edge connects may be used to specify an edge in a basic directed graph. We state that both v_i and v_j are next to one another. Due to the fact that each edge contributes two degrees (one for each terminal vertex), the sum of the degrees of the vertices in G equals two times the amount of edges in G. According to the Handshading theorem, we may determine the indegree and outdegree frame of the two Matrix Representations of the i^{th} item's vertices and the j^{th} receiver's indegree and outdegree.

The assignment method that is being presented is the "subtract row and add one" method.

1.6 Game theory

The game theory works as follows

(a) If the reliable fundamentals of one column (let's assume the i^{th} column) are smaller or comparable to those of another column (let's assume the j^{th} column), then the j^{th} column has been defeated by the i^{th} column. Thus, the column j is separated from the overall payout table.

(b) If the whole fundamentals of a particular row (let's claim the i^{th} row) are lesser than or equally identical to the constant fundamentals of every other row (let's claim the j^{th} row), the i^{th} row predominates. Consequently, the i^{th} row is separated shortly after the payout table.

(c) A participant's untainted strategy could be conquered if it is inferior to a curved composite of two or more undiluted methods or inferior to the mean of two or more pristine tactics. In this circumstance, the original approach is eliminated.

(d) It is also possible for one row (or column) to lead the mean of two other rows. (or columns). In this case, every row (column) which was intricately involved in calculating the mean will be eliminated.

2. Methods

In this research work we can find the result for Assignment problem, Decision under uncertainty using symmetric matrix non symmetric matrix values and comparting with game theory result. Using directed and undirected graph based on problem in graph theory. We can frame the problem using subgraph and we get strategy of the result.

Consider the graph with four vertices. From the main graph, the three subgraphs can be obtained and indegree and outdegree for the four graphs are computed.

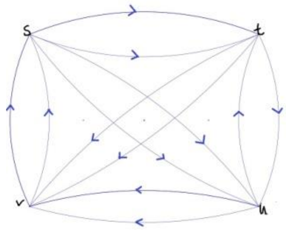

Figure 1

Then we can find the route of the graph into three subgraph. We can frame indegree and outdegree of the matrix. By the above four routes we can find the different direction ways indegree and outdegree of the routes. Based on this we can frame the following matrices for the indegree and outdegree of our graphs.

Problem :1 The following computation describes the result for Assignment problem, Decision under uncertaintly and comparing with Game theory using domiance rule. Then using indegree and outdegree we can frame matrix table.

Vertices- degree	1	2	3	4
A	2	1	0	1
B	3	1	1	1
C	4	1	2	1
D	3	1	1	1

Table: 2.1 Frame the matrix

Clarification:
No.of vertices= 4 and degree = 4 are equal

Vertices-degree	1	2	3	4
A	2	1	0	1
B	3	1	1	1
C	4	1	2	1
D	3	1	1	1

Table: 2.2 showing the square matrix

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Here given problem is equivalent.

Step-1: Considering the least element in each vertices and subtracting the same from that vetices **.**

Vertices-degree	1	2	3	4	
A	2	1	0	1	(-0)
B	2	0	0	0	(-1)
C	3	0	1	0	(-1)
D	2	0	0	0	(-1)

Table : 2.3 Reduced the row minimum

Step-2: Find out the each degree minimum element and subtract degree.

Vertices-degree	1	2	3	4
A	0	1	0	1
B	0	0	0	0
C	1	0	1	0
D	0	0	0	0
	(-2)	(-0)	(-0)	(-0)

Table : 2.4 Reduced the column minimum

Row wise & column wise assignment shown in table5.

Vertices-degree	1	2	3	4
A	**[0]**	1	0	1
B	0	0	**[0]**	0
C	1	**[0]**	1	0
D	0	0	0	**[0]**

Table :2.5 row wise and coloumn wise allotted one zero.

Step-4: Number of assignments = 4, number of rows = 4,Which is equal, so solution is optimal. Optimal assignments are

Vertices-degree	1	2	3	4
A	**[0]**	1	0	1
B	0	0	**[0]**	0
C	1	**[0]**	1	0
D	0	0	0	**[0]**

Table :2.6 Optimal cost allottedOptimal solution is

Work	**Job**	**Cost**
A	1	2
B	3	1
C	2	1
D	4	1
	Total	5

Table: 2.7 work are assign cost

Problem : 2 Find (a) Laplace Criterion (b) Criterion of Optimism (c) Minimax regret Criterion (d) Hurwicz Criterion

a)Laplace Criterion/Equal probability/Rationality-(Bayes Criterion)

Table: 2.8 Expected value for Baye's criterion

	v1	v2	v3	v4	Expected payoff	value
v1	2	1	0	1	4	1
v2	3	1	1	1	6	1.5
v3	4	1	2	1	8	2
v4	3	1	1	1	6	1.5
Maximum payoff						2

Result: v2=2 is Maximum payoff

v1=1 is minimum cost

b)Criterion of Optimism

(i) Maximax criterion

Table: 2.9 Value for Maximax criterion

Status	**v1**	**v2**	**v3**	**v4**	**Maximum**
v1	2	1	0	1	2
v2	3	1	1	1	3
v3	4	1	2	1	4
v4	3	1	1	1	3
Maximum value of payoff					4

v3=4 is maximum

(ii) Minimin criterion

Table: 2.10 Value for Minimin criterion

Status	**v1**	**v2**	**v3**	**v4**	**Minimum**
v1	2	1	0	1	0
v2	3	1	1	1	1
v3	4	1	2	1	1
v4	3	1	1	1	1
Maximum value of payoff					0

v1=0 is maximum

c) Minimax regret criterion

(i) maximization problem

Table: 2.11 Value for Maximization problem based regret criterion

Status	**v1**	**v2**	**v3**	**v4**	**Minimax**
v1	2	1	0	1	1
v2	3	1	1	1	2
v3	4	1	2	1	0
v4	3	1	1	1	1
Minimum value of payoff					0

v3=0 is minimum

(ii) minimization problem

Table: 2.12 Value for Minimization problem based regret criterion

Status	**v1**	**v2**	**v3**	**v4**	**Minimax**
v1	0	0	0	0	0
v2	1	0	1	0	1
v3	2	0	2	0	2
v4	1	0	1	0	1
Minimum value of payoff					0

v3=0 is minimum

d) Hurwicz Criterion

(i) maximization problem(0.6)

Table :2.13 Value for Maximization problem based Hurwicz criterion

Status	**v1**	**v2**	**v3**	**v4**	**Max**	**Min**	**WO**
v1	2	1	0	1	2	0	1.2
v2	3	1	1	1	3	1	2.2
v3	4	1	2	1	4	1	2.8
v4	3	1	1	1	3	1	2.2
Maximum value of payoff							2.8

V3=2.8 Maximum

(ii) minimization problem(0.5)

Table: 2.14 Value for Minimization problem based Hurwicz criterion

Status	**v1**	**v2**	**v3**	**v4**	**Min**	**Max**	**WO**
v1	2	1	0	1	0	2	1
v2	3	1	1	1	1	3	2
v3	4	1	2	1	1	4	2.5
v4	3	1	1	1	1	3	2
Minimum value of payoff							1

V1=1 is minimum

Problem 3. Find Solution of game theory problem using dominance method

Table 2.15 dominance rule

Player A\Player B	**B1**	**B2**	**B3**	**B4**
A1	2	1	0	1
A2	3	1	1	1
A3	4	1	2	1
A4	3	1	1	1

Solution:

Using dominance property

Table: 2.16 using dominance property

Players		Player B			
		B1	B2	B3	B4
Player A	A1	2	1	0	1
	A2	3	1	1	1
	A3	4	1	2	1
	A4	3	1	1	1

Step: 1 row-4 is dominated by row-3 (row-4 ≤ row-3), so row-4 is deleted, (A4≤A3:3≤4,1≤1,1≤2, 1≤1)

Table: 2.17 reduced row dominated and deleted A4

Players	Player B				
	-	B1	B2	B3	B4
Player A	A1	2	1	0	1
	A2	3	1	1	1
	A3	4	1	2	1

Step: 2 row-2 is dominated by row-3 (row-2 ≤ row-3), so row-2 is deleted, (A2≤A3:3≤4,1≤1,1≤2,1≤1)

Table: 2.18 reduced row dominated and deleted A2

Players	Player B				
	-	B1	B2	B3	B4
Player A	A1	2	1	0	1
	A3	4	1	2	1

Step:3 row-1 is dominated by row-2 (row-1 ≤ row-2), so row-1 is deleted, (A1≤A3:2≤4,1≤1,0≤2, 1≤1)

Table: 2.19 reduced row dominated and deleted A1

Players	Player B				
	-	B1	B2	B3	B4
Player A	A3	4	1	2	1

Step: 4 column-4 is dominated by column-2 (column-4 ≥ column-2), so column-4 is deleted. (B4≥B2:1≥1)

Table:2.20 Reduced column and deleted B4

Players	Player B			
	-	B1	B2	B3
Player A	A3	4	1	2

Step: 5 column-3 is dominated by column-2 (column-3 ≥ column-2), so column-3 is deleted. (B3≥B2:2≥1)

Table:2.21 Reduced column dominated and deleted B3

Players	Player B		
	-	B1	B2
Player A	A3	4	1

Step: 6 column-1 is dominated by column-2 (column-1 ≥ column-2), so column-1 is deleted. (B1≥B2:4≥1)

Table:2.22 Reduced column dominated deleted B1

Players	Player B	
	-	B2
Player A	A3	1

Result: Comparing the result in this frame matrix we get effective value in Game theory compared with assignment problem and decision under uncertaintly.

Problem.4 Then we can find the result for non symmetric problem for assignment problem, Decision under uncertaintly and Game theory in below Table :2.23

Table 2.23 non symmetrix matrix

Work\Job	1	2	3	4
A	3	0	2	1
B	3	2	0	1
C	4	2	1	1

Solution:

The number of rows = 3 and columns = 4

Table :2.24 unbalance matrix

	1	2	3	4
A	3	0	2	1
B	3	2	0	1
C	4	2	1	1

Here given problem is unbalanced and add 1 new row to convert it into a balance.

Table :2.25 balanced matrix

	1	2	3	4
A	3	0	2	1
B	3	2	0	1
C	4	2	1	1
W4	0	0	0	0

Step-1: Find out the each row minimum element and subtract it from that row

Table: 2.26 each row minimum

	1	2	3	4
A	3	0	2	1
B	3	2	0	1
C	3	1	0	0
W4	0	0	0	0

Step-2: Find out the each column minimum element and subtract it from that column.

Table: 2.27 Allocated matrix

	1	2	3	4
A	3	0	2	1
B	3	2	0	1
C	3	1	0	0
W4	0	0	0	0
	(-0)	(-0)	(-0)	(-0)

Step-3: Cover all zeros with a minimum number of lines

Determine the minimum number of lines, required to cover all zeros in the matrix. There are **4** lines required to cover all zeros, which is equal to size of matrix (**4**), so an optimal assignment exists and the algorithm stops.

Table :2.28 each row one allocated cells

	1	2	3	4
A	3	0	2	1
B	3	2	0	1

C	3	1	0	0
W4	0	0	0	0

The Optimal assignments are

Table: 2.29 There is one cell in each row alloacted matrix

	1	2	3	4
A	3	**[0]**	2	1
B	3	2	**[0]**	1
C	3	1	0	**[0]**
W4	**[0]**	0	0	0

Table 2.29**,** We get Optimal Solution

Table 2.30 Total cost

Work	Job	Cost
A	2	0
B	3	0
C	4	1
W4	1	0
	Total	1

Total cost of assignment problem is 1.

Problem : 5 Find (a) Laplace Criterion (b) Criterion of Optimism (c) Minimax regret Criterion (d) Hurwicz Criterion

a)Laplace Criterion/Equal probability/Rationality-(Bayes Criterion)

Table: 2.31 Expected value for Baye's criterion

	v1	v2	v3	v4	Expected payoff	value
v1	3	0	2	1	6	1.5
v2	3	2	0	1	6	1.5
v3	4	2	1	1	8	2
Maximum payoff						2

Result: v2=2 is Maximum payoff

v1=1.5 is minimum cost

b)Criterion of Optimism

(i) Maximax criterion

Table: 2.32 value for Maximax criterion

Status	**v1**	**v2**	**v3**	**v4**	**Maximum**
v1	3	0	2	1	2
v2	3	2	0	1	3
v3	4	2	1	1	4
Maximum value of payoff					4

v3=4 is maximum

(ii) Minimin criterion

Table: 2.33 Value for Minimin criterion

Status	**v1**	**v2**	**v3**	**v4**	**Minimum**
v1	3	0	2	1	0
v2	3	2	0	1	1
v3	4	2	1	1	1
Maximum value of payoff					0

v1=0 is maximum

c) Minimax regret criterion

(i) maximization problem

Table: 2.34 Value for Maximization problem based regret criterion

Status	**v1**	**v2**	**v3**	**v4**	**Minimax**
v1	3	0	2	1	1
v2	3	2	0	1	2
v3	4	2	1	1	0
Minimum value of payoff					0

v3=0 is minimum

(ii) minimization problem

Table: 2.35 Value for Minimization problem based regret criterion

Status	**v1**	**v2**	**v3**	**v4**	**maximin**
v1	3	0	2	1	3
v2	3	2	0	1	3
v3	4	2	1	1	4
Minimum value of payoff					4

v3=4is minimum

d) Hurwicz Criterion

(i) maximization problem(0.6)

Table :2.36 Value for Maximization problem based Hurwicz criterion

Status	**v1**	**v2**	**v3**	**v4**	**Max**	**Min**	**WO**
v1	3	0	2	1	3	0	1.2
v2	3	2	0	1	3	1	2.2
v3	4	2	1	1	4	1	2.8
Maximum value of payoff							2.8

V3=2.8 Maximum

(ii) minimization problem(0.5)

Table: 2.37 Value for Minimization problem based Hurwicz criterion

Status	**v1**	**v2**	**v3**	**v4**	**Min**	**Max**	**WO**
v1	3	0	2	1	0	3	1
v2	3	2	0	1	1	3	2
v3	4	2	1	1	1	4	2.5
Minimum value of payoff							1

V1=1 is minimum

Problem 6. Dominance rule to reduce the size of the payoff matrix Using dominance property in below table 2.23.

Table 2.23. Dominance rule

	Player B				
	-	B1	B2	B3	B4
Player A	A1	3	0	2	1
	A2	3	2	0	1
	A3	4	2	1	1

Step: 1 row-2 is dominated by row-3 (row-2 ≤ row-3), so row-2 is deleted, (A2≤A3:3≤4,2≤2,0≤1,1≤1)

Table 2.38 Reduced into payoff matrix

	Player B				
		B1	B2	B3	B4
Player A	A1	3	0	2	1
	A3	4	2	1	1

Step: 2 column-3 is dominated by column-4 (column-3 ≥ column-4), so column-3 is deleted. (B3≥B4:2≥1,1≥1)

Table: 2.39 reduced into 3 X 2 matrix

	Player B			
		B1	B2	B4
Player A	A1	3	0	1
	A3	4	2	1

Step: 3 column-1 is dominated by column-3 (column-1 ≥ column-3), so column-1 is deleted. (B1≥B4:3≥1,4≥1)

Table:2.40 reduced into 2 X 2 Matrix

	Player B		
		B2	B4
Player A	A1	0	1
	A3	2	1

Step: 4 row-1 is dominated by row-2 (row-1 ≤ row-2), so row-1 is deleted, (A1≤A3:0≤2,1≤1)

Table:2.41 reduced into payoff matrix

	Player B				
			B2	B4	
Player A	A3	**[**	2	1	**]**

Step: 5 column-1 is dominated by column-2 (column-1 ≥ column-2), so column-1 is deleted. (B2≥B4:2≥1)

Table :2.42 value of the game.

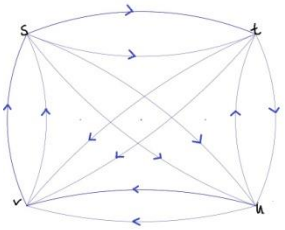
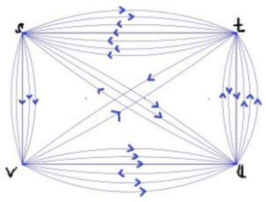
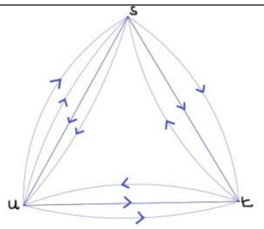
			Player B	
			B4	
Player A	A3	**[**	1	**]**

Result: Comparing the result in this frame nonsymmetrix matrix we get effective value in Game theory compared with assignment problem and decision under uncertaintly.

3. Comparative Study

In order to evaluate the performance efficacy of the all the three methods, viz., Game theory, graph theory and decision under uncertainty the following comparative study is done as listed in the Table.2.

Table.2. Comparative study

S.No.	Example	Directed graph frame the matrix	Decision under uncertainty					Game theory	Assignem nt cost
			Baye's probability	Minimax/maximin	wald	savage	Realism		
1.		$\begin{bmatrix} a\backslash b & b1 & b2 & b3 & b4 \\ a1 & 2 & 1 & 0 & 1 \\ a2 & 3 & 1 & 1 & 1 \\ a3 & 4 & 1 & 2 & 1 \\ a4 & 3 & 1 & 1 & 1 \end{bmatrix}$	Ad=2 Ex=1	Ad=4 Ex=0	Ad=1 Ex=2	Ad=0 Ex=0	Ad=2.8 Ex=1	V=1	5
2.		$\begin{bmatrix} a\backslash b & b1 & b2 & b3 & b4 \\ a1 & 3 & 0 & 1 & 1 \\ a2 & 5 & 1 & 0 & 1 \\ a3 & 2 & 1 & 1 & 0 \\ a4 & 3 & 1 & 1 & 2 \end{bmatrix}$	Ad=1.75 Ex=1	Ad=5 Ex=0	Ad=1 Ex=2	Ad=1 Ex=1	Ad=2.5 Ex=1	V=1	3
3.		$\begin{bmatrix} A/B & b1 & b2 & b3 \\ a1 & 3 & 1 & 2 \\ a2 & 3 & 2 & 1 \\ a3 & 4 & 2 & 2 \end{bmatrix}$	Ad=2.66 Ex=2	Ad=4 Ex=3	Ad=2 Ex=3	Ad=0 Ex=1	Ad=3.2 Ex=1.8	V=2	6

The efficacy of the two methods that have been developed to address the problem of resource allocation has been evaluated by applying them to sets of problems of varying sizes that have been produced at random. In order to carry out the evaluation according to the Table.2, three different sizes of RAP are taken into consideration.

In order to carry out the experiment, software is utilized. When applied to a diverse range of problems in engineering, economics, and management, MATLAB emerges as a potent tool. It is able to deal with a variety of computing algorithms within a fair amount of time. In other words, a wider variety of algorithms could be analyzed and tested in a shorter amount of time. The utilization of simulation results in the acquisition of more knowledge and insight, both of which may be utilized to improve the RAP solution.

All the algorithms were executed ten times, and the statistical data for each algorithm, including the optimal cost and optimal value was recorded, as shown in Table.3.

Table.3. Comparison for undirected Graph

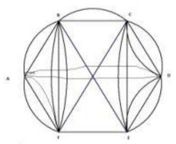
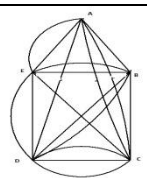
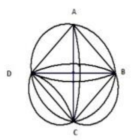
Graph	UnDirected graph frame the matrix	Decision under uncertainty					Game theory	Assignment problem
	degree	Baye's probability	Minimax/maximin	wald	savage	Realism	Optimal value	Optimal cost
	A 52 B 5 3 C 4 4 D 5 3 E 52 F 4 4	Ad=5 Ex=2	Ad=5 Ex=2	Ad=4 Ex=4	Ad=1 Ex=1	Ad=4.2 Ex=3.8	4	6
	A 4 2 B 4 2 C 5 2 D 5 3 E 4 3	Ad=4 Ex=3	Ad=5 Ex=4	Ad=3 Ex=4	Ad=0 Ex=0	Ad=4 Ex=3	3	6
	A 3 3 B 5 3 C 4 5 D 5 4	Ad=4.5 Ex=3	Ad=6 Ex=2	Ad=4 Ex=4	Ad=1 Ex=1	Ad=4.5 Ex=3	4	5

Table 3 is the solution to the symmetric and non-symmetric problems found in directed graphs and undirected graphs. The values of the variables are determined to be as follows: 2,1.1,1,1 as an indegree and 1.33,1,1,1.33 as an outdegree of GameTheory utilizing for direct graph in Graph as shown in Fig.1. The possible values for the parameters are determined to be 12 costs, 4 costs, 6 costs, and 6 costs as an assignment problem, as well as 2,4,4.5,4 as GameTheory utilizing for an undirected graph in Fig.2. In a comparison research that included a number of difficulties, the "Assignment problem and Game theory" solution was used to solve the problems, and the values that were obtained as a consequence are shown in the line chart and graph that are provided below in Fig.2.

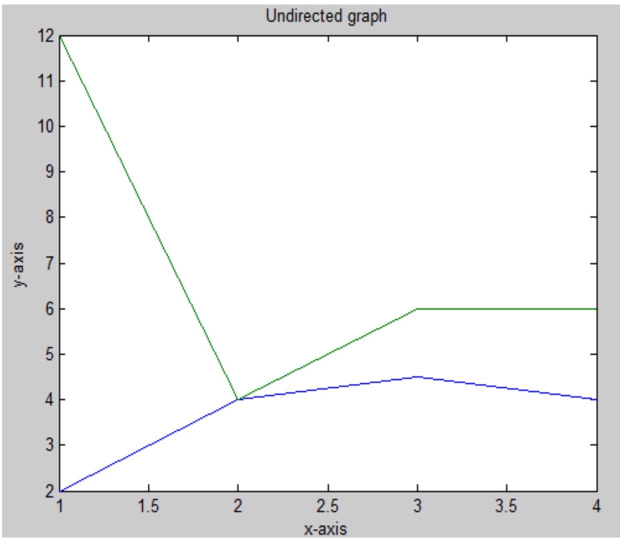


Fig.1. Graph generated for table.1

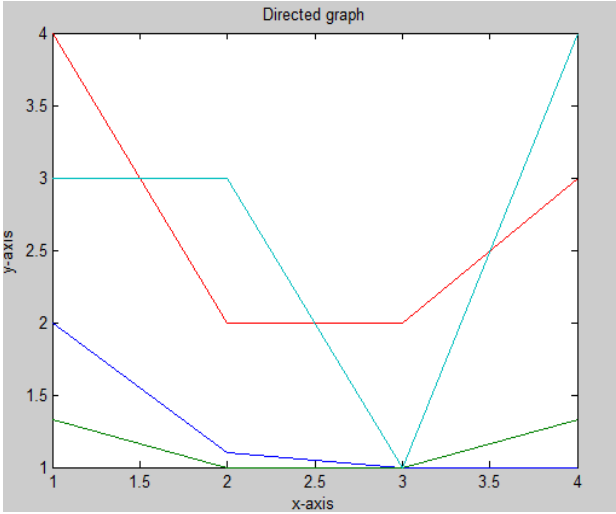


Fig.2. Graph generated for table.2

Fig 1 and Fig 2 are the respective graphs generated for the data in Table 1 and 2 using the Matlab simulation tool. Every simulation was run on a computer that had an Intel Core i7-4500 processor and 8 gigabytes of random access memory (RAM). Table.1 contains the performance parameters that have been analyzed. Both the size of the population and the number of iterations have been equalized for the purpose of making a fair comparison between the three different algorithms. (Mirjalili and Lewis, 2016)[13] suggested that the value of the coefficient (a) be set to 2, thus that is what was done. In the same manner, the value of the switch probability, denoted by the symbol p, has been set to 0.5 as per the recommendation made by Yang. (2012)[14].

4. Conclusion

Using direction graphs and undirected graphs, which both depend on graph theory, researchers may determine the best answer for the assignment problem and game theory by applying graph theory. By evaluating the indegree matrix with the outdegree matrix, researchers are able to identify the numerical outcome that is the closest to having the ideal cost. So that we can obtain the results of given the fact that one can structure matrices using directed graphs, indegree graph connections have a higher priority than outdegree graph connections. In the exact same manner, one can contrast directed graphs with undirected graphs. The results of the degree are available. Therefore, the takeaway of the present investigation is that a directed graph has fewer edges than an undirected graph. Coding in Matlab the researchers to draw graphs of both directed and undirected graphs. The novel approach involves generating ones within the assignment matrix and subsequently determining an assignment based on the presence of said ones. A significant number of methods have been presented for the assignment problem, among which the Hungarian Method is considered the most convenient. Furthermore, the paper presents a comparison of both methods. This paper endeavours to propose a novel approach for resolving the assignment problem that deviates from previous methodologies. The optimal value indicates that the game theory utilised in the Hungarian method is the most effective approach for resolving the issue of resource allocation.

Refrences

[1] Ammar, A., Pierreval, H., & Elkosentini, S. (2013, October). Workers assignment problems in manufacturing systems: A literature analysis. In Proceedings of 2013 international conference on industrial engineering and systems management (IESM) (pp. 1-7). IEEE.

[2] Bouajaja, S., & Dridi, N. (2017). A survey on human resource allocation problem and its applications. Operational Research, 17, 339-369.

[3] Burkard, R. E. (1999). Optimierung und Kontrolle: Selected Topics on Assignment Problems. Karl-Franzens-Universita" t Graz & Technische Universita" t Graz.

[4] Bisen, S. K. (2017). Application of Graph Theory in Transportation Networks. International Journal of Scientific research and Management, 5(07), 6197-6201.

[5] Ostadi, B., & Hamedankhah, R. (2021). A two-stage reliability optimization approach for solving series–parallel redundancy allocation problem considering the sale of worn-out parts. Annals of Operations Research, 304(1-2), 381-396.

[6] Lin, J. T., & Chiu, C. C. (2018). A hybrid particle swarm optimization with local search for stochastic resource allocation problem. Journal of Intelligent Manufacturing, 29(3), 481-495.

[7] Cattrysse, D. G., Salomon, M., & Van Wassenhove, L. N. (1994). A set partitioning heuristic for the generalized assignment problem. European Journal of Operational Research, 72(1), 167-174.

[8] Demiral, M. F. (2017, December). Ant colony optimization for a variety of classic assignment problems. In International Turkish World Engineering and Science Congress, Antalya.

[9] Suliman, A. S. A. (2019). Using ant colony algorithm to find the optimal assignment. AL-Anbar University journal of Economic and Administration Sciences, 11(25).

[10] Chu, P. C., & Beasley, J. E. (1997). A genetic algorithm for the generalised assignment problem. Computers & Operations Research, 24(1), 17-23.

[11] Jia, Z., & Gong, L. (2008, December). Multi-criteria human resource allocation for optimization problems using multi-objective particle swarm optimization algorithm. In 2008 International Conference on Computer Science and Software Engineering (Vol. 1, pp. 1187-1190). IEEE.

[12] Zamfir, C. (2008). Decision-Making under Persistent Uncertainty. A New Paradigm of Decision-Making and its Multiple Explanatory Capacities. Calitatea vieţii, 19(1-2), 185-192.

[13] Mirjalili, S., & Lewis, A. (2016). The whale optimization algorithm. Advances in engineering software, 95, 51-67.

[14] Yang, X. S. (2012). Flower pollination algorithm for global optimization. In Unconventional Computation and Natural Computation: 11th International Conference, UCNC 2012, Orléan, France, September 3-7, 2012. Proceedings 11 (pp. 240-249). Springer Berlin Heidelberg.

[15] Eneh, A. H., & Arinze, U. C. (2017). Comparative analysis and implementation of dijkstra's shortest path algorithm for emergency response and logistic planning. Nigerian Journal of Technology, 36(3), 876-888.

[16] Shirinivas, S. G., Vetrivel, S., & Elango, N. M. (2010). Applications of graph theory in computer science an overview. International journal of engineering science and technology, 2(9), 4610-4621.

[17] Attaway, S. (2013). Matlab: a practical introduction to programming and problem solving. Butterworth-Heinemann.

[18] Burkard, R. E. (1999). Optimierung und Kontrolle: Selected Topics on Assignment Problems. Karl-Franzens-Universita" t Graz & Technische Universita" t Graz.

[19] Balakrishnan, R., & Ranganathan, K. (2012). A textbook of graph theory. Springer Science & Business Media.