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The Stochastic Analysis of a Diabetic Person's Blood Glucose Levels During Treatment

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Abstract:

This paper assumes a diabetic person with n stages and the absorption stage is a controlled stage. Blood glucose level of a patient with a damaged pancreas may pass through the fair control stage, poor control stage, and very poor control stage several times before it reaches the controlled stage during the treatment. Blood glucose level can identify from any one of the n stages and it can be moved to other stages easily. This may form number of cycles of fair control stage to poor control stage to fair control stage based on the changes in the blood glucose level of the patient. This paper provides two types of models. In model 1, r to s to r cycle, and in model 2 u to v to w to u cycle. This paper obtains the probability generating function of the number of cycles before the glucose level is controlled and their numerical illustrations are provided.

Introduction: In model 1,Blood glucose level of a patient with damaged pancreas may pass through fair control level (say stage r) and poor control level (say stage s) several times, between them before the glucose level reaches controlled level due to the treatment. This may form number of cycles of fair control level to poor control level to fair control level based on the changes in the blood glucose level of the patient. This is taken as one r-s-r cycle. In model 2, Blood glucose level of a patient with damaged pancreas may pass through fair control level (say stage u) ,poor control level (say stage v)and very poor control level (say stage w) several times, between them before the glucose level reaches controlled level due to the treatment. This may form number of cycles of fair control level to poor control level to very poor control level to fair control level based on the changes in the blood glucose level of the patient. This is taken as one u-v-w-u cycle. For more details on such deficiency and disorder, one may refer to [3], [4], [5]. Using the Mathematical approach given in [1], [8], [9], and [10], we study the case of a diabetic person.

Results: The results of model 1 in section 2 and model 2 in section 3. In this sections are derive the probability generating function, mean and variance of number of cycles occur in our model during treatment time.

Conclusions: In Model 1, if a diabetic person start the treatment in fair control stage then the person can reduce the expected value of number of r-s-r cycle. In Model 2, when the transition probability of controlled blood

glucose level (absorption stage) is high, the expected value of r-s-r cycle is decrease

Keywords: Generating function, Markov chain, *r* to *s* to *r* cycle, *u* to *v* to *w* to *u* cycle, Transition matrix.

1. Introduction

Diabetes is one of the world's leading chronic diseases and has serious social and economic considerations. Tight glucose control significantly decrease the risk of developing both micro and macro vascular complications. S.K.Bhattacharya, R.Biswas, M.M.Ghosh, P.Banerjee, have studied a risk factors of diabetes mellitus[2]. Glucotoxicity is defined as the non physiologic alteration in a celluar function caused by chronic exposure to high blood glucose concentrations. Hyperglycaemia arising from any primary cause further worsens the glycaemic status. It is toxic to the β −cell; it affects the glucose sensor mechanisms where by insulin secretion in response to rising blood glucose becomes inappropriate and it impairs the glucose transporter system. The Diabetes Control and Complications Motocross Research group have discussed the long term complications in insulin dependent diabetes mellitus [7].

There are many reasons for which the blood glucose level of the patient to shift from one stage to another stage or equivalently percentage of one glucose level to another. Various factors are responsible for this rising prevalence of diabetes which include: increasing prevalence of obesity, the relatively low levels physical activity, increasing age of population, rapid growth of population that are particularly susceptible to diabetes and improved medical care which prolongs life, thus increasing the risk for development of diabetes and its cardiovascular disease complications. In this model here transfers from one stage to another stage of the patient is considered. All complications of organs may move to different stages during treatment, they may move to complicated stages when patient is careless and may move to better stages when the patient is sufficiently careful.

In model 1 ,Blood glucose level of a patient with damaged pancreas may pass through fair control level (say stage r) and poor control level (say stage s) several times, between them before the glucose level reaches controlled level due to the treatment. This may form number of cycles of fair control level to poor control level to fair control level based on the changes in the blood glucose level of the patient. This is taken as one r-s-r cycle. In model 2, Blood glucose level of a patient with damaged pancreas may pass through fair control level (say stage u) ,poor control level (say stage v)and very poor control level (say stage w) several times, between them before the glucose level reaches controlled level due to the treatment. This may form number of cycles of fair control level to poor control level to very poor control level to fair control level based on the changes in the blood glucose level of the patient. This is taken as one u-v-w-u cycle. For more details on such deficiency and disorder, one may refer to [3], [4], [5]. Using the Mathematical approach given in [1], [8], [9], and [10], we study the case of a diabetic person.

The results of model 1 in section 2 and model 2 in section 3. In this sections are derive the probability generating function, mean and variance of number of cycles occur in our model during treatment time.

2. SECTION

2.1.MODEL -1 ASSUMPTION

The general assumption of models studied are given below.

1. There are n+1 stages with n+1 as absorption stage, i.e controlled stage.
2. Blood glucose level starts from stage *i* during the treatment with probability a_i where the starting probability vector is $(a_1,a_2,\ldots a_n)$ with $\sum_{i=1}^{n} a_i = 1$.
3. The probability of moving from stage i to stage j is denoted by p_{ij} ; $\sum_{j=1}^{n+1} p_{ij}=1$ for all i ,$1 \leq i \leq n$ and $1 \leq j \leq n+1$.
4. Here (n+1)th stage is our controlled absorption stage and its probability is denoted by $p_{i,\,n+1}$, $1 \leq i \leq n$ when the treatment moves from i to n+1.
5. The state space of our model is W= {1, 2, ...n, n+1} ,where(n+1) is controlled stage which is absorption stage.

The transition probability matrix is below

$$P=\begin{pmatrix} p_{1,1} & p_{1,2} \ldots \ldots & p_{1,n} & p_{1,n+1} \\ \vdots & & & \vdots \\ p_{n,1} & p_{n,2} \ldots \ldots & p_{n,n} & p_{n,n+1} \\ 0 & 0 \ldots \ldots & 0 & 1 \end{pmatrix} \quad (1)$$

where $p_{i,j}$ means the probability of moving from stage i to stage j, the last row denote the absorption row and last column denotes the absorption probabilities from various stages.

In this model assume the treatment for one cycle starts from fair control stage *r* then can move to any stage, it passes through the poor control stage *s* and finishing their moving in fair control stage *r*.

2.2 ANALYSIS

Let N be the random variable indicating the number of cycles *r*-*s*-*r* before blood glucose level reaches the controlled stage during the treatment.

Now form a matrix R of order (n-1) x (n-1) .which is formed by deleting *r*-th row, *r*-th column, last row and last column from P matrix. Similarly form a matrix S of order (n-1) x (n-1).

This is formed by deleting *s*-th row, *s*-th column, last row and last column from P matrix.

$$R=\begin{pmatrix} p_{1,1} \ldots & p_{1,r-1} & p_{1,r+1} \ldots & p_{1,n} \\ \vdots & & & \vdots \\ p_{r-1,1} \ldots & p_{r-1,r-1} & p_{r-1,r+1} \ldots & p_{r-1,n} \\ p_{r+1,1} \ldots & p_{r+1,r-1} & p_{r+1,r+1} \ldots & p_{r+1,n} \\ \vdots & & & \vdots \\ p_{n,1} \ldots & p_{n,r-1} & p_{n,r+1} \ldots \ldots & p_{n,n} \end{pmatrix} \quad (2)$$

$$S=\begin{pmatrix} p_{1,1} \cdots & p_{1,s-1} & p_{1,s+1} \cdots & p_{1,n} \\ \vdots & & & \vdots \\ p_{s-1,1} \cdots & p_{s-1,s-1} & p_{s-1,s+1} \cdots & p_{s-1,n} \\ p_{s+1,1} \cdots & p_{s+1,r-1} & p_{s+1,s+1} \cdots & p_{s+1,n} \\ \vdots & & & \vdots \\ p_{n,1} \cdots & p_{n,s-1} & p_{n,s+1} \cdots \cdots & p_{n,n} \end{pmatrix} \quad (3)$$

Let R_r be the next transition probability from stage *r* to stage in W-{*s*}.

$R_r = (p_{r,1} \ldots . p_{r,s-1}\ p_{r,s+1} \ldots . . p_{r,n})$. (4)

Let R_s be the next transition probability from stage *s* to stage in W-{*r*}.

$R_s = (p_{s,1} \ldots . p_{s,r-1}, p_{s,r+1} \ldots . . p_{s,n})$. (5)

Let P_s be the next transition probability of entries from stages W-{*s*}to *s* .

$P_s=(p_{1,s} \ldots . p_{s-1,s}, p_{s+1,s} \ldots p_{n,s})^T$ where T denotes transpose matrix (6)

Let P_r be the next transition probability of entries from stages W-{*r*}to r.

$P_r=(p_{1,r} \ldots . p_{r-1,r}, p_{r+1,r} \ldots . p_{n,,r})^T$ (7)

Let $\hat{A}$ be the initial transition probability for all stages. $\hat{A}=(a_1, a_2, \ldots a_r \ldots a_n)$ with $\sum a_i=1$ (8)

Let *a* be the initial transition probability for W-{*r*}.

$a = (a_1, a_{2 \ldots . .} a_{r-1}, a_{r+1}, \ldots a_n)$ (9)

Let S_s be transition probabilities of absorption in n+1 from stages W-{*s*}.

$S_s = (p_{1,n+1,} \ldots p_{s-1,n+1}, p_{s+1,n+1}, \ldots . p_{n,n+1})^T$ (10)

Let S_r be transition probabilities of absorption in n+1 from stages W-{r}.

$S_r=(p_{1,n+1}, \ldots p_{r-1,n+1}, p_{r+1,n+1}, \ldots . p_{n,n+1})^T$ (11)

The above equations (6), (7), (10) and (11) are all column vectors with order (n-1) x1

Let C_k be the probability that the treatment makes exactly k cycles before it reaches controlled stage ($0 \leq k < \infty$) that is absorption stage .Let N be the number of cycles r-s-r completes before the absorption controlled stage occurs.

Case (I) Number of r-s-r cycle is zero before glucose level reaches absorption controlled stage.

First assume that there is no cycle of *r-s-r* exists.

$P(N=0) = C_0 = (a(I-R)^{-1}S_r) + (a_r + a(I-R)^{-1} P_r)$

$[(p_{r,n+1} + R_r (I-S)^{-1}S_s) + (p_{r,s} + R_r (I-S)^{-1}P_s)(p_{s,n+1} + R_s (I-R)^{-1}S_r)]$ (12)

In the above equation there are three types of events considered before absorption, namely (i) the treatment does not enter fair control stage *r* or (ii) the treatment enters fair control stage *r* but does not enter poor control stage *s* during the treatment completion or (iii) the treatment enters fair control stage *r* and moves to poor control stage *s* but does not enter again fair control stage *r* before the treatment completion.

The first term is the probability that the treatment enters the stage in W-{*r*} and after making number of transitions in W-{*r*} absorption occurs. Second term is the probability that it enters stage *r* or it starts from W-{*r*}, moves in W-{*r*} and enters fair control stage *r*, after that it moves to absorption stage. Third term is the probability that the treatment goes to poor control stage *s* from fair control stage *r* after moving in W-{*s*} and the treatment moves to directly absorption stage or travel in W-{*r*} stages and it exits.

Case (II) Number of r-s-r cycle is one

Now we get the probability of cycle as follows.exactly one *r-s-r*

$$P(N=1) = C_1 = (a_r + a\,(I-R)^{-1}P_r)\ (p_{r,s} + R_r\,(I-S)^{-1}P_s)\,(p_{s,r} + R_s(I-R)^{-1}P_r)$$
$$[\,(p_{r,n+1} + R_r\,(I-S)^{-1}S_s) + (p_{r,s} + R_r\,(I-S)^{-1}P_s)\,(p_{s,n+1} + R_s\,(I-R)^{-1}S_r)\,] \quad (13)$$

In the above equation there are four types of events considered before controlled absorption stage occurs, namely, (i) the treatment enters fair control stage *r*, (ii) the treatment moves from fair control stage *r* to poor control stage *s* then (iii) the treatment moves from poor control stage *s* to again fair control stage *r* and (iv) the treatment absorption occurs from fair control stage *r* or from poor control stage *s*.

The first bracket is the probability that the treatment starts from fair control stage *r* or it starts from W-{*r*} moves in W-{*r*} and enters fair control stage *r*. The second bracket is the probability that the treatment goes to poor control stage s in the next transition from fair control stage *r* or it goes to stages in W-{*s*}, moves there and it enters poor control stage *s*. The third bracket is the probability that the treatment goes to fair control stage *r* or it goes to stage in W-{*r*}, moves there and it enters again fair control stage *r*. The first term of the fourth square bracket is the probability that the treatment is absorbed from fair control stage *r* or it goes to stage in W-{*s*}, moves there and it is absorbed from stage W-{*s*}. The second term product of the fourth term is already explained in equation (12).

Case (III) Number of r-s-r cycle is two

We can write C_2 the probability of exactly 2- cycles (*r-s-r*) as follows

$$P(N=2) = C_2 = (a_r + a\,(I-R)^{-1}P_r)\ [\,(p_{r,s} + R_r\,(I-S)^{-1}P_s)\,(p_{s,r} + R_s(I-R)^{-1}P_r)\,]^2$$
$$[\,(p_{r,n+1} + R_r\,(I-S)^{-1}S_s) + (p_{r,s} + R_r\,(I-S)^{-1}P_s)\,(p_{s,n+1} + R_s\,(I-R)^{-1}S_r)\,] \quad (14)$$

Using similar argument, We can write C_k the probability of exactly k cycles (*r-s-r*) as follows

$$P(N=k) = C_k = (a_r + a\,(I-R)^{-1}P_r)\ [\,(p_{r,s} + R_r\,(I-S)^{-1}P_s)\,(p_{s,r} + R_s(I-R)^{-1}P_r)\,]^k$$
$$[\,(p_{r,n+1} + R_r\,(I-S)^{-1}S_s) + (p_{r,s} + R_r\,(I-S)^{-1}P_s)\,(p_{s,n+1} + R_s\,(I-R)^{-1}S_r)\,] \quad (15)$$

The probability generating function of C_k is $\Phi(x)=\sum_{k=0}^{\infty} C_k x^k, 0 \le x \le 1$ (16)

Now substitute the equation (12), (13), (14) and (15) in (16)

$$\Phi(x) = \{\,(a(I-R)^{-1}S_r) + (a_r + a\,(I-R)^{-1}P_r)[\,(p_{r,n+1} + R_r\,(I-S)^{-1}S_s) + (p_{r,s} + R_r\,(I-S)^{-1}P_s)(p_{s,n+1} + R_s\,(I-R)^{-1}S_r)\,]\}$$
$$+\{\,(a_r + a\,(I-R)^{-1}P_r)(p_{r,s} + R_r\,(I-S)^{-1}P_s)\,(p_{s,r} + R_s(I-R)^{-1}P_r)$$

$$[\ (p_{r,n+1} + R_r\,(I\text{-}S)^{-1}S_s\ ) + (p_{r,s} + R_r\,(I\text{-}S)^{-1}P_s\ )\ (\ p_{s,n+1} + R_s\,(I\text{-}R)^{-1}S_r\ )\ ]\ \}x\ +\ldots.$$

$$+\{(a_r + a\,(I\text{-}R)^{-1}\,P_r)\ [\ (p_{r,s} + R_r\,(I\text{-}S)^{-1}P_s\ )\,(p_{s,r} + R_s(I\text{-}R)^{-1}P_r\ )]^k$$

$$[\ (p_{r,n+1} + R_r\,(I\text{-}S)^{-1}S_s\ ) + (p_{r,s} + R_r\,(I\text{-}S)^{-1}P_s\ )\ (\ p_{s,n+1} + R_s\,(I\text{-}R)^{-1}S_r\ )\ ]\ \}x^k + \ldots \quad (17)$$

$$\Phi(x) = (a(I\text{-}R)^{-1}S_r) + (a_r + a\,(I\text{-}R)^{-1}\,P_r)$$

$$[\ (p_{r,n+1} + R_r\,(I\text{-}S)^{-1}S_s\ ) + (p_{r,s} + R_r\,(I\text{-}S)^{-1}P_s\ )(\ p_{s,n+1} + R_s\,(I\text{-}R)^{-1}S_r\ )\ ]$$

$$\{1/1\text{-}\ (p_{r,s} + R_r\,(I\text{-}S)^{-1}P_s\ )\,(p_{s,r} + R_s(I\text{-}R)^{-1}P_r\ )x\} \quad (18)$$

Here we note that

$$(p_{r,n+1} + R_r\,(I\text{-}S)^{-1}S_s\ ) + (p_{r,s} + R_r\,(I\text{-}S)^{-1}P_s\ )(\ p_{s,n+1} + R_s\,(I\text{-}R)^{-1}S_r\ )$$

$$= 1\text{-}\ (p_{r,s} + R_r\,(I\text{-}S)^{-1}P_s\ )\,(p_{s,r} + R_s(I\text{-}R)^{-1}P_r\ ) = 1\text{-}\ \beta, \quad \text{where } \beta = (p_{r,s} + R_r\,(I\text{-}S)^{-1}P_s\ )\,(p_{s,r} + R_s(I\text{-}R)^{-1}P_r\ )$$

$$\Phi(x) = (a(I\text{-}R)^{-1}S_r) + (a_r + a\,(I\text{-}R)^{-1}\,P_r)\,[1\text{-}\beta]\{1/1\text{-}x\,\beta\} \quad (19)$$

To check that $\Phi(x)$ is generating function of probabilities we can derive $\Phi(1)=1$ by putting $x=1$

Now we can calculate the expected number of cycle(*r-s-r*) completed before absorption and variance using $\Phi(x)$.

$$E(N) = (d\Phi/dx)_{\text{at } x=1} = (a_r + a\,(I\text{-}R)^{-1}\,P_r)\,(\beta/1\text{-}\beta) \quad (20)$$

$$\text{Var}(N) = E(N)[(2\beta/1\text{-}\beta) + 1\text{-}E(N)] \quad (21)$$

2.3 NUMERICAL EXAMPLES

Example 1:

Now consider 5stages of Blood glucose level in this transition probability matrix for the application of the result.

In the following discrete Markov chain assume the cycle 1-4-1and 5-th stage is control stage.

Also stage 1 is fair control stage and stage 4 is poor control stage.

$$\text{Let matrix } P = \begin{pmatrix} 0.2 & 0.1 & 0.2 & 0.3 & 0.2 \\ 0.2 & 0.2 & 0.2 & 0.1 & 0.3 \\ 0.2 & 0.2 & 0.2 & 0.2 & 0.2 \\ 0.2 & 0.1 & 0.2 & 0.2 & 0.3 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

Initial probability vector $\hat{A} = (0.25 \quad 0.25 \quad 0.25 \quad 0.25)$,$a = (0.25 \ \ 0.25 \ \ 0.25)$

We note that matrix

$$R = \begin{pmatrix} 0.2 & 0.2 & 0.1 \\ 0.2 & 0.2 & 0.2 \\ 0.1 & 0.2 & 0.2 \end{pmatrix} \& \quad (I\text{-}R)^{-1} = \begin{pmatrix} 1.389 & 0.417 & 0.278 \\ 0.417 & 1.458 & 0.417 \\ 0.278 & 0.417 & 1.389 \end{pmatrix}$$

$$S = \begin{pmatrix} 0.2 & 0.1 & 0.2 \\ 0.2 & 0.2 & 0.2 \\ 0.2 & 0.2 & 0.2 \end{pmatrix} \& \quad (I-S)^{-1} = \begin{pmatrix} 1.429 & 0.286 & 0.429 \\ 0.476 & 1.429 & 0.476 \\ 0.476 & 0.429 & 1.476 \end{pmatrix}$$

Also $P_r = (0.2 \quad 0.2 \quad 0.2)^T$, $P_s = (0.3 \quad 0.1 \quad 0.2)^T$

$S_r = (0.3 \quad 0.2 \quad 0.3)^T$, $S_s = (0.2 \quad 0.3 \quad 0.2)^T$

$R_r = (0.2 \quad 0.1 \quad 0.2)$, $R_s = (0.1 \quad 0.2 \quad 0.2)$

Using equation (19) we get $\Phi(1) = 1$ as follows

$\Phi(1) = 0.427+0.573(1-0.543)x1/0.457=1$

Using equation (20)& (21) we get,

E(N)=0.167467 **.**

Var(N)=0.237326.

Example 2

Now consider 5stage transition probability matrix for the application of the result.

In the following discrete Markov chain we assume the cycle 1-4-1and 5-th stage is absorption stage. Also stage 1 is fair control stage and stage 4 is poor control stage. Now consider the same transition matrix in example 1.

Case 1

Let initial probability vector $\hat{A} = (1 \ 0 \ 0 \ 0)$. & $a = (0 \ 0 \ 0)$.In this case the probability of treatment starting time is high in fair control stage(r).

Using equation (20)&(21) we get,

E(N)=0.07307 & Var(N)=0.11045.

Case 2

Let initial probability vector $\hat{A} = (0 \ 1 \ 0 \ 0)$. & $a = (1 \ 0 \ 0)$. In this case the probability of treatment starting time is high in stage 2.

Using equation (20)&(21) we get,

E(N)=0.12179 & Var(N)= 0.17816

Case 3

Let initial probability vector $\hat{A} = (0 \ 0 \ 1 \ 0)$. & $a = (0 \ 1 \ 0)$. In this case the probability of treatment starting time is high in stage 3.

Using equation (20)&(21) we get,

E(N)=0.13397 & Var(N)=0.19434

Case 4

Let initial probability vector Â= (0 0 0 1) . & a = (0 0 1). In this case the probability of treatment starting time is high in poor control stage(s).

Using equation (20)&(21) we get,

E(N)=0.12179 & Var(N)= 0.17816

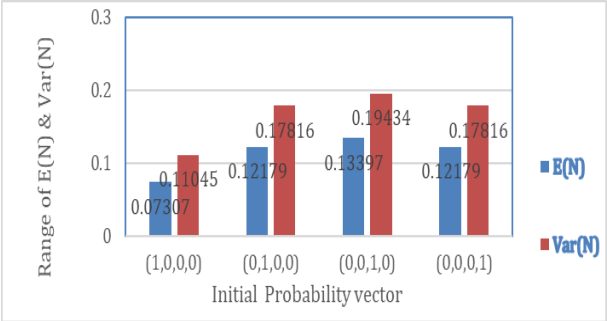

Fig 2.1

From example 2 observe in Fig 2.1 the behaviour of E(N) and Var(N).In case1 the initial probability vector denote the treatment start from fair control stage in that case the value of E(N) and Var(N) is less than that value in case4 person start the treatment initially then the person can reduce the value of E(N) and Var(N).

In case4 the initial probability vector denote the treatment start from poor control stage in that case the value of E(N) and Var(N) is greater than that values in case 1 .If we start the treatment in poor control stage then the value of E(N) is high for this r-s-r cycle.

SECTION-3

3.1 MODEL-2 ASSUMPTION

The general assumption of models studied are given below.

1. There are n+1 stages with n+1 as absorption stage, i.e controlled stage.

2. Blood glucose level starts from stage i during the treatment with probability a_i where the starting probability vector is $(a_1,a_2,\ldots a_n)$ with $\sum_{i=1}^{n} a_i = 1$.

3. The probability of moving from stage i to stage j is denoted by p_{ij} ; $\sum_{j=1}^{n+1} p_{ij}=1$ for all i ,$1 \leq i \leq n$ and $1 \leq j \leq n+1$.

4. Here (n+1)th stage is our controlled absorption stage and its probability is denoted by $p_{i,\,n+1}$, $1 \leq i \leq n$ when the treatment moves from i to n+1.

5. The state space of our model is W= {1, 2, ...n, n+1} ,where(n+1) is controlled stage which is absorption stage.

The transition probability matrix is below

$$P=\begin{pmatrix} p_{1,1} & p_{1,2} \cdots \cdots & p_{1,n} & p_{1,n+1} \\ \vdots & & & \vdots \\ p_{n,1} & p_{n,2} \cdots \cdots & p_{n,n} & p_{n,n+1} \\ 0 & 0 \ldots \ldots & 0 & 1 \end{pmatrix} \quad (22)$$

where $p_{i,j}$ means the probability of moving from stage i to stage j, the last row denote the absorption row and last column denotes the absorption probabilities from various stages.

In our model we consider the treatment for one cycle starts from fair control stage *u* then can move to any stage, it passes through the poor control stage *v*, very poor control stage *w* and finishing their moving in fair control stage *u*

3.2 ANALYSIS :

Let N be the random variable indicating the number of loops *u*-*v*-*w*-*u* are completed before its absorption.

Now we form a matrix U of order *(n-1)* X *(n-1)* which is formed by deleting *u*-th row, *u*-th column, last row and last column from P matrix.

Then we form a matrix V of order *(n-1)* X *(n-1)* which is formed by deleting *v*-th row, *v*-th column, last row and last column from P matrix.

Similarly we form a matrix W of order *(n-1)* X *(n-1)* which is formed by deleting *w*-th row, *w*-th column, last row and last column from P matrix.

$$U=\begin{pmatrix} p_{1,1} \cdots & p_{1,u-1} & p_{1,u+1} \cdots & p_{1,n} \\ \vdots & & & \vdots \\ p_{u-1,1} \cdots & p_{u-1,u-1} & p_{u-1,u+1} \cdots & p_{u-1,n} \\ p_{u+1,1} \cdots & p_{u+1,u-1} & p_{u+1,u+1} \cdots & p_{u+1,n} \\ \vdots & & & \vdots \\ p_{n,1} \cdots & p_{n,u-1} & p_{n,u+1} \cdots \cdots & p_{n,n} \end{pmatrix} \quad (23)$$

$$V=\begin{pmatrix} p_{1,1} \cdots & p_{1,v-1} & p_{1,v+1} \cdots & p_{1,n} \\ \vdots & & & \vdots \\ p_{v-1,1} \cdots & p_{v-1,v-1} & p_{v-1,v+1} \cdots & p_{v-1,n} \\ p_{v+1,1} \cdots & p_{v+1,v-1} & p_{v+1,v+1} \cdots & p_{v+1,n} \\ \vdots & & & \vdots \\ p_{n,1} \cdots & p_{n,v-1} & p_{n,v+1} \cdots \cdots & p_{n,n} \end{pmatrix} \quad (24)$$

$$W=\begin{pmatrix} p_{1,1} \cdots & p_{1,w-1} & p_{1,w+1} \cdots & p_{1,n} \\ \vdots & & & \vdots \\ p_{w-1,1} \cdots & p_{w-1,w-1} & p_{w-1,w+1} \cdots & p_{w-1,n} \\ p_{w+1,1} \cdots & p_{w+1,w-1} & p_{w+1,w+1} \cdots & p_{w+1,n} \\ \vdots & & & \vdots \\ p_{n,1} \cdots & p_{n,w-1} & p_{n,v+1} \cdots \cdots & p_{n,n} \end{pmatrix} \quad (25)$$

Let U_u be the next transition probability vector of transition from stage *u* to stages in S/ {*v*},

$$U_u=(p_{u,1},p_{u,2},\ldots p_{u,v-1},p_{u,v+1},..p_{u,n}) \quad (26)$$

Let V_v be the next transition probability vector of transition from stage *v* to stages in S/ {*w*},

$$V_v=(p_{v,1},p_{v,2},\ldots p_{v,w-1},p_{v,w+1},..p_{v,n}) \quad (27)$$

Let W_w be the next transition probability vector of transition from stage *w* to stages in S/ {*u*},

$$W_w=(p_{w,1},p_{w,2},\ldots p_{w,u-1},p_{w,u+1},..p_{w,n}) \quad (28)$$

Let R_u be the next transition probability of entries from stages S/ {*u*} to stage *u*,

$$R_u=(p_{1,u},p_{2,u},\ldots p_{u-1,u},p_{u+1,u},..p_{n,u})^T \quad (29)$$

Let R_v be the next transition probability of entries from stages S/{*v*} to stage *v*,

$$R_v=(p_{1,v},p_{2,v},\ldots..p_{v-1,v},p_{v+1,v},\ldots..p_{n,v})^T \quad (30)$$

Let R_w be the next transition probability of entries from stages S/{*w*} to stage *w*,

$$R_w=(p_{1,w},p_{2,w},\ldots..p_{w-1,w},p_{w+1,w},\ldots..p_{n,,w})^T \quad (31)$$

Equation (8), (9) and (10) are column vectors of (n-1) x 1 type.

The project starts from any stage i with starting probability a_i where

$$\hat{A}=(a_1,a_2,\ldots\ldots\ldots,a_n) \text{ with } \Sigma a_i =1 \quad (32)$$

Let $a= (a_1,a_2\ldots.. a_{u-1},a_{u+1},\ldots a_n)$ (33)

be the starting probability vector from any stage in S/{u}.

Let S_u, S_v, and S_w be the column vectors of type (n-1)x1 of probabilities of absorption in stage n+1 from stages S/{*u*},S/{*v*} and S/{*w*} respectively.

$$S_u =(p_{1,n+1},p_{2,n+1},\ldots\ldots p_{u-1,n+1},p_{u+1,n+1},\ldots..p_{n,,n+1})^T \quad (34)$$

$$S_v =(p_{1,n+1},p_{2,n+1},\ldots\ldots p_{v-1,n+1},p_{v+1,n+1},\ldots..p_{n,,n+1})^T \quad (35)$$

$$S_w =(p_{1,n+1},p_{2,n+1},\ldots\ldots p_{w-1,n+1},p_{w+1,n+1},\ldots..p_{n,,n+1})^T \quad (36)$$

Case (I) Number of u-v-w-u loops is zero before absorption

Let q_k be the probability that the treatment makes exactly k triangular loops u-v-w-u before it enters absorption stage (n+1). We write down expressions for q_k ,$0 \leq k < \infty$. Let N be the number of triangular loops the treatment completes before it enters absorption stage (n+1). The probability q_0 of making no triangular loop is given by

$$P(N=0)= (a\,(I - U)^{-1} S_u)+(a_u + a\,(I - U)^{-1} R_u)[(p_{u,n+1} +U_u\,(I - V)^{-1} S_v)+(p_{u,v}+U_u\,(I - V)^{-1} R_v)$$

$$[\,(p_{v,n+1} +V_v\,(I - W)^{-1} S_w) + (p_{v,w} + V_v\,(I - W)^{-1} R_w)\,(p_{w,n+1} + W_w\,(I - U)^{-1} S_u)]\,] \quad (37)$$

In the above equation there are four types of events considered before absorption. They are namely

(i) Blood glucose level does not enter stage *u* ,

(ii) Blood glucose level enters stage *u* but does not enter stage *v* ,

(iii) Blood glucose level enters stage *u* and moves to stage *v* but does not enter stage *w* ,

(iv) Blood glucose level enters stage *u* then moves to stage *v* and stage *w* but does not enter again stage *u* before the treatment absorption.

The first term of (16), (a (I - U)$^{-1}$ S_u) is the probability that blood glucose level enters stage in S/{u} and after making number of transition in S/{u} absorption occurs which is the probability of the event stated as (i). The probability of the event (ii) is given by (a_u + a (I - U)$^{-1}$ R_u) ($p_{u,n+1}$ +U_u (I - V)$^{-1}$ S_v). Here the treatment enters stage u but does not enter stage v before absorption. We may note that the probability of the event (iii) is given by (a_u + a (I - U)$^{-1}$ R_u) ($p_{u,v}$ +U_u (I - V)$^{-1}$ R_v) ($p_{v,n+1}$ +V_v (I – W)$^{-1}$ S_w). Here the treatment enters stage u and moves to stage v but does not enter stage w before absorption. Similarly we easily note that the probability of the event (iv) is given by (a_u + a (I - U)$^{-1}$ R_u)($p_{u,v}$ +U_u (I - V)$^{-1}$ R_v) ($p_{v,w}$ + V_v (I – W)$^{-1}$ R_w) ($p_{w,n+1}$ + W_w (I – U)$^{-1}$ S_u). Here the treatment enters stage u then moves to stage v and stage w but does not enter again stage u before absorption.

Case(II) Number of u-v-w-u loops is one before absorption.

$$P(N=1)= (a_u + a (I - U)^{-1} R_u) (p_{u,v} + U_u (I – V)^{-1} R_v) (p_{v,w} +V_v (I –W)^{-1}R_w)$$
$$(p_{w,,u} +W_w (I – U)^{-1} R_u) [(p_{u,n+1} +U_u (I - V)^{-1} S_v)+(p_{u,v}+U_u (I – V)^{-1} R_v)$$
$$[(p_{v,n+1} +V_v (I – W)^{-1} S_w) + (p_{v,w} + V_v (I – W)^{-1} R_w) (p_{w,n+1} + W_w (I – U)^{-1} S_u)]] \quad (38)$$

The first bracket is the probability that the treatment starts from stage u or it starts from S/{u} moves in S/{u} and enter stage u. The second bracket is the probability that the treatment in the next transition from stage u goes to stage v or it goes to stages in S/{v} , moves there and it enters stage v. The third bracket is the probability that the treatment in the next transition from stage v goes to stage w or it goes to stage S/{w}, moves there and it enters stage w. The fourth bracket is the probability the treatment in the next transition from stage w goes to stage u or it goes to stages in S/{u}, moves there and it enters again stage u. We may note here that the probability of one *u-v-w-u* loop is the product of the three probabilities which is as follows given by ($p_{u,v}$ + U_u (I – V)$^{-1}$ R_v) ($p_{v,w}$ +V_v (I –W)$^{-1}$$R_w$) ($p_{w,u}$ +W_w (I – U)$^{-1}$ R_u).

The term ($p_{u,n+1}$ +U_u (I - V)$^{-1}$ S_v) of the first square bracket in (38) is the probability that the treatment is absorbed from stage u or it goes to stage in S/{v}, moves there, it is absorbed from stages S/{v}.

The term given by ($p_{u,v}$ + U_u (I – V)$^{-1}$ R_v)($p_{v,n+1}$ +V_v (I – W)$^{-1}$ S_w) is the probability that the treatment after stage u visits v but does not enter stage w before absorption.

The term given by ($p_{u,v}$ + U_u (I – V)$^{-1}$ R_v)($p_{v,w}$ + V_v (I – W)$^{-1}$ R_w) ($p_{w,n+1}$ + W_w (I – U)$^{-1}$ S_u) is the probability that the treatment after stage u visits v and after visiting v visits stage w but after that it does not enter stage u before absorption.

Case(III) Number of u-v-w-u loops is two before absorption.

$$P(N=2)= (a_u + a (I - U)^{-1} R_u)[(p_{u,v} + U_u (I – V)^{-1} R_v) (p_{v,w} +V_v (I –W)^{-1}R_w)$$
$$(p_{w,,u} +W_w (I – U)^{-1} R_u)]^2 [(p_{u,n+1} +U_u (I - V)^{-1} S_v)+(p_{u,v}+U_u (I – V)^{-1} R_v)$$
$$[(p_{v,n+1} +V_v (I – W)^{-1} S_w) + (p_{v,w} + V_v (I – W)^{-1} R_w) (p_{w,n+1} + W_w (I – U)^{-1} S_u)]] \quad (39)$$

In (18) (a_u + a (I - U)$^{-1}$ R_u) is the probability that the treatment enters stage u. Then we may note that [($p_{u,v}$ + U_u (I – V)$^{-1}$ R_v) ($p_{v,w}$ +V_v (I –W)$^{-1}$$R_w$) ($p_{w,,u}$ +W_w (I – U)$^{-1}$ R_u)]^2 is the probability of two loops. Other remaining factors are explained earlier for equation (38).

Number of u-v-w-u loops is k before absorption.

Using similar argument we can write q_k the probability of exactly k loops (*u-v-w-u*) as follows

$$P(N=k)= (a_u + a (I - U)^{-1} R_u)[(p_{u,v} + U_u (I – V)^{-1} R_v) (p_{v,w} +V_v (I –W)^{-1}R_w)$$
$$(p_{w,u} +W_w (I – U)^{-1} R_u)]^k [(p_{u,n+1} +U_u (I - V)^{-1} S_v)+(p_{u,v}+U_u (I – V)^{-1} R_v)$$
$$[(p_{v,n+1} +V_v (I – W)^{-1} S_w) + (p_{v,w} + V_v (I – W)^{-1} R_w) (p_{w,n+1} + W_w (I – U)^{-1} S_u)]] \quad (40)$$

The probability generating function of q_k is $\Phi(x) = \sum_{k=0}^{\infty} q_k x^k$, $0 \le x \le 1$ (41)

Then we substitute the equation (37),(38),(39) & (40) in (41)

$$\Phi(x)= (a (I - U)^{-1} S_u)+[(a_u + a (I - U)^{-1} R_u)[(p_{u,n+1} +U_u (I - V)^{-1} S_v)+(p_{u,v}+U_u (I – V)^{-1} R_v)$$
$$[(p_{v,n+1} + V_v (I – W)^{-1} S_w) + (p_{v,w} + V_v (I – W)^{-1} R_w) (p_{w,n+1} + W_w (I – U)^{-1} S_u)]]$$
$$[1/1-[(p_{u,v} + U_u (I – V)^{-1} R_v) (p_{v,w} +V_v (I –W)^{-1}R_w) (p_{w,,u} +W_w (I – U)^{-1} R_u)]x]] \quad (42)$$

$$\Phi(x)=\alpha +[(1- \alpha) \beta [1/1-(1- \beta)x]] \quad (43)$$

Where $\alpha= (a (I - U)^{-1} S_u)$, $1- \alpha = (a_u + a (I - U)^{-1} R_u)$

$$\beta= (p_{u,n+1} +U_u (I - V)^{-1} S_v)+(p_{u,v}+U_u (I – V)^{-1} R_v)$$
$$[(p_{v,n+1} V_v (I – W)^{-1} S_w) + (p_{v,w} + V_v (I – W)^{-1} R_w) (p_{w,n+1} + W_w (I – U)^{-1} S_u)]$$

$$1- \beta = (p_{u,v} + U_u (I – V)^{-1} R_v) (p_{v,w} +V_v (I –W)^{-1}R_w) (p_{w,,u} +W_w (I – U)^{-1} R_u)$$

To check that $\Phi(x)$ is generating function of probabilities we can derive $\Phi(1)=1$ by putting $x = 1$.

Now we can calculate the expected number of u-v-w-u loops completed before absorption and variance using $\Phi(x)$.

$$E(N) = (1- \alpha)(1- \beta) / \beta . \quad (44)$$

$$Var(N) = (1- \alpha)(1- \beta) (1 + \alpha - \alpha \beta) / \beta^2 . \quad (45)$$

3.3 NUMERICAL EXAMPLES

Example 1:

Now we consider the following transition probability matrix for the application of results. We consider a discrete Markov chain with 5 stages where the 5-th stage is absorption stage. we assume the triangular loop 1-2-3-1 .

$$P=\begin{pmatrix} 0.2 & 0.2 & 0.2 & 0.3 & 0.1 \\ 0.2 & 0.2 & 0.2 & 0.3 & 0.1 \\ 0.2 & 0.2 & 0.2 & 0.3 & 0.1 \\ 0.2 & 0.2 & 0.2 & 0.3 & 0.1 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} \quad (46)$$

We work out the example for two initial probability vectors:

Case I Â=(1, 0, 0, 0) and Case II Â=(.25,0.25,0.25,0.25)

Case I

In this case the treatment starts only from stage 1. That is fair control stage .Now we calculate the probability generating function, mean and variance of number of cycle 1-2-3-1 Completed before the controlled stage. Here stage 2 denote poor control stage and stage 3 denote very poor control stage.

We note that matrix

$$U=\begin{pmatrix} 0.2 & 0.2 & 0.3 \\ 0.2 & 0.2 & 0.3 \\ 0.2 & 0.2 & 0.3 \end{pmatrix} \text{ \& } (I-U)^{-1}=\begin{pmatrix} 1.667 & 0.667 & 1 \\ 0.667 & 1.667 & 1 \\ 0.667 & 0.667 & 2 \end{pmatrix}$$

Similarly we can calculate $(I-V)^{-1}$ and $(I-W)^{-1}$

Also $S_u=(0.1\ 0.1\ 01)^T$; $U_u=(0.2\ 0.2\ 0.3)$; $R_u=(0.2\ 0.2\ 0.2)^T$.

Similarly we can calculate S_v, S_w, U_v, U_w, R_v and R_w

Using equation (21) we get ,

$\alpha=0$, $1-\alpha=1$, $\beta=0.703259$, $1-\beta=0.29674$;

$E(N)=(1-\alpha)(1-\beta)/\beta=0.42149$;

$Var(N)=(1-\alpha)(1-\beta)(1+\alpha-\alpha\beta)/\beta^2=0.59999$;

Case II

In this case it starts from any stage n ,for $1\leq n\leq 4$ with probability 0.25.

That is Â=(.25,0.25,0.25,0.25) and we take the same transition matrix P in

equation (25).

E(N)=0.750 x 0.29674/0.703259=0.316462

Var(N)=(0.750 x0.29674 x1.0741)/0.49457=0.64160.

Example 2

Now we consider the following transition probability matrix with 5 stages and

5-th stage is controlled blood glucose level. In example 2 we increase the probability of controlled blood glucose in the given transition matrix.

$$P=\begin{pmatrix} 0.1 & 0.1 & 0.1 & 0.1 & 0.6 \\ 0.1 & 0.1 & 0.1 & 0.1 & 0.6 \\ 0.1 & 0.1 & 0.1 & 0.1 & 0.6 \\ 0.1 & 0.1 & 0.1 & 0.1 & 0.6 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

Let the initial probability vector: Â=(1, 0, 0, 0)

We note that matrix

$$U=\begin{pmatrix} 0.1 & 0.1 & 0.1 \\ 0.1 & 0.1 & 0.1 \\ 0.1 & 0.1 & 0.1 \end{pmatrix} \text{ ; } (I-U)^{-1}=\begin{pmatrix} 1.143 & 0.143 & 0.143 \\ 0143 & 1.143 & 0.143 \\ 0.143 & 0.143 & 1.143 \end{pmatrix}$$

Similarly we can calculate $(I - V)^{-1}$ and $(I - W)^{-1}$

Also note that

$S_u = (0.1\ 0.1\ 0.1)^T$; $U_u = (0.1\ 0.1\ 0.1)$; $R_u = (0.1\ 0.1\ 0.1)^T$;

Similarly we can calculate S_v, S_w, U_v, U_w, R_v and R_w.

Using equation (21) we get ,

E(N) = 0.00293.

Var(N)= 0.002937.

From example 1& 2 we observed the behavior of E(N) and Var(N). When the transition probability increases in absorption stage, the value of E(N) and Var(N) decrease .

In example 1, we worked out the two types of initial probability vectors in which the value of E(N) & Var(N) is nearly equal in both cases.

Conclusion

In Model 1, if a diabetic person start the treatment in fair control stage then the person can reduce the expected value of number of r-s-r cycle. In Model 2, when the transition probability of controlled blood glucose level (absorption stage) is high, the expected value of r-s-r cycle is decrease.

Conflict of interest:

In this research paper no conflicts were identified or observed throughout the study

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