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A Study On Vibrations of Nanorod using Timoshenko Beam Theory Under the Influence of Magnetic Field

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Abstract:

The transverse vibrations of a thin graphene rod under the influence of a magnetic field are investigated. An analytical method is developed to solve the frequencies of the transversely vibrating graphene rod. Explicit results are obtained using Timoshenko beam theory as a function of wavenumber and phase velocity and numerical results are performed. It is observed that when the difference in magnetic field strength is minor, the influence of beam slenderness on frequencies becomes magnified. Finally, it is found that the hardness or softening impact of the beam model can be controlled by varying the field strength.

Keywords: Magnetic field, Timoshenko Beam, Vibrations, Phase Velocity, Frequency.

1. Introduction

Magnetic fields are particularly useful for generating colloidal nanostructured materials because they not only allow for controlled synthesis but also guide self-assembly into different superstructures. Elishakoff [1] discussed the beam theory with both the rotary inertia and shear deformation in addition to the shear correction factor, termed the Timoshenko-Ehrenfest beam theory. Su et al. [2] introduced a magnetothermal process for synthesizing hybrid nanostructures composed of ferrimagnetic magnetite nanorods coated with fluorescent perovskite nanocrystals, which self-assemble into superstructures emitting polarized light. Using nonlocal strain gradient theory and shear deformation, Bargozini et al. [3] examined that the sandwich composite beam reinforced with carbon nanorods synthesized from potato waste to increase the stiffness of the beam and buckling load is more economical than carbon nanotubes. Arda et al. [4] investigated the longitudinal wave propagation in axially graded nanotubes. The nonlocal Raylegh-Bishop rod theory was applied to investigate the influence of shear deformation and lateral inertia on nanorods. More recently, Nazemnezhad et al. [5] investigated the influence of functionally graded materials on internal resonances in nanorods

undergoing torsional vibration, incorporating von-Karman type nonlinearity and surface elasticity theory.

For the first time, Hosseini [6] developed a reduced-order nonlocal Timoshenko beam model for accurately modeling micro-elastic beams with layer-wise heterogeneous nonlocal effects. Ding et al. [7] developed a nonlocal elasticity model to examine size effects in a functionally graded (FG) nanorod under an axial magnetic field and torsional foundation. Zarezadeh et al. [8] investigated the nonlocal elasticity theory for capturing size effects in functionally graded (FG) nanorods under a magnetic field supported by a torsional foundation. When the material length scale parameter is set to zero, the model can transform into the classical model. Zhao et al. [9] analyzed the longitudinal vibration responses of a double rod system coupled with a nonlinear stiffness using Hamilton's principle and the Galerkin truncation method. The model reveals that nonlinear elements affect the system’s dynamic behavior. Novakovic et al. [10] discussed the stability and optimization of an axially loaded nonlocal beam with one end simply supported and the other elastically constrained against rotation. Singh et al. [11] constructed the variational formulation of the microstructure-dependent Timoshenko beam model using Hamilton's principle and modified couple stress theory (MCST) incorporated with the material length-scale parameter.

Junot et al. [12] explained that the magnetic nanorods in rotating fields transition from rolling to kayak-like motion at higher frequencies, generating localized hydrodynamic flows that enable efficient trapping and expulsion of microscopic cargoes. Hohenau et al. [13] studied the angle-invariant resonances of nano-rod dimers that can promote controlled application in surface enhanced spectroscopy or sensing. Cui et al. [14] discussed how the gold nanorods demonstrate the synchronized rotational motion at kilohertz frequencies when self-assembled in water using circularly polarized light, which is driven by strong optical interactions.

The article highlights that applying a magnetic field to structures increases their rigidity and stability. It also demonstrates that the application of a magnetic field can affect the stability of structures without altering their geometrical or material properties. The study of these effects on nanorods is in high demand, the extensive analysis and research have been conducted on the static stability of nanostructures under magnetic fields, no research has yet been done on the dynamic stability of Timoshenko beams with magnetic fields in the literature. Investigating the influence of magnetic fields on the dynamic instability of nanostructures plays an important role in optimizing the design of these structures. The numerical results illustrating dynamic instability behaviour are presented.

2. Formulation of the Problem

Consider a graphene rod based on Timoshenko beam model subjected to external magnetic force as shown in the Figure 1. For finite element analysis the Timoshenko beam model gives the stiffness matrix that includes both bending and shear stiffness for more accurate modelling in deflections.

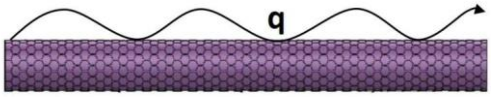

Figure 1.: A Graphene rod subjected to magnetic field

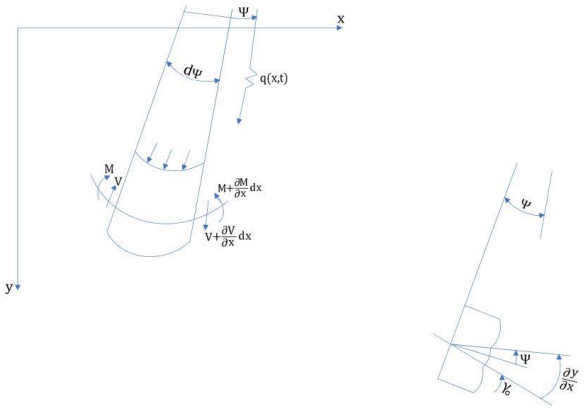

Figure 2.: Cartesian co-ordinate of rod subjected to external force and additional deformations.

During the bending, the plane is perpendicular to the horizontal axis. The relation between the moment of bending and curvature is as follows

$$\frac{\partial y}{\partial x} = \frac{\psi}{\gamma_o} \qquad [2.1.1]$$

where y is the displacement to the centroidal plane, x is the axial coordinate, t is the time, ψ is the effect of bending and γ_o is the shear effects. The expressions for bending moment is given by

$$\frac{M}{EI} = \frac{\partial \psi}{\partial x} \qquad [2.1.2]$$

Where M denotes moment of bending, E denotes Youngs Modulus and I is the moment of inertia. Shear strain γ and shear modulus G are applied for expressing the shear force V at the cross section, respectively as

$$V = G\gamma_o A$$

Where A represents the cross sectional area. Since γ_o represents the centroidal axis for the shear strain, then $G\gamma_o A$ is the shear force. The adjustment coefficient κ is introduced to balance the equation and is given by

$$V = GA\kappa\left(\frac{\partial y}{\partial x} - \psi\right) \qquad [2.1.3]$$

Equation of motion in the vertical direction for the element is of the form

$$\frac{\partial V}{\partial x} + q = \rho A \frac{\partial^2 y}{\partial t^2} \qquad [2.1.4]$$

Where ρ is the element of mass density and q is the external force. Summing moments about axis that is perpendicular to x,y-plane and passing through the element centre, we get

$$V - \frac{\partial M}{\partial x} = \rho I \frac{\partial^2 \psi}{\partial t^2} \quad [2.1.5]$$

Substituting the bending moment Eqn.[2.1.2] and shear force Eqn. [2.1.3] into Eqns.[2.1.4] and [2.1.5], the equations of the motion becomes

$$GA\kappa\left(\frac{\partial \psi}{\partial x} - \frac{\partial^2 y}{\partial x^2}\right) + \rho A \frac{\partial^2 y}{\partial t^2} = q(x,t)$$

$$GA\kappa\left(\frac{\partial y}{\partial x} - \psi\right) + EI \frac{\partial^2 \psi}{\partial t^2} = \rho I \frac{\partial^2 \psi}{\partial t^2} \quad [2.1.6]$$

Magnetic force is exerted by a magnetic field on moving charges in magnetic materials acting perpendicular to the velocity of the magnetic interactions. Due to magnetic effects [15], the constant axial force q is given by

$$q = \rho A \eta H_x^2 \frac{\partial^2 y}{\partial x^2} \quad [2.1.7]$$

where η is the magnetic field strength and the H_x is the magnetic permeability. Substituting the Eqns.[2.1.8] in [2.1.6], the governing equations of the motion for Timoshenko beam theory of the graphene rod becomes

$$GA\kappa\left(\frac{\partial \psi}{\partial x} - \frac{\partial^2 y}{\partial x^2}\right) + \rho A \frac{\partial^2 y}{\partial t^2} = \rho A \eta H_x^2 \frac{\partial^2 y}{\partial x^2}$$

$$GA\kappa\left(\frac{\partial y}{\partial x} - \psi\right) + EI \frac{\partial^2 \psi}{\partial t^2} = \rho I \frac{\partial^2 \psi}{\partial t^2} \quad [2.1.8]$$

3. Solution of the Problem

Distribution of waves in graphene nanorod under a magnetic field is studied by considering the harmonic wave in the infinite rod. Assuming the solutions in the form

$$y = B_1 e^{i(\gamma x - \omega t)}$$

$$\psi = B_2 e^{i(\gamma x - \omega t)} \quad [3.1.1]$$

Where B_1 and B_2 are amplitudes, γ is the wavenumber and ω is the frequency. Substituting the harmonic solution Eqn[3.1.1] in the equation of motion Eqn[2.1.8],we get

$$(GA_\kappa\gamma^2 - \rho A\omega^2 - \rho A\eta H_x^2\gamma^2)B_1 + iGA_\kappa\gamma B_2 = 0 \quad [3.1.2]$$

$$iGA_\kappa\gamma B_1 - (GA_\kappa - \rho I\omega^2 + EI\gamma^2)B_2 = 0 \quad [3.1.3]$$

Eqns(3.1.2)-(3.1.3)can be written in the form of matrix as

$$\begin{bmatrix} (GA_\kappa\gamma^2 - \rho A\omega^2 + \rho A\eta H_x^2\gamma^2) & iGA_\kappa\gamma \\ iGA_\kappa\gamma & -(GA_\kappa - \rho I\omega^2 + EI\gamma^2) \end{bmatrix}\begin{bmatrix} B_1 \\ B_2 \end{bmatrix} = [0] \quad [3.1.4]$$

A trivial solution is obtained by solving the matrix given in Eqn.(3.1.4), so as to achieve a significant solution compare the coefficient of the determinant arrangement to nonexistent as follows

$$\begin{vmatrix} (GA_\kappa\gamma^2 - \rho A\omega^2 + \rho A\eta H_x^2\gamma^2) & iGA_\kappa\gamma \\ iGA_\kappa\gamma & -(GA_\kappa - \rho I\omega^2 + EI\gamma^2) \end{vmatrix} = 0 \qquad [3.1.5]$$

By solving the determinant given in Eqn.(3.1.5), a fourth order frequency equation is obtained in the form

$$\frac{EI}{\rho A}\left(1 + \frac{\eta H_x^2}{G_\kappa}\right)\gamma^4 - \frac{I}{A}\left(1 + \frac{\eta H_x^2}{G_\kappa} + \frac{E}{G_\kappa}\right)\gamma^2\omega^2 - \omega^2 + \frac{\rho I}{GA_\kappa}\omega^4 - \frac{\eta H_x^2\gamma^2}{\rho} = 0 \qquad [3.1.6]$$

Eqn.(3.1.6) represents the relationship between the frequency and wavenumber of the graphene nanorod based on Timoshenko beam theory. Letting the wave propagation's phase velocity $C = \frac{\omega}{\gamma}$, the disperive characteristics of the nanorod are analyzed and introducing the non dimensional quantities such as $\varepsilon = \frac{E}{G_\kappa}, \bar{M} = \frac{\eta H_x^2}{G_\kappa}$ and $\bar{\gamma} = \frac{I\gamma^2}{A}$, then Eqn(3.1.6) becomes,

$$\varepsilon\bar{\gamma}\,\bar{c}^4 - (\bar{\gamma}(1 + \varepsilon + \bar{M}) - 1)\bar{c}^2 + \bar{\gamma}(1 + \bar{M}) - \bar{M} = 0 \qquad [3.1.7]$$

Using non dimensional term $\bar{\omega} = \frac{A\omega^2}{C_O^2}, \bar{I} = \frac{A^2}{I}$ in eqn(3.1.7), we obtain the frequency equation of nanorod under the effect of magnetic field as interms of $\bar{\omega}$.

$$\bar{\omega}^2 - \frac{\bar{\gamma}(1+\varepsilon+\bar{M})\bar{\omega}}{\varepsilon} - \frac{\bar{\gamma}^2(1+\bar{M})}{\varepsilon} - \frac{\bar{I}\bar{M}}{\varepsilon^2} = 0 \qquad [3.1.8]$$

4. Numerical Results

The frequency equation is derived using beam theory and parameters of the beam are taken in the order from Davies[16] . The parameters as ε = 20/9(1 + σ), Where σ = 0.29 is poison ratio.

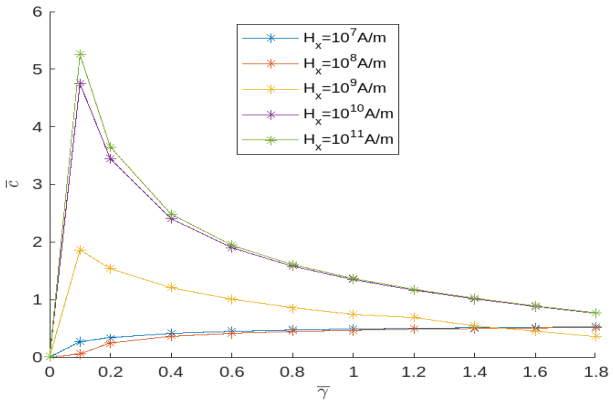

Figure 3.: Scattered relation of dimensionless phase speed as a function of non-dimensional wavenumber of nanorod under the effect of magnetic field for lower modes of vibrations.

Dispersion curve for nanorod under different magnetic field for the relation phase speed and wavenumber is drawn and is shown in Figure 3. From the Figure 3, it is observed that the magnetic field can influence the material's permittivity and permeability. This changes the

dispersion relation of the waves traveling through the material. In some materials, the presence of a magnetic field induces changes in the refractive index, which modifies the wave's velocity. In mechanical or elastic waves, a magnetic field may cause shifts in the elastic properties of materials, which also affects the propagation speed.

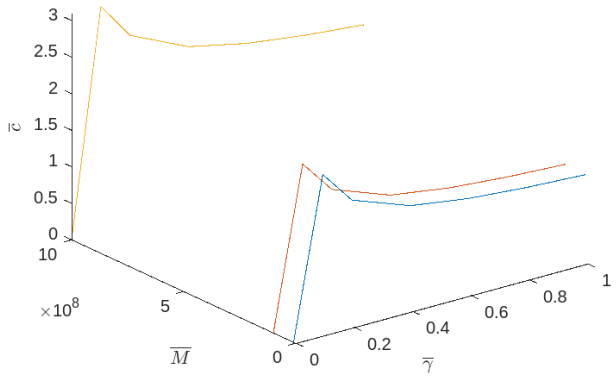

Figure 4.: Scattered relation of dimensionless phase speed as a function of non-dimensional wavenumber of nanorod under the effect of magnetic field for higher modes of vibration.

Dispersion curve for nanorod under different magnetic field for the relation phase speed and wavenumber is drawn and is shown in Figure 4. From the Figure 4, it is observed that the wavenumber increases the magnetic field strength has a noticeable influence by increasing the phase velocity. However, when increasing the wavenumbers, the phase velocity gets accelerates. It suggests that magnetic fields have a greater impact on wave propagation at longer wavelengths, which could be useful in many different kinds of applications that influences the wave speed.

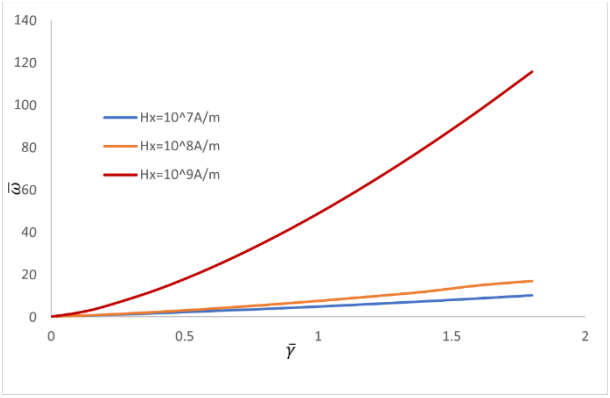

Figure 5: Dispersion curve of nanorod for dimensionless wavenumber and under different magnetic field for higher modes of vibrations.

Dispersion curve for nanorod under different magnetic field for higher modes is drawn and is shown in Figure 5. From the Figure 5, it is observed that the frequency increases linearly with increasing the wavenumber for $H_x = 10^7\ A/m\ and\ H_x = 10^8\ A/m$. When a magnetic field interacts with an elastic medium, it can combine with mechanical waves including the shear or longitudinal waves to produce magnetoelastic waves. As the magnetic field increases the waves propagates faster which sharp slope at $H_x = 10^9 A/m$. This indicates that the magnetic field

has a substantial influence on the frequency of waves as the wavenumber increases. The impact is week in low magnetic fields but it is strong in high magnetic fields, the frequency increases fast with wavenumber, indicating substantial magneto-mechanical coupling. This has prominent result for materials and systems that use magnetic fields to control wave propagation especially in advanced engineering applications such as nanodevices. This result can be useful for creating waveguides whenever the wave propagation speed is significant. By generating a magnetic field, the phase speed can be modulated for specific applications, potentially influencing wave transmission or reflection across a medium.

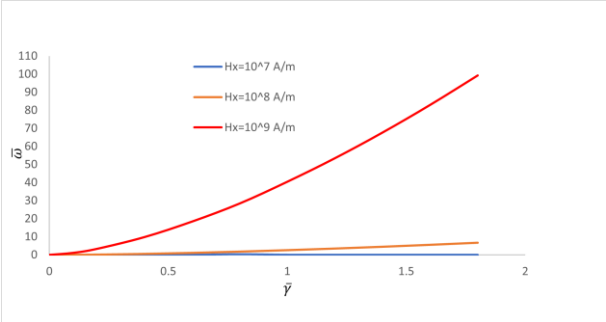

Figure 6: Dispersion curve for dimensionless wavenumber under different magnetic field for lower modes of vibrations.

Dispersion curve for nanorod under different magnetic field for lower modes is drawn and is shown in Figure 6. From the Figure 6, it is observed that the response of the system is proportional to the wavenumber. For $H_x = 10^9 A/m$, the magnetic fields make the system more sensitive to vibrations. The response develops rapidly as the wavenumber increases, this indicating the nonlinearity. Stronger the magneto-mechanical coupling at high magnetic fields, the material's magnetic fields or internal structures may converge, substantially enhancing the mechanical responses might take place in ferromagnetic materials or other systems where the magnetic field directly changes the structure at the microscopic level.This implies that larger the magnetic fields, there is a strong connection between the magnetic field and the material's mechanical properties possibly due to material softening.

5. Conclusion

In this study, the vibration of nanorod with magnetic field effects for Timoshenko beam theory are examined. The scattered relations between the phase velocity and frequency is derived by finding the solution in matrix form. The frequency equations are determined by solving the governing equations of motion. The magnetic field is found to have a significant impact on the stability of the nanorod and nanosystem. This study's findings can be utilized to design and manufacture double nano-micromechanical systems for advanced applications

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