Advances in Nonlinear Variational Inequalities
ISSN: 1092-910X
Vol 28 No. 4s (2025)

Neutrosophic Nano Topology: An Analysis of Neut.nano αg#ψ-Open and Closed Functions and Their Properties

[1]T.Nandhini, [2]Dr M Nazreen Banu, [3]Dr. D. Savithiri, [4]Dr.J.Pandima Devi, [5]R. Rajalakshmi , [6]Dr.G. Suganthi, [7]Dr.R.Brindha

[1]Assistant Professor, Department of mathematics,Karpagam Institute of Technology, Coimbatore-641 105, Tamil Nadu, India. [1]Email ID: nandhini.mat@karpagamtech.ac.in

[2]Associate Professor, Department of Mathematics,Thassim Beevi Abdul Kader College for Women, Ramanathapuram.[2]Email ID: nazreenmathematics@gmail.com

,[3]Assistant Professor, Sree Narayana Guru College, Coimbatore.[3]Email ID : savithirisngcmat@gmail.com

[4]Assistant Professor, Department of Mathematics, NPR College of Engineering and Technology,Natham

[4]Email ID:sabarimadhu84@gmail.com

[5]Assistant Professor, Department of Mathematics, Panimalar Engineering College,Chennai

[5]Email ID: rajimat2020@gmail.com

[6]Assistant professor, Department of Mathematics, Sona College of Technology, Salem,

[6]Email ID:suganthig@sonatech.ac.in

[7]Assistant Professor, Department of Mathematics , Mahendra Engineering College, Namakkal

[7]Corresponding Author Email ID: brindhaaramasamy@gmail.com

Article History:

Received: *17-10-2024*

Revised: *26-11-2024*

Accepted: *03-12-2024*

Abstract:

This article provides an in-depth introduction to the fundamental concepts of neutrosophic nano αg^# ψ -open and closed functions within the context of neutrosophic nano topological spaces. These concepts play a significant role in understanding the intricate relationships and behaviors in this specialized field of topology. In addition to presenting these definitions, the article also delves into a broader discussion, exploring the various properties and implications associated with these functions. This extended examination aims to offer readers a comprehensive view of the subject, highlighting potential applications and theoretical advancements that arise from studying these unique functions within neutrosophic nano topological spaces

Keywords: Neutrosopic topology, neutrosophic nano αg^# ψ-open function; neutrosophic nano αg^# ψ-closed function;

1 Introduction

Rough set theory was extended to add Nano topology, which was introduced by Lellis Thivagar[6] in 2013. Additionally, he proposed Nano topological spaces, which were defined by an equivalency relation on a subset of the universe. The terms N-open set and N-closed sets refer to the constituents of a Nano topological space and their complements, respectively. Nano refers to an extremely small scale. The study of extremely small surfaces is thus literally defined as nano topology. The concepts of indiscernibility relation and approximations are the cornerstones of nano topology. Following that, in 2018 Lellis Thivagar[7] investigated a novel idea of neutrosophic nano topological space (NNTS). The topics he covered in that work were neut. nano closure and interior. Neutosophic $\alpha g^{\#}\psi$-closed sets were presented and some basic characteristics were investigated by Vigneshwaran et al. in 2019. They expanded this idea to include neutrosophic nano topological spaces and neutrosophic nano $\alpha g^{\#}\psi$-closed sets. (NNTSs). Here, we define and establish some basic features of neut. nano $\alpha g^{\#}\psi$-open and neutrosophic nano $\alpha g^{\#}\psi$-closed functions within neut. nano topological spaces (NNTSs). The initial definitions are mentioned in 1,7,9,10 and 12.

2 Neutrosophic Nano $\alpha g^{\#}\psi$-open function

Definition 2.1 *A func $\mathcal{T}:(\mathcal{X},\eta) \to (\mathcal{Y},\upsilon)$ is neut. nano $\alpha g^{\#}\psi$-open($\mathcal{N}_{\mathcal{N}-\alpha g^{\#}\psi}$-OF) if image of every neut. nano open set($\mathcal{NN}$ $-$OS) of $(\mathcal{P},\gamma)$ is neut. nano $\alpha g^{\#}\psi$-open set($\mathcal{N}_{\mathcal{N}-\alpha g^{\#}\psi}$-OS) in $(\mathcal{Y},\upsilon)$.*

Theorem 2.2 *Each neut. nano open func($\mathcal{NN}$ $-$OF) is a $\mathcal{N}_{\mathcal{N}-\alpha g^{\#}\psi}$-OF.*

Proof: Assume $\mathcal{T}:(\mathcal{X},\eta) \to (\mathcal{Y},\upsilon)$ is $\mathcal{NN}$ $-$OF, thus and so $\mathcal{J}$ is a $\mathcal{NN}$ $-$OS in $(\mathcal{X},\eta)$. Since $\mathcal{T}$ is a $\mathcal{NN}$ $-$OF, $\mathcal{T}(\mathcal{J})$ is $\mathcal{NN}$-OS in $(\mathcal{Y},\upsilon)$. We know $\mathcal{NN}$ $-$OS is $\mathcal{N}_{\mathcal{N}-\alpha g^{\#}\psi}$-OS. Therefore, $\mathcal{T}(\mathcal{J})$ is a $\mathcal{N}_{\mathcal{N}-\alpha g^{\#}\psi}$-OS in $(\mathcal{Y},\upsilon)$. Hence, $\mathcal{T}$ is a $\mathcal{N}_{\mathcal{N}-\alpha g^{\#}\psi}$-OF.

The below example shows $\mathcal{N}_{\mathcal{N}-\alpha g^{\#}\psi}$-OF need not be a $\mathcal{NN}$ $-$OF .

Example 2.3 *Let $\mathcal{U}_1 = \{\alpha,\beta,\lambda\}$ and the equivalence relation is $\mathcal{U}_1/\mathcal{R}_1 = \{\{\alpha,\lambda\},\{\lambda\}\}$. Let*

$$\mathcal{A}_1 = \{\langle\alpha,(4/10,4/10,3/10)\rangle,\langle\beta,(3/10,4/10,2/10)\rangle,\langle\lambda,(4/10,3/10,4/10)\rangle\}$$

be neut. nano subset of $\mathcal{U}_1$. Then

$$\underline{\mathcal{N}}(\mathcal{A}_1) = \{\langle\alpha,(3/10,4/10,3/10)\rangle,\langle\beta,(3/10,4/10,3/10)\rangle,\langle\lambda,(4/10,3/10,4/10)\rangle\}$$

$$\overline{\mathcal{N}}(\mathcal{A}_1) = \{\langle\alpha,(4/10,4/10,2/10)\rangle,\langle\beta,(4/10,4/10,2/10)\rangle,\langle\lambda,(4/10,3/10,4/10)\rangle\}$$

$$\mathcal{B}(\mathcal{A}_1) = \{\langle\alpha,(2/10,4/10,4/10)\rangle,\langle\beta,(2/10,4/10,4/10)\rangle,\langle\lambda,(4/10,3/10,4/10)\rangle\}$$

Then $\tau 1_{NN} = \{0_N,\underline{\mathcal{N}}(\mathcal{A}_1),\overline{\mathcal{N}}(\mathcal{A}_1),\mathcal{B}(\mathcal{A}_1),1_N\}$ be neut. nano topology on $(\mathcal{X},\eta)$ and let $\mathcal{U}_2 = \{\alpha,\beta,\lambda\}$ and the equivalence relation is $\mathcal{U}_2/\mathcal{R}_2 = \{\{\alpha,\lambda\},\{\lambda\}\}$.

Let

$$\mathcal{A}_2 = \{\langle\alpha,(5/10,4/10,3/10)\rangle,\langle\beta,(3/10,5/10,2/10)\rangle,\langle\lambda,(4/10,3/10,4/10)\rangle\}$$

be neut. nano subset of $\mathcal{U}_2$. Then

$\underline{\mathcal{N}}(\mathcal{A}_2) = \{\langle\alpha,(3/10,4/10,3/10)\rangle,\langle\beta,(3/10,4/10,3/10)\rangle,\langle\lambda,(4/10,3/10,4/10)\rangle\}$

$\overline{\mathcal{N}}(\mathcal{A}_2) = \{\langle\alpha,(4/10,4/10,2/10)\rangle,\langle\beta,(4/10,4/10,2/10)\rangle,\langle\lambda,(4/10,3/10,4/10)\rangle\}$

$\mathcal{B}(\mathcal{A}_2) = \{\langle\alpha,(2/10,4/10,4/10)\rangle,\langle\beta,(2/10,4/10,4/10)\rangle,\langle\lambda,(4/10,3/10,4/10)\rangle\}$

Then $\tau 2_{NN} = \{0_N, \underline{\mathcal{N}}(\mathcal{A}_2), \overline{\mathcal{N}}(\mathcal{A}_2), \mathcal{B}(\mathcal{A}_2), 1_N\}$ be neut. nano topology on $(\mathcal{Y},\upsilon)$

Define $\mathcal{T}:(\mathcal{X},\eta)\to(\mathcal{Y},\upsilon)$ by $\mathcal{T}(\alpha)$=α, $\mathcal{T}(\beta)$=β, $\mathcal{T}(\lambda)$=λ.

$\mathcal{N}_{\mathcal{N}-\alpha g^{\#}\psi}$-OS of $(\mathcal{Y},\upsilon)$ = $\langle\alpha,(3/10,4/10,3/10)\rangle,\langle\beta,(3/10,4/10,3/10)\rangle,\langle\lambda,(4/10,3/10,4/10)\rangle\}$.

Here $\mathcal{T}(N_{A_1})$ is $\mathcal{N}_{\mathcal{N}-\alpha g^{\#}\psi}$-open.

Then $\mathcal{T}$ is $\mathcal{N}_{\mathcal{N}-\alpha g^{\#}\psi}$-OF. However, it is not a $\mathcal{N}\mathcal{N}$ $-$OF because $\mathcal{T}(N^c_{A_1})$ is not $\mathcal{N}\mathcal{N}$-OS in $(\mathcal{Y},\upsilon$.

Theorem 2.4 *A func $\mathcal{T}:(\mathcal{X},\eta)\to(\mathcal{Y},\upsilon)$ is $\mathcal{N}_{\mathcal{N}-\alpha g^{\#}\psi}$-open iff for each NNS $\mathcal{C}$ of $(\mathcal{X},\eta)$, $\mathcal{T}(H^\circ(\mathcal{A}_1)) \subseteq \mathcal{N}_{\alpha g^{\#}\psi}(H^\circ(\mathcal{T}(\mathcal{A}_1))$.*

Proof: Let $\mathcal{T}$ be a $\mathcal{N}_{\mathcal{N}-\alpha g^{\#}\psi}$-OF and $\mathcal{A}_1$ is a $\mathcal{N}\mathcal{N}$ $-$OS in $(\mathcal{X},\eta)$. Now, $H^\circ(\mathcal{A}_1)\subseteq\mathcal{A}_1$ which implies that $\mathcal{T}(H^\circ(\mathcal{A}_1))\subseteq\mathcal{T}(\mathcal{A}_1)$. Since $\mathcal{T}$ is a $\mathcal{N}_{\mathcal{N}-\alpha g^{\#}\psi}$ -OF, $\mathcal{T}(H^\circ(\mathcal{A}_1))$ is $\mathcal{N}_{\mathcal{N}-\alpha g^{\#}\psi}$ -OS in $(\mathcal{Y},\upsilon)$ thus and so $\mathcal{T}(H^\circ(\mathcal{A}_1))\subseteq\mathcal{T}(\mathcal{A}_1)$ therefore $\mathcal{T}(H^\circ(\mathcal{A}_1))\subseteq \mathcal{N}_{\alpha g^{\#}\psi}(H^\circ\mathcal{T}(\mathcal{A}_1))$.

Suppose $\mathcal{A}_1$ is an $\mathcal{N}\mathcal{N}$ $-$OS of $(\mathcal{X},\eta)$. Then $\mathcal{T}(\mathcal{A}_1)=\mathcal{T}(H^\circ(\mathcal{A}_1))\subseteq\mathcal{N}_{\alpha g^{\#}\psi}(H^\circ\mathcal{T}(\mathcal{A}_1))$. But neut. nano $\alpha g^{\#}\psi(H^\circ(\mathcal{T}(\mathcal{A}_1)))\subseteq\mathcal{T}(\mathcal{A}_1)$. Consequently $\mathcal{T}(\mathcal{A}_1)=\mathcal{N}_{\alpha g^{\#}\psi}(H^\circ(\mathcal{A}_1))$ which implies that $\mathcal{T}(\mathcal{A}_1)$ is a $\mathcal{N}_{\mathcal{N}-\alpha g^{\#}\psi}$-OS of $(\mathcal{Y},\upsilon)$. Hence $\mathcal{T}$ is a $\mathcal{N}_{\mathcal{N}-\alpha g^{\#}\psi}$-open.

Theorem 2.5 *If $\mathcal{T}:(\mathcal{X},\eta)\to(\mathcal{Y},\upsilon)$ is a $\mathcal{N}_{\mathcal{N}-\alpha g^{\#}\psi}$-OF then $H^\circ(\mathcal{T}^{-I}(G))\subseteq \mathcal{T}^{-I}(\mathcal{N}_{\alpha g^{\#}\psi}(H^\circ(G))$ for every NNS $\mathcal{G}$ of $(\mathcal{Y},\upsilon)$.*

Proof: Let $\mathcal{G}$ is a NNS of $(\mathcal{Y},\upsilon)$. Then $H^\circ(\mathcal{T}^{-I})(\mathcal{G})$ is a $\mathcal{N}\mathcal{N}$ $-$OS in $(\mathcal{X},\eta)$. Since $\mathcal{T}$ is $\mathcal{N}_{\mathcal{N}-\alpha g^{\#}\psi}$-open $\mathcal{T}(H^\circ(\mathcal{T}^{-I}(\mathcal{G})))$ is $\mathcal{N}_{\mathcal{N}-\alpha g^{\#}\psi}$-open in $(\mathcal{Y},\upsilon)$ and hence $\mathcal{T}(H^\circ(\mathcal{T}^{-I}(\mathcal{G})))\subseteq \mathcal{N}_{\alpha g^{\#}\psi}(H^\circ(\mathcal{T}(\mathcal{T}^{-I}(\mathcal{G})))\subseteq\mathcal{N}_{\alpha g^{\#}\psi}(H^\circ(\mathcal{G}))$. Thus $H^\circ(\mathcal{T}^{-I}(\mathcal{G}))\subseteq\mathcal{T}^{-I}(\mathcal{N}_{\alpha g^{\#}\psi}(H^\circ(\mathcal{G})))$.

Theorem 2.6 *A func $\mathcal{T}:(\mathcal{X},\eta)\to(\mathcal{Y},\upsilon)$ is $\mathcal{N}_{\mathcal{N}-\alpha g^{\#}\psi}$-open iff for each NNS $\mathcal{F}$ of $(\mathcal{Y},\upsilon)$ and for each $\mathcal{N}\mathcal{N}$-OF $\mathcal{A}_1$ of $(\mathcal{X},\eta)$ containing $\mathcal{T}^{-I}(\mathcal{F})$ there is a $\mathcal{N}_{\mathcal{N}-\alpha g^{\#}\psi}$-CS $\mathcal{B}$ of $(\mathcal{Y},\upsilon)$ thus and so $\mathcal{F}\subseteq\mathcal{B}$ and $\mathcal{T}^{-I}(\mathcal{B})\subseteq\mathcal{A}_1$.*

Proof: Suppose $\mathcal{T}$ is a $\mathcal{N}_{\mathcal{N}-\alpha g^{\#}\psi}$-OF. Let $\mathcal{F}$ be the $\mathcal{N}\mathcal{N}$-OF of $(\mathcal{Y},\upsilon)$ and $\mathcal{A}_1$ is a $\mathcal{N}\mathcal{N}$-OF of $(\mathcal{X},\eta)$ thus and so $\mathcal{T}^{-I}(\mathcal{F})\subseteq\mathcal{A}_1$. Then $\mathcal{B}=(\mathcal{T}^{-I}(\mathcal{A}_1^c))^c$ is $\mathcal{N}_{\mathcal{N}-\alpha g^{\#}\psi}$-CS of $(\mathcal{Y},\upsilon)$ thus and so $\mathcal{T}^{-I}(\mathcal{B}) \subseteq \mathcal{A}_1$.

Suppose $\mathcal{G}$ is a $\mathcal{NN}$ −OS of $(\mathcal{X},\eta)$. Then $\mathcal{T}^{-I}((\mathcal{T}(\mathcal{G}))^c \subseteq \mathcal{G}^c$ and $\mathcal{G}^c$ is $\mathcal{NN}$-OF in $(\mathcal{X},\eta)$. By hypothesis $\mathcal{N}_{\mathcal{N}-\alpha g^{\#}\psi}$ -CS $\mathcal{B}$ of $(\mathcal{Y},\upsilon)$ thus and so $(\mathcal{T}(\mathcal{G}))^c \subseteq \mathcal{B}$ and $\mathcal{T}^{-I}(\mathcal{B}) \subseteq \mathcal{G}^c$. Therefore $\mathcal{G} \subseteq (\mathcal{T}^{-I}(\mathcal{B}))^c$. Hence $\mathcal{B}^c \subseteq \mathcal{T}(\mathcal{G}) \subseteq \mathcal{T}((\mathcal{T}^{-I}(\mathcal{B}))^c) \subseteq \mathcal{B}^c$ then $\mathcal{T}(\mathcal{G}) = \mathcal{B}^c$. Since $\mathcal{A}^c$ is $\mathcal{N}_{\mathcal{N}-\alpha g^{\#}\psi}$-OS of $(\mathcal{Y},\upsilon)$. Hence $\mathcal{T}(\mathcal{G})$ is $\mathcal{N}_{\mathcal{N}-\alpha g^{\#}\psi}$-open in $(\mathcal{Y},\upsilon)$ and thus $\mathcal{T}$ is $\mathcal{N}_{\mathcal{N}-\alpha g^{\#}\psi}$-OF.

Theorem 2.7 *If $\mathcal{T}:(\mathcal{X},\eta) \to (\mathcal{Y},\upsilon)$ and $\mathcal{S}:(\mathcal{Y},\upsilon) \to (\mathcal{Z},\omega)$ be two neut. nano funcs and $\mathcal{S} \circ \mathcal{T}:(\mathcal{X},\eta) \longrightarrow (\mathcal{Z},\omega)$ is $\mathcal{N}_{\mathcal{N}-\alpha g^{\#}\psi}$-open. If $\mathcal{S}:(\mathcal{Y},\upsilon) \to (\mathcal{Z},\omega)$ is $\mathcal{N}_{\mathcal{N}-\alpha g^{\#}\psi}$-irresolute then $\mathcal{T}:(\mathcal{X},\eta) \to (\mathcal{Y},\upsilon)$ is $\mathcal{N}_{\mathcal{N}-\alpha g^{\#}\psi}$-OF.*

Proof: Let $\mathcal{H}$ be an $\mathcal{NN}$ −OS of NNTS $(\mathcal{X},\eta)$. Then $(\mathcal{S} \circ \mathcal{T})(\mathcal{H})$ is $\mathcal{N}_{\mathcal{N}-\alpha g^{\#}\psi}$-OS of $(\mathcal{Z},\omega)$ because $\mathcal{S} \circ \mathcal{T}$ is $\mathcal{N}_{\mathcal{N}-\alpha g^{\#}\psi}$-OF. Now since $\mathcal{S}:(\mathcal{Y},\upsilon) \to (\mathcal{Z},\omega)$ is $\mathcal{N}_{\mathcal{N}-\alpha g^{\#}\psi}$-irresolute and $(\mathcal{S} \circ \mathcal{T})(\mathcal{H})$ is $\mathcal{N}_{\mathcal{N}-\alpha g^{\#}\psi}$-OS of $(\mathcal{Z},\omega)$ therefore $\mathcal{S}^{-1}(\mathcal{S} \circ \mathcal{T}(\mathcal{H})) = \mathcal{T}(\mathcal{H})$ is $\mathcal{N}_{\mathcal{N}-\alpha g^{\#}\psi}$-OS in NNTS $(\mathcal{Y},\upsilon)$. Hence $\mathcal{T}$ is $\mathcal{N}_{\mathcal{N}-\alpha g^{\#}\psi}$-OF.

Theorem 2.8 *If $\mathcal{T}:(\mathcal{X},\eta) \to (\mathcal{Y},\upsilon)$ is $\mathcal{NN}$-OS and $\mathcal{S}:(\mathcal{Y},\upsilon) \to (\mathcal{Z},\omega)$ is $\mathcal{N}_{\mathcal{N}-\alpha g^{\#}\psi}$-OFs then $\mathcal{S} \circ \mathcal{T}:(\mathcal{X},\eta) \longrightarrow (\mathcal{Z},\omega)$ is $\mathcal{N}_{\mathcal{N}-\alpha g^{\#}\psi}$-open.*

Proof: Let $\mathcal{H}$ be $\mathcal{NN}$ −OS of NNTS $(\mathcal{X},\eta)$. Then $\mathcal{T}(\mathcal{H})$ is a $\mathcal{NN}$ −OS of $(\mathcal{Y},\upsilon)$ because $\mathcal{T}$ is a $\mathcal{NN}$ −OF. Now since $\mathcal{S}:(\mathcal{Y},\upsilon) \longrightarrow (\mathcal{Z},\omega)$ is $\mathcal{N}_{\mathcal{N}-\alpha g^{\#}\psi}$–open, $\mathcal{S}(\mathcal{T}(\mathcal{H})) = (\mathcal{S} \circ \mathcal{T})(\mathcal{H})$ is $\mathcal{N}_{\mathcal{N}-\alpha g^{\#}\psi}$-OS of $(\mathcal{Z},\omega)$. Hence $\mathcal{S} \circ \mathcal{T}$ is $\mathcal{N}_{\mathcal{N}-\alpha g^{\#}\psi}$-OF.

3 $\mathcal{N}_{\mathcal{N}-\alpha g^{\#}\psi}$-closed function

Definition 3.1 *A func $\mathcal{T}:(\mathcal{X},\eta) \to (\mathcal{Y},\upsilon)$ is $\mathcal{N}_{\mathcal{N}-\alpha g^{\#}\psi}$-closed if image of each $\mathcal{NN}$-OF of $(\mathcal{X},\eta)$ is $\mathcal{N}_{\mathcal{N}-\alpha g^{\#}\psi}$-CS in $(\mathcal{Y},\upsilon)$.*

Theorem 3.2 *Each neut. nano closed func($\mathcal{NN}$ −CF) is a $\mathcal{N}_{\mathcal{N}-\alpha g^{\#}\psi}$-OF.*

Proof: Assume that $\mathcal{T}:(\mathcal{X},\eta) \to (\mathcal{Y},\upsilon)$ is $\mathcal{NN}$-OF, thus and so $\mathcal{H}$ is $\mathcal{NN}$-OF in $(\mathcal{X},\eta)$. Since $\mathcal{T}$ is a $\mathcal{NN}$-OF, $\mathcal{T}(\mathcal{H})$ is $\mathcal{NN}$-CS in $(\mathcal{Y},\upsilon)$. But every $\mathcal{NN}$-OF is a $\mathcal{N}_{\mathcal{N}-\alpha g^{\#}\psi}$-CS. Therefore, $\mathcal{T}(\mathcal{H})$ is a $\mathcal{N}_{\mathcal{N}-\alpha g^{\#}\psi}$-CS in $(\mathcal{Y},\upsilon)$. Hence, $\mathcal{T}$ is an $\mathcal{N}_{\mathcal{N}-\alpha g^{\#}\psi}$-OF.

The below example shows $\mathcal{N}_{\mathcal{N}-\alpha g^{\#}\psi}$-OF need not be $\mathcal{NN}$-OF.

Example 3.3 *Let $\mathcal{U}_1 = \{\alpha_1, \beta_{1),\lambda_1\}}$ and the equivalence relation is $\mathcal{U}_1/\mathcal{R}_1 = \{\{\alpha_1,\lambda_1\},\{\lambda_1\}\}$. Let*

$$\mathcal{A}_1 = \{\langle\alpha_1,(4/10,4/10,3/10)\rangle,\langle\beta_1,(3/10,4/10,2/10)\rangle,\langle\lambda_1,(4/10,3/10,4/10)\rangle\}$$

be neut. nano subset of $\mathcal{U}_1$. Then

Advances in Nonlinear Variational Inequalities
ISSN: 1092-910X
Vol 28 No. 4s (2025)

$\underline{\mathcal{N}}(\mathcal{A}_1) = \{\langle\alpha_1, (3/10,4/10,3/10)\rangle, \langle\beta_1, (3/10,4/10,3/10)\rangle, \langle\lambda_1, (4/10,3/10,4/10)\rangle\}$

$\overline{\mathcal{N}}(\mathcal{A}_1) = \{\langle\alpha_1, (4/10,4/10,2/10)\rangle, \langle\beta_1, (4/10,4/10,2/10)\rangle, \langle\lambda_1, (4/10,3/10,4/10)\rangle\}$

$\mathcal{B}(\mathcal{A}_1) = \{\langle\alpha_1, (2/10,4/10,4/10)\rangle, \langle\beta_1, (2/10,4/10,4/10)\rangle, \langle\lambda_1, (4/10,3/10,4/10)\rangle\}$

Then $\tau 1_{NN} = \{0_N, \underline{\mathcal{N}}(\mathcal{A}_1), \overline{\mathcal{N}}(\mathcal{A}_1), \mathcal{B}(\mathcal{A}_1), 1_N\}$ be neut. nano topology on $(\mathcal{X}, \eta)$ and let $\mathcal{U}_2 = \{\alpha_1, \beta_1, \lambda_1\}$ and the equivalence relation is $\mathcal{U}_2/\mathcal{R}_2 = \{\{\alpha_1, \lambda_1\}, \{\lambda_1\}\}$.

Let

$$\mathcal{A}_2 = \{\langle\alpha_1, (5/10,4/10,3/10)\rangle, \langle\beta_1, (3/10,5/10,2/10)\rangle, \langle\lambda_1, (4/10,3/10,4/10)\rangle\}$$

be neut. nano subset of $\mathcal{U}_2$. Then

$\underline{\mathcal{N}}(\mathcal{A}_2) = \{\langle\alpha_1, (3/10,4/10,3/10)\rangle, \langle\beta_1, (3/10,4/10,3/10)\rangle, \langle\lambda_1, (4/10,3/10,4/10)\rangle\}$

$\overline{\mathcal{N}}(\mathcal{A}_2) = \{\langle\alpha_1, (4/10,4/10,2/10)\rangle, \langle\beta_1, (4/10,4/10,2/10)\rangle, \langle\lambda_1, (4/10,3/10,4/10)\rangle\}$

$\mathcal{B}(\mathcal{A}_2) = \{\langle\alpha_1, (2/10,4/10,4/10)\rangle, \langle\beta_1, (2/10,4/10,4/10)\rangle, \langle\lambda_1, (4/10,3/10,4/10)\rangle\}$

Then $\tau 2_{NN} = \{0_N, \underline{\mathcal{N}}(\mathcal{A}_2), \overline{\mathcal{N}}(\mathcal{A}_2), \mathcal{B}(\mathcal{A}_2), 1_N\}$ be neut. nano topology on $(\mathcal{Y}, \upsilon)$

Define $\mathcal{T}: (\mathcal{X}, \eta) \to (\mathcal{Y}, \upsilon)$ by $\mathcal{T}(\alpha_1)=\alpha_1$, $\mathcal{T}(\beta_1)=\beta_1$, $\mathcal{T}(\lambda_1)=\lambda_1$.

$\mathcal{N}_{\mathcal{N}-\alpha g^{\#}\psi}$-CS of $(\mathcal{Y}, \upsilon)$ = $\{\langle\alpha_1, (3/10,4/10,3/10)\rangle, \langle\beta_1, (3/10,4/10,3/10)\rangle, \langle\lambda_1, (4/10,3/10,4/10)\rangle\}$.

Here $\mathcal{T}(N^c_{A_1})$ is $\mathcal{N}_{\mathcal{N}-\alpha g^{\#}\psi}$-closed in $(\mathcal{Y}, \upsilon)$.

Therefore $\mathcal{T}$ is $\mathcal{N}_{\mathcal{N}-\alpha g^{\#}\psi}$-OF. However, $\mathcal{NN}$ is not OF because $\mathcal{T}(N^c_{A_1})$ is not $\mathcal{NN}$-CS in $(\mathcal{Y}, \upsilon)$.

Theorem 3.4 *A func $\mathcal{T}: (\mathcal{X}, \eta) \to (\mathcal{Y}, \upsilon)$ is $\mathcal{N}_{\mathcal{N}-\alpha g^{\#}\psi}$-closed iff for each ($\mathcal{NNS}$) $\mathcal{S}$ of $(\mathcal{Y}, \upsilon)$ and for each $\mathcal{NN}$ $-OS$ $\mathcal{U}$ of $(\mathcal{X}, \eta)$ containing $\mathcal{T}^{-I}(\mathcal{S})$ there is a $\mathcal{N}_{\mathcal{N}-\alpha g^{\#}\psi}$-OS $\mathcal{A}_1$ of $(\mathcal{Y}, \upsilon)$ thus and so $\mathcal{S} \subseteq \mathcal{A}_1$ and $\mathcal{T}^{-I}(\mathcal{A}_1) \subseteq \mathcal{U}$.*

Proof: Suppose $\mathcal{T}$ is a $\mathcal{N}_{\mathcal{N}-\alpha g^{\#}\psi}$-OF. Let $\mathcal{S}$ be the $\mathcal{NN}$-OF of $(\mathcal{Y}, \upsilon)$ and $\mathcal{U}$ is a $\mathcal{NN}$ $-$OS of $(\mathcal{X}, \eta)$ thus and so $\mathcal{T}^{-I}(\mathcal{S}) \subseteq \mathcal{U}$. Then $\mathcal{A}_1 = \mathcal{T} - \mathcal{T}^{-I}(\mathcal{U}^c)$ is $\mathcal{N}_{\mathcal{N}-\alpha g^{\#}\psi}$-OS of $(\mathcal{Y}, \upsilon)$ thus and so $\mathcal{T}^{-I}(\mathcal{A}_1) \subseteq \mathcal{U}$.

Suppose $\mathcal{F}$ is a $\mathcal{NN}$-OF of $(\mathcal{X}, \eta)$. Then $(\mathcal{T}(\mathcal{F}))^c$ is a NNS of $(\mathcal{Y}, \upsilon)$ and $\mathcal{F}^c$ is $\mathcal{NN}$ $-$OS in $(\mathcal{X}, \eta)$ thus and so $\mathcal{T}^{-I}((\mathcal{T}(\mathcal{F}))^c) \subseteq \mathcal{F}^c$. By hypothesis nano $\alpha g^{\#}\psi$-OS $\mathcal{A}_1$ of $(\mathcal{Y}, \upsilon)$ thus and so $(\mathcal{T}(\mathcal{F}))^c \subseteq \mathcal{A}_1$ and $\mathcal{T}^{-I}(\mathcal{A}_1) \subseteq \mathcal{F}^c$. Therefore $\mathcal{F} \subseteq (\mathcal{T}^{-I}(\mathcal{A}_1))^c$. Hence $\mathcal{A}_1^c \subseteq \mathcal{T}(\mathcal{F}) \subseteq \mathcal{T}((\mathcal{T}^{-I}(\mathcal{A}_1))^c) \subseteq \mathcal{A}_1^c$ which indicates $\mathcal{T}(\mathcal{F}) = \mathcal{A}_1^c$. Since $\mathcal{A}_1^c$ is $\mathcal{N}_{\mathcal{N}-\alpha g^{\#}\psi}$-CS of $(\mathcal{Y}, \upsilon)$. Hence $\mathcal{T}(\mathcal{F})$ is $\mathcal{N}_{\mathcal{N}-\alpha g^{\#}\psi}$-closed in $(\mathcal{Y}, \upsilon)$ and thus $\mathcal{T}$ is $\mathcal{N}_{\mathcal{N}-\alpha g^{\#}\psi}$-OF.

Theorem 3.5 *If $\mathcal{T}:(\mathcal{X},\eta)\to(\mathcal{Y},\upsilon)$ is $\mathcal{NN}$-CS and $\mathcal{S}:(\mathcal{Y},\upsilon)\to(\mathcal{Z},\omega)$ is $\mathcal{N}_{\mathcal{N}-\alpha g^{\#}\psi}$-closed. Then $\mathcal{S}\circ\mathcal{T}:(\mathcal{X},\eta)\longrightarrow(\mathcal{Z},\omega)$ is $\mathcal{N}_{\mathcal{N}-\alpha g^{\#}\psi}$-closed.*

Proof: Let $\mathcal{H}$ be an $\mathcal{NN}$-OF of NNTS $(\mathcal{X},\eta)$. Then $\mathcal{T}(\mathcal{H})$ is $\mathcal{NN}$-OF of $(\mathcal{Y},\upsilon)$ because $\mathcal{T}$ is $\mathcal{NN}$-OF. Now $(\mathcal{S}\circ\mathcal{T})(\mathcal{H})=\mathcal{ST}(\mathcal{H}))$ is $\mathcal{N}_{\mathcal{N}-\alpha g^{\#}\psi}$-CS in NNTS $(\mathcal{Z},\omega)$ because $\mathcal{S}$ is $\mathcal{N}_{\mathcal{N}-\alpha g^{\#}\psi}$-OF. Thus $\mathcal{S}\circ\mathcal{T}:(\mathcal{X},\eta)\longrightarrow(\mathcal{Z},\omega)$ is $\mathcal{N}_{\mathcal{N}-\alpha g^{\#}\psi}$-OF.

Theorem 3.6 *If $\mathcal{T}:(\mathcal{X},\eta)\to(\mathcal{Y},\upsilon)$ is $\mathcal{N}_{\mathcal{N}-\alpha g^{\#}\psi}$–OF, then neut. nano $\alpha g^{\#}\psi(H^{\circ}(\mathcal{T}(\mathcal{A}_1))\subseteq\mathcal{T}(H^{\circ}(\mathcal{A}_1))$.*

Proof: Let $\mathcal{A}_1$ is any NNS of $(\mathcal{X},\eta)$ and $\mathcal{T}:(\mathcal{X},\eta)\to(\mathcal{Y},\upsilon)$ is $\mathcal{N}_{\mathcal{N}-\alpha g^{\#}\psi}$ –OF, then $\mathcal{T}(H^{\circ}(\mathcal{A}_1))$ is $\mathcal{N}_{\mathcal{N}-\alpha g^{\#}\psi}$-closed in $(\mathcal{Y},\upsilon)$. Now $\mathcal{T}(\mathcal{A}_1)\subseteq\mathcal{T}(H^{\circ}(\mathcal{A}_1))$ implies neut. nano $\alpha g^{\#}\psi(H^{\circ}(\mathcal{T}(\mathcal{A}_1)))\subseteq\mathcal{N}_{\alpha g^{\#}\psi}(H^{\circ}(\mathcal{T}(H^{\circ}(\mathcal{A}_1))))$. Since $\mathcal{T}(H^{\circ}(\mathcal{A}_1))$ is $\mathcal{N}_{\mathcal{N}-\alpha g^{\#}\psi}$-closed in $(\mathcal{Y},\upsilon)$, neut. nano $\alpha g^{\#}\psi(H^{\circ}(\mathcal{T}(H^{\circ}(\mathcal{A}_1)))=\mathcal{T}(H^{\circ}(\mathcal{A}_1))$. Hence neut. nano $\alpha g^{\#}\psi(H^{\circ}(\mathcal{T}(\mathcal{A}_1))\subseteq\mathcal{T}(H^{\circ}(\mathcal{A}_1))$.

Theorem 3.7 *Let $\mathcal{T}:(\mathcal{X},\eta)\to(\mathcal{Y},\upsilon)$ and $\mathcal{S}:(\mathcal{Y},\upsilon)\to(\mathcal{Z},\omega)$ are $\mathcal{N}_{\mathcal{N}-\alpha g^{\#}\psi}$-OFs. If every $\mathcal{N}_{\mathcal{N}-\alpha g^{\#}\psi}$-CS of $(\mathcal{Y},\upsilon)$ is neut. nano α-closed then, $\mathcal{S}\circ\mathcal{T}:(\mathcal{X},\eta)\longrightarrow(\mathcal{Z},\omega)$ is $\mathcal{N}_{\mathcal{N}-\alpha g^{\#}\psi}$-closed.*

Proof: Let $\mathcal{H}$ be $\mathcal{NN}$-OF of NNTS $(\mathcal{X},\eta)$. Then $\mathcal{T}(\mathcal{H})$ is $\mathcal{N}_{\mathcal{N}-\alpha g^{\#}\psi}$-CS of $(\mathcal{Y},\upsilon)$ because $\mathcal{T}$ is $\mathcal{N}_{\mathcal{N}-\alpha g^{\#}\psi}$-OF. By hypothesis $\mathcal{T}(\mathcal{H})$ is neutrosophic nano α-CS of $(\mathcal{Y},\upsilon)$. Now $\mathcal{S}(\mathcal{T}(\mathcal{H}))=(\mathcal{S}o\mathcal{T})(\mathcal{H})$ is $\mathcal{N}_{\mathcal{N}-\alpha g^{\#}\psi}$-CS in NNTS $(\mathcal{Z},\omega)$ because $\mathcal{S}$ is $\mathcal{N}_{\mathcal{N}-\alpha g^{\#}\psi}$-OF. Thus $\mathcal{S}\circ\mathcal{T}:(\mathcal{X},\eta)\longrightarrow(\mathcal{Z},\omega)$ is $\mathcal{N}_{\mathcal{N}-\alpha g^{\#}\psi}$-OF.

Theorem 3.8 *If $\mathcal{T}:(\mathcal{X},\eta)\to(\mathcal{Y},\upsilon)$ be bijective func, then*

(p) $\mathcal{T}$ is $\mathcal{N}_{\mathcal{N}-\alpha g^{\#}\psi}$-OF.

(q) $\mathcal{T}$ is $\mathcal{N}_{\mathcal{N}-\alpha g^{\#}\psi}$-OF.

(r) $\mathcal{T}^{-I}$ is $\mathcal{N}_{\mathcal{N}-\alpha g^{\#}\psi}$-continuous func are equivalent.

Proof: (p)⇒(q): Let $\mathcal{T}$ is a $\mathcal{N}_{\mathcal{N}-\alpha g^{\#}\psi}$-OF. By defn, $\mathcal{B}_1$ is a $\mathcal{NN}-$OS in $(\mathcal{X},\eta)$, then the image $\mathcal{T}(\mathcal{B}_1)$ is a $\mathcal{N}_{\mathcal{N}-\alpha g^{\#}\psi}$-OS in $(\mathcal{Y},\upsilon)$. Here, $\mathcal{B}_1$ is $\mathcal{NN}$-OF in $(\mathcal{X},\eta)$, then $\mathcal{S}-\mathcal{B}_1$ is a $\mathcal{NN}-$OS in $(\mathcal{X},\eta)$. By assumption, we have $\mathcal{T}(\mathcal{S}-\mathcal{B}_1)$ is a $\mathcal{N}_{\mathcal{N}-\alpha g^{\#}\psi}$-OS in $(\mathcal{Y},\upsilon)$. Hence,$\mathcal{T}-\mathcal{T}(\mathcal{S}-\mathcal{B}_1)$ is a $\mathcal{N}_{\mathcal{N}-\alpha g^{\#}\psi}$-CS in $(\mathcal{Y},\upsilon)$. Therefore, $\mathcal{T}$ is a $\mathcal{N}_{\mathcal{N}-\alpha g^{\#}\psi}$-OF.

(q)⇒(r): Let $\mathcal{B}_1$ be $\mathcal{NN}$-OF in $(\mathcal{X},\eta)$. By (q), $\mathcal{T}(\mathcal{B}_1)$ is a $\mathcal{N}_{\mathcal{N}-\alpha g^{\#}\psi}$-CS in $(\mathcal{Y},\upsilon)$. Hence, $\mathcal{T}(\mathcal{B}_1)=(\mathcal{T}^{-I})^{-I}(\mathcal{B}_1)$, so $\mathcal{T}^{-I}$ is a $\mathcal{N}_{\mathcal{N}-\alpha g^{\#}\psi}$-CS in $(\mathcal{Y},\upsilon)$. Hence, $\mathcal{T}^{-I}$ is $\mathcal{N}_{\mathcal{N}-\alpha g^{\#}\psi}$-continuous.

(r)⇒(s): Let $\mathcal{B}_1$ be $\mathcal{NN}-$OS in $(\mathcal{X},\eta)$. By (r), $(\mathcal{T}^{-I})^{-1}(\mathcal{B}_1)=\mathcal{T}(\mathcal{B}_1)$ is a $\mathcal{N}_{\mathcal{N}-\alpha g^{\#}\psi}$-OF.

Conclusion:

To conclude, this article has provided a thorough exploration of neutrosophic nano $\alpha g^{\#}\psi$ -open and closed functions, underscoring their importance in the study of neutrosophic nano topological spaces. By examining both foundational concepts and their broader implications, it has highlighted how these functions contribute to a deeper understanding of relationships within this specialized area of topology. The insights gained here not only enhance comprehension of these functions but also open pathways for further research and applications, underscoring their potential impact on advancements in theoretical and applied topology.

References

[1] I. Arokiarani, R. Dhavaseelan, S. Jafari and M. Parimala, "'On some new notions and functions in neutrosophic topological spaces"', Neutrosophic Sets Systems, (2017), 16, pp.16-19.
[2] Atanassov, K. "'Intuitionstic fuzzy sets"', Fuzzy Sets Systems, 1986, 20, pp.87-96.
[3] Atanassov, K. Stoeva, S. "'Intuitionistic fuzzy sets"', In Proceedings of the Polish Symposium on Interval and Fuzzy Mathematics, Poznan, Poland, 26-29 August 1983, pp.23-26.
[4] Chang, C.L. "'Fuzzy Topological Spaces"', J. Math. Anal. Appl. 24 (1968), pp.182-190.
[5] Dogan Coker, "'An introduction to intuitionistic fuzzy topological spaces"', Fuzzy Sets and Systems, Vol 88, No.1, 1997, pp.81-89.
[6] Kungumaraj. E, Durgadevi S and Tharani.N, "Neutrosophic Heptagonal Topology" in Neutrosophic Sets and Systems, Scopus Indexed and Web of Science journal., Vol 60, 2023, pp 335-356.
[7] Kungumaraj. E, Lathanayagam. E, Saikia.U, Clement Joe Anand. M , Khanna. S.T, Nivetha Martin, Mohit Tiwari and Edalatpanah. S.A, "Neutrosophic Topological Vector Spaces and its Properties", International Journal Science, Scopus Indexed journal, Vol 23, No. 02, 2024, pp 63-76.
[8] Lellis Thivagar, M. and Carmel Richard, "'On nano forms of weakly open sets"', International journal of mathematics and statistics invention, Volume 1, Issue 1, (2013), pp.31-37.
[9] Lellis Thivagar, M. Jafari, S. Sudha devi, V. and Antonysamy, V. "'A novel approach to nano topology via neutrosophic sets"', Neutrosophic sets and systems, Vol 20, (2018), pp.86-94.
[10] Nandhini, T. Vigneshwaran, M. "' $\mathcal{N}_{\alpha g^{\#}\psi}$ -closed sets in neutrosophic topological spaces"', American International Journal of Research in Science, Technology, Engineering and Mathematics, Special issue of 2nd International Conference on Current Scenario in Pure and Applied Mathematics, 3rd January, 2019, pp.370-373.
[11] Nandhini, T. Vigneshwaran, M. "'On $\mathcal{N}_{\mathcal{N}-\alpha g^{\#}\psi}$-closed sets and continuous functions in neutrosophic nano topological spaces "'.(Communicated.)
[12] Priyadharsini.S, Kungumaraj. E and Santhi. R, An Evaluation of Triangular Neutrosophic PERT Analysis for Real-life Project Time and Cost Estimation", Neutrosophic Sets and Systems, Scopus Indexed Journal, Vol.63, 2024, pp 62-81
[13] Smarandache, F. Neutrosophy and Neutrosophic Logic, First International Conference on Neutrosophy, Neutrosophic Logic, Set, Probability and Statistics; University of New Mexico: Gallup, NM, USA, 2002.
[14] Smarandache, F.; A Unifying Field in Logics: Neutrosophic Logic. Neutrosophy, Neutrosophic Set, Neutrosophic Probability; American Research Press: Rehoboth, NM, USA, 1999.
[15] Salama, A.A. Alblowi, S.A. "'Neutrosophic Set and Neutrosophic Topological Spaces"'. IOSR J. Math. 2012, 3, pp.31-35.
[16] Zadeh, L.A. "Fuzzy Sets," Inf. Control, 8, pp.338-353, 1965.