Advances in Nonlinear Variational Inequalities
ISSN: 1092-910X
Vol 26 No. 4 (2023)
Received: 24-08-2023 Revised: 22-10-2023 Accepted: 30-10-2023

Mathematical Modelling Of Municipal Solid Waste Management In Spherical Fuzzy Environment

Manbir Kaur[a)] and Rakesh Yadav[b)]

Department of Mathematics, School of Chemical Engineering & Physical Sciences, Lovely Professional University, Phagwara-143005, Punjab, India

[a)]*kmanbir20@gmail.com*
[b)]*rakesh.21798@lpu.ac.in*

Abstract. A Spherical fuzzy model induced with teaching learning based optimization technique is developed for supporting the municipal solid waste management under fuzzy environment. Spherical fuzzy set's ability to capture imprecise and contradictory information results in a substantial contribution to decision-making issues. Thus, we introduce SFLPP in a spherical fuzzy environment in this article, which entails maximization of truthiness and minimization of indeterminacy and falsity membership functions. In present era TLBO is gaining the popularity of being less complex and only two algorithmic parameters based algorithm. This study introduced a mathematical model to include all of the major components of municipal solid waste management. To deal with uncertainty, the mathematical model of municipal solid waste management is defined using a spherical fuzzy environment. The goal of this research is to determine the current state of waste management in the Dinanagar area of Punjab, India. Finally, the mathematical model is in possession of long-term waste management in the study area, Dinanagar city in Punjab, India. The findings of comparing the suggested model to the current framework show that the new model provides better solutions in terms of sustainability.

INTRODUCTION

Humans are frequently confronted with several decision-making challenges, including uncertainty or ambiguity, in their daily lives. The information in decision-making situations is not always easy to explain in terms of crisp numbers, thus fuzzy sets are a better option. Spherical fuzzy sets are the newest extensions of Intuitionistic fuzzy sets, proposing an independent neutral degree from the other parameters [1, 2, 3, 4, 5, 6]. Most academics are likely to prefer spherical fuzzy sets in the near future since their principles are strong enough to develop further. In spherical fuzzy optimization, we look for the best possible solution that can be found in the face of inadequate, imprecise, or ambiguous data. Most real-world optimization issues have fuzzy mathematical models [7, 8, 9], despite the fact that they are frequently supposed to be crisp for the sake of solving them quickly. In such instances, one normally strives to find a solution that is as approachable as possible and feasible given the decision maker's (DM) requirements. To arrive at such a satisfactory result, fuzzy optimization in an interactive manner is required, with the DM being asked to first describe his or her preferences and expectations. The constraints of intuitionistic, Pythagorean, and neutrosophic fuzzy sets can be described using the following illustration: In multi-attribute decision-making problems, if the acceptance degree of selecting an alternative is 0.7, the rejection degree of the alternatives is 0.6, and the indeterminacy or neutral degree of selecting the alternative is found to be 0.9, the situation is outside the coverage of the above uncertain sets. As a result, the spherical fuzzy set provides a valid decision-making tool in a fuzzy environment in these cases. The basic concept of this article is introduced from [10].

The studies [11, 12, 13, 14, 15, 16] contribute to evaluating the current status of MSWM in India and suggest some significant improvements for better waste management. For articles on MSWM modeling, see [17] for a review of previously published models on MSW generation and to propose implementation guidelines that will provide a compromise between environmental and economically efficient model development.The authors of [18, 19] also contribute their work to the mathematical modelling of MSWM.

We investigate the LPP in a spherical fuzzy environment in this paper. The deterministic version is calculated using SFS theory after the parameters are converted to SF numbers. Many SF optimization models are proposed to find the best SFLPP solution. A numerical example of a real-life problem, municipal solid waste management, is solved to demonstrate the applicability and validity of the SF optimization models. Based on the offered work, conclusions and future scope are also considered. In [20], an efficient algorithm is developed by the authors with the help of combination theory and the combined fuzzy TOPSIS method to choose the best suitable alternative out of all possible single and hybrid energy resources in Turkey. The latest articles based on MSWM in Indian cities [21, 22, 23, 24, 25, 26, 27, 28] discussed about challenges, current status, sustainable waste management, life-cycle assessment and so on but the concept of mathematical modelling is not introduced by any author. Moreover, some authors worked on waste

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management issues at global level like [29, 30, 31, 32, 33, 34, 35, 36]. In article [37] the authors worked on Forecasting of MSW generation using non-linear auto regressive neural models. The authors of article [38] introduced LandGEM mathematical model for quantification of landfill gas emissions and energy production potential in Tirupati Municipal solid waste disposal. In paper [39] the authors used multi-criteria decision making under fuzzy environment for the evaluation of municipal solid waste management scenarios. In article [40] the authors write about detailed literature review on application of deterministic, stochastic and fuzzy linear programming models in solid waste management studies. In paper [41] the authors introduced method based on spherical fuzzy to optimize the transportation problem. One more article [42] based on spherical fuzzy is prepared to prioritize the indicators responsible for sustainable municipal solid waste management using SF-AHP and SF-TOPSIS after working and writing an article [43] over the implementation analysis of municipal solid waste management in study area.

PRELIMINARIES

Fuzzy set

Definition: Ordinary Fuzzy set : [44] Let **U** be the universe of discourse then a fuzzy set $\widetilde{X_f}$ in **U** is defined as follows:

$$\widetilde{X_f} = \{(x, t_{\widetilde{X_f}}(x)) \,|\, x \in \mathbf{U}\}$$

such that $t_{\widetilde{X_f}}(x) : \mathbf{U} \longrightarrow [0,1]$ is the membership function and $0 \le t_{\widetilde{X_f}}(x) \le 1 \; \forall x \in \mathbf{U}$, represents the membership degree of each $x \in \mathbf{U}$ to $\widetilde{X_f}$.

Definition: Triangular fuzzy number: The ordered triplets $\widetilde{X_f}(t_1, t_2, t_3)$ denoting lower value, middle value & upper value of a m.f, is said to be triangular fuzzy number if its m.f is defined as:

$$t_{\widetilde{X_f}}(x) = \begin{cases} \dfrac{x - t_1}{t_2 - t_1} & if\ t_1 \le x \le t_2 \\ \dfrac{t_3 - x}{t_3 - t_2} & if\ t_2 \le x \le t_3 \\ 0 & otherwise \end{cases} \tag{1}$$

Defuzzification of triangular fuzzy number

In literature, there are various methods available to defuzzify the fuzzy number [45]. Among all, the centroid method is most widely used as it gives a deterministic value on the basis of center of gravity of fuzzy numbers. In this article, the same method is used to obtain the defuzzified version of the triangular fuzzy number which is defined as follows:

$$def(\widetilde{X}) = \frac{\int_x x t_{\widetilde{X}}(x) dx}{\int_x t_{\widetilde{X}}(x) dx} \tag{2}$$

where x is the output variable and $t_{\widetilde{X}}(x)$ is the m.f.

Hence, by calculating the integrals of (2), the defuzzified version of the TFN $\widetilde{X}(t_1, t_2, t_3)$ is:

$$def(\widetilde{X}) = \frac{(\frac{t_3 - t_1}{2})(\frac{3t_1 + t_2 + 3t_3}{3})}{(\frac{t_3 - t_1}{2})} = \frac{3t_1 + t_2 + 3t_3}{3} \tag{3}$$

Definition: Spherical fuzzy sets: [1] Let **U** be the universal set then a fuzzy set $\widetilde{X}$ in **U** is defined as follows:

$$\widetilde{X_s} = \{(x; t_{\widetilde{X_s}}(x), i_{\widetilde{X_s}}(x), f_{\widetilde{X_s}}(x)) \,|\, x \in \mathbf{U}\}$$

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such that $t_{\widetilde{X_s}}(x) : \mathbf{U} \longrightarrow [0,1]$,$i_{\widetilde{X_s}}(x) : \mathbf{U} \longrightarrow [0,1]$ and $f_{\widetilde{X_s}}(x) : \mathbf{U} \longrightarrow [0,1]$ are the truthiness m.f , indeterminacy m.f and falsity m.f respectively. Also $0 \leq t^2_{\widetilde{X_s}}(x) + i^2_{\widetilde{X_s}}(x) + f^2_{\widetilde{X_s}}(x) \leq 1 \; \forall x \in \mathbf{U}$, represents the membership degree for each element $x \in \mathbf{U}$ to $\widetilde{X_s}$.
Definition: Spherical triangular fuzzy number(STFN) The spherical triangular fuzzy number $\widetilde{X_s} = (t,i,f) = (t_1,t_2,t_3;i_1,i_2,i_3;f_1,f_2,f_3)$ s.t $t,i,f \in [01]$

The m.f for t, i, and f can be defined by using (1):

$$t_{\widetilde{X_s}}(x) = \begin{cases} \dfrac{x-t_1}{t_2-t_1} & if\ t_1 \leq x \leq t_2 \\ \dfrac{t_3-x}{t_3-t_2} & if\ t_2 \leq x \leq t_3 \\ 0 & otherwise \end{cases} \tag{4}$$

$$i_{\widetilde{X_s}}(x) = \begin{cases} \dfrac{x-i_1}{i_2-i_1} & if\ i_1 \leq x \leq i_2 \\ \dfrac{i_3-x}{i_3-i_2} & if\ i_2 \leq x \leq i_3 \\ 0 & otherwise \end{cases} \tag{5}$$

$$f_{\widetilde{X_s}}(x) = \begin{cases} \dfrac{x-f_1}{f_2-f_1} & if\ f_1 \leq x \leq f_2 \\ \dfrac{f_3-x}{f_3-f_2} & if\ f_2 \leq x \leq f_3 \\ 0 & otherwise \end{cases} \tag{6}$$

RANKING OF STFN

In the literature, the [46] authors proposed ranking functions for ordering the SFNs but the procedure is not clear, so the existing ranking functions are not universal and cannot be used for ordering or defuzzify the SFNs. To overcome this situation, we develop a new score function using the centroid method (2) and used this to develop an algorithm to optimize the transportation problems.
Definition: Score function & Accuracy function Let $\widetilde{X_s} = (t,i,f) = (t_1,t_2,t_3;i_1,i_2,i_3;f_1,f_2,f_3)$ such that $t,i,f \in [01]$ be a STFN. The score functions for the m.f $t_{\widetilde{X_s}}(x), i_{\widetilde{X_s}}(x)$, and$f_{\widetilde{X_s}}(x)$ are denoted and defined respectively as follows:

$$Sc(t_{\widetilde{X_s}}) = \frac{3t_1+t_2+3t_3}{3}; Sc(i_{\widetilde{X_s}}) = \frac{3i_1+i_2+3i_3}{3}; Sc(f_{\widetilde{X_s}}) = \frac{3f_1+f_2+3f_3}{3} \tag{7}$$

Now, the accuracy function of $\widetilde{X_s}$ is denoted and defined by:

$$\begin{aligned} Acc(\widetilde{X_s}) &= \frac{Sc(t_{\widetilde{X_s}}) + Sc(i_{\widetilde{X_s}}) + Sc(f_{\widetilde{X_s}})}{3} \\ &= \frac{(3t_1+t_2+3t_3)+(3i_1+i_2+3i_3)+(3f_1+f_2+3f_3)}{9} \end{aligned} \tag{8}$$

Example: Let $\widetilde{X_s} = (2.5,3,4.5;2.4,3,4.8;2.3,3,5)$ and $\widetilde{Y_s} = (4.5,5,6.3;4.3,5,6.5;4,5,6.7)$ be the two STFNs, then their respective accuracy functions using proposed method (8) are 2.5000 and 3.922

Theorem: Let the score functions for truthiness, indeterminacy, & falsity are linear functions and accuracy function is the average of their score functions then the accuracy function is also a linear function.
proof: Let $\tilde{X} = (t_1,t_2,t_3;i_1,i_2,i_3;f_1,f_2,f_3)$ & $\tilde{Y} = (t_1',t_2',t_3';i_1',i_2',i_3';f_1',f_2',f_3')$ are any two STFNs. Then for any scalar

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ISSN: 1092-910X
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a, we have

$$\begin{aligned}
Acc(a\tilde{X}+\tilde{Y}) =& Acc(a(t_1,t_2,t_3;i_1,i_2,i_3;f_1,f_2,f_3)+(t_1',t_2',t_3';i_1',i_2',i_3';f_1',f_2',f_3'))\\
=& Acc((at_1,at_2,at_3;ai_1,ai_2,ai_3;af_1,af_2,af_3)\\
&+(t_1',t_2',t_3';i_1',i_2',i_3';f_1',f_2',f_3'))\\
=& Acc(at_1+t_1',at_2+t_2',at_3+t_3';ai_1+i_1',ai_2+i_2',ai_3+i_3';\\
& af_1+f_1',af_2+f_2',af_3+f_3')\\
=& \{(3(at_1+t_1')+(at_2+t_2')+3(at_3+t_3'))+(3(ai_1+i_1')+(ai_2+i_2')+3(ai_3+i_3'))\\
&+(3(af_1+f_1')+(af_2+f_2')+3(af_3+f_3'))\}/9\\
=& \frac{(3t_1+t_2+3t_3)+(3i_1+i_2+3i_3)+(3f_1+f_2+3f_3)}{9}\\
&+\frac{(3t_1'+t_2'+3t_3')+(3i_1'+i_2'+3i_3')+(3f_1'+f_2'+3f_3')}{9}\\
=& aAcc(\tilde{X})+Acc(\tilde{Y})
\end{aligned}$$

Hence, Acc() is linear function.

Definition: Ordering of STFNs using accuracy function
Let $\tilde{X}=(t_1,t_2,t_3;i_1,i_2,i_3;f_1,f_2,f_3)$ & $\tilde{Y}=(t_1',t_2',t_3';i_1',i_2',i_3';f_1',f_2',f_3')$ are any two STFNs. Then,

1. If $Acc(\tilde{X})\geq Acc(\tilde{Y})$ then $\tilde{X}\geq\tilde{Y}$
2. If $Acc(\tilde{X})\leq Acc(\tilde{Y})$ then $\tilde{X}\leq\tilde{Y}$
3. If $Acc(\tilde{X})=Acc(\tilde{Y})$ then $\tilde{X}=\tilde{Y}$
4. If $\tilde{X}\geq\tilde{Y}$ then $\max(\tilde{X},\tilde{Y})=\tilde{X}$
5. If $\tilde{X}\leq\tilde{Y}$ then $\min(\tilde{X},\tilde{Y})=\tilde{X}$

METHODOLOGY

The linear model, sometimes known as the linear programming problem (LPP), is the most popular, most straightforward, and frequently used mathematical programming model. The LPP model is straightforward and may be used to a variety of real-world problems, including those involving transportation, supply chain management, job assignment, manufacturing and production planning, supplier selection, and other issues. Over several decades, traditional LPP has changed and expanded. Uncertainty is included in the LPP, which is often used by academics. A typical and frequently used mathematical programming issue is the linear programming problem. Numerous academicians have researched the numerous fuzzy environment extensions of the linear programming issue, including ordinary fuzzy, intuitionistic, Pythagorean, neutrosophic, and others.

Spherical fuzzy linear programming problem [10]

The introduction of a spherical fuzzy idea known as the spherical fuzzy linear programming problem (SFLPP) is presented as a further expansion of LPP.
The extension of LPP by introducing SF concept named as SFLPP and can be expressed as:
Model-I: This model presents the SFLPP in which coefficients of objective functions are represented in spherical fuzzy number(SFN) but the coefficients of variables and right hand side constants of constraints are represented in real numbers.

$$Optimize\ Z=\sum_{k=1}^{K}\widetilde{c}_k x_k$$

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Subject to

$$\sum_{k=1}^{K} a_{ik}x_k \leq,=,\geq b_i, \quad \forall i=1,2,....,I$$

$$x_k \geq 0, \quad \forall k=1,2,....,K$$

where $\widetilde{c}_k$ denotes a SFN and a_{ik}, b_i are real numbers.

Model-II: This model presents the SFLPP in which coefficients of objective functions are represented by real numbers but the coefficients of variables and right hand side constants of constraints are represented in SF numbers.

$$Optimize\ Z = \sum_{k=1}^{K} c_k x_k$$

Subject to

$$\sum_{k=1}^{K} \widetilde{a}_{ik}x_k \leq,=,\geq \widetilde{b}_i, \quad \forall i=1,2,....,I$$

$$x_k \geq 0, \quad \forall k=1,2,....,K$$

where c_k is a real number and $\widetilde{a}_{ik}$ and $\widetilde{b}_i$ are SF numbers.

Model-III: This model presents the SFLPP in which coefficients of objective functions, the coefficients of variables and right hand side constants of constraints all are represented in SF numbers.

$$Optimize\ Z = \sum_{k=1}^{K} \widetilde{c}_k x_k$$

Subject to

$$\sum_{k=1}^{K} \widetilde{a}_{ik}x_k \leq,=,\geq \widetilde{b}_i, \quad \forall i=1,2,....,I$$

$$x_k \geq 0, \quad \forall k=1,2,....,K$$

where $\widetilde{c}_k$ denotes a SFN and $\widetilde{a}_{ik}$, $\widetilde{b}_i$ are also SF numbers.

The different spherical fuzzy parameters are converted into the crisp version by using Eqs. 4,5,6 for each membership degree assigned by the decision makers. The obtained crisp version can solved by using any suitable optimization method.

Algorithm

- **Step-I** Define the problem and examine the parameters which contains uncertainty.
- **Step-II** Formulate the mathematical model in terms of spherical fuzzy environment.
- **Step-III** Formulate the truthiness, indeterminacy and falsity membership functions under spherical fuzzy environment.
- **Step-IV** Decide the degree of confirmation based on the previous knowledge of decision maker for the truthiness, indeterminacy and falsity memberships of spherical fuzzy numbers.
- **Step-V** Convert the defined SFLPP into the crisp version using equations 4,5,6.
- **Step-VI** Solve the obtained optimization model by using robust optimization method or suitable technique to obtain the desired solution of SFLPP.
- **Step-VII** Application of novel method to solve mathematically modeled Municipal Solid Waste Management(MSWM) problem.

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TABLE I. Estimated ward wise population and waste generation[a].

Ward No.	Total Houses	Population	No. of houses paying for WM	Waste generated (in kg/day)
1.	497	2236	120	457
2.	466	2097	125	429
3.	523	2354	150	481
4.	425	1912	130	391
5.	388	1746	165	357
6.	438	1971	125	403
7.	393	1768	50	362
8.	415	1868	120	382
9.	256	1152	100	236
10.	276	1242	75	254
11.	428	1926	80	394
12.	264	1188	50	243
13.	283	1274	70	260
14.	231	1040	90	213
15.	354	1593	50	326
Total	5637	25376	1500	5188

[a] Source: Municipal corporation of Dinanagar

Assessment of MSWM system in study area [47]

Waste management in Punjab, India: Data is taken from cpcb.nic.in "Total 167 ULBs are responsible for MSW management in the Punjab state. There are 26 Class-I, 47 Class-II, 25 Class-III cities/towns, 56 Nagar Panchayats & 13 Municipal Corporations in the State. Total Solid Waste generation in Punjab is around 4338.37 TPD, out of which 4278.86 TPD of waste is collected, 1894.04 TPD is treated and 2384.82is land-filled. House to house collection is practiced in 142 ULBs, segregation is practiced in 113 ULBs, storage facility is available in 98ULBs and covered transportation is being practiced in 143 ULBs. There are a total of 1572 composting facilities operation in the State along with 1 Vermi-composting (at Shamchaurassi) and 2 RDF/palletization facilities (at Bathinda and Ludhiana). The Department of Local Govt. (DLG) has adopted the decentralized approach for management of solid waste. Total 1572 processing sites (composting pits) have been setup in the State till the end of year 2020 for processing of wet waste. Channelization of recyclable waste is being done through 235 Material Recovery Facilities (MRFs) and only inert waste will go to landfill sites. 2 Waste to Energy plant (at Bathinda and Ludhiana) are installed but yet not in operation. Total 143 no. of solid waste dumping sites have been setup by the ULBs in the State. Waste management strategies in Punjab are more difficult to implement due to a variety of factors limiting the performance of SWM processes in cities, including human resources, financial, and political restraints."
Waste management in Dinanagar: Insufficient money for maintaining waste management services, non-supportive conduct of urban local bodies, unawareness, and lack of enthusiasm among residents are all obstacles that the Municipal Corporation of Dinanagar(MCD) in Punjab, India is facing. The Indian government establishes new targets to minimize the quantity of biodegradable waste in landfill or dumping sites. To achieve this target, the composting is primarily solution in small municipality like Dinanagar. Using this technique the waste volume is reduced by 50-65%. Composting can be done either manually or mechanically. Presently, 33 number of manual composting pits at different locations are successfully maintained by Municipal Corporation of Dinanagar(MCD). There is need to improve the collection, treatment and disposal rate of generated waste. The population of Dinanagar is 25376 (table I) inhabitants and 15 square km land area which is further distributed in 15 wards with average of 376 inhabitants each. The MSW generated is only about 0.2-0.25 kg/capita/day, of which 60% is wet waste, 40% is dry waste and only 50-60% of generated waste in the city is collected with the utilization of presently provided collection services. Material recovery facility(MRF) is also adapted by the MCD to treat all recoverable material from the perspective of cost management. The need of effective and efficient MSWM is increasing as the poor management contributes adverse effects on economy, health, environment and one major threat is increase in Green House Gas(GHG) emissions which further responsible for global warming due to uncollected/untreated waste lying in open dump sites. The inefficient collection and treatment services are the reasons for gap existence in present practices of MCD.

The technique of five R's is responsible for sustainable and zero waste objective of solid waste management. The

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definition of 5 R's are: REFUSE, REDUCE, REUSE, RECYCLE, RECOVER.

- **Refuse:** Say no to non-biodegradable material or products.
- **Reduce:** Replace the non-biodegradable with biodegradable material.
- **Reuse:** Do not use disposable products. Replace them with more sustainable alternatives.
- **Recycle:** Use the material which can be transform into another usable form.
- **Recover:** Convert the organic waste into compost.

By adopting these 5R's policy in daily routine life, the challenges of waste management can be achieved effectively and efficiently.

MATHEMATICAL FORMULATION OF MSWM: MULTI OBJECTIVE PROBLEM

Indices notation:

$i-$ Generators of solid waste, $i = 1$ to I
$j-$ Segregation stations(SS), $j = 1$ to J
$k-$ Material recovery stations(MRS), $k = 1$ to K
$l-$ Composting stations(CS), $l = 1$ to L
$m-$ Incineration stations(IS), $m = 1$ to M
$n-$ Anaerobic digestion stations(ADS), $n = 1$ to N
$o-$ Landfill sites(LF) , $o = 1$ to O

Decision variables & Binary variables:

$x_{ij}^{seg}-$ Amount of waste transferred from waste generators to segregation stations per day
$x_{jk}^{mrf}-$ Amount of waste transferred from segregation stations to MRS per day
$x_{jl}^{comp}-$ Amount of waste transferred from segregation stations to composting stations per day
$x_{jm}^{inc}-$ Amount of waste transferred from segregation stations to incineration stations per day
$x_{jn}^{anb}-$ Amount of waste transferred from segregation stations to ADS per day
$x_{jo}^{lf}-$ Amount of waste transferred from segregation stations to landfill sites per day
$x_j^{seg}, x_k^{mrf}, x_l^{comp}, x_m^{inc}, x_n^{anb}, x_o^{lf}$ are the **binary variables** , which takes value 1 if SS, MRS, CS, IS, ADS, LF facilities are provided in the area under study and 0 otherwise.

Input values/parameters:

Fixed costs such as maintenance etc at various stations of waste management are denoted as follows:

$c_j^{seg}-$ Fixed cost related to segregation stations per unit weight
$c_k^{mrf}-$ Fixed cost related to MRS per unit weight
$c_l^{comp}-$ Fixed cost related to composting stations per unit weight
$c_m^{inc}-$ Fixed cost related to incineration stations per unit weight
$c_n^{anb}-$ Fixed cost related to ADS per unit weight
$c_o^{lf}-$ Fixed cost related to landfill sites per unit weight

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Capacity of various waste management stations to process the waste are denoted as follows:

p_j^{seg} – Waste processing capacity at segregation stations per day
p_k^{mrf} – Waste processing capacity at MRS per day
p_l^{comp} – Waste processing capacity at composting stations per day
p_m^{inc} – Waste processing capacity at incineration stations per day
p_n^{anb} – Waste processing capacity at ADS per day
p_o^{lf} – Waste processing capacity at landfill sites per day

Processing costs of waste at different stations of waste management are denoted as follows:

$\tilde{pc}_j^{seg}$ – Waste processing cost at segregation stations per unit weight
$\tilde{pc}_k^{mrf}$ – Waste processing cost at MRS per unit weight
$\tilde{pc}_l^{comp}$ – Waste processing cost at composting stations per unit weight
$\tilde{pc}_m^{inc}$ – Waste processing cost at incineration stations per unit weight
$\tilde{pc}_n^{anb}$ – Waste processing cost at ADS per unit weight
$\tilde{pc}_o^{lf}$ – Waste processing cost at landfill sites per unit weight

Transportation costs of waste from one station to another station are denoted as follows:

$\tilde{tc}_{ij}^{seg}$ – Transportation cost of waste transferred from waste generators to segregation stations per day
$\tilde{tc}_{jk}^{mrf}$ – Transportation cost of waste transferred from segregation stations to MRS per day
$\tilde{tc}_{jl}^{comp}$ – Transportation cost of waste transferred from segregation stations to composting stations per unit weight
$\tilde{tc}_{jm}^{inc}$ – Transportation cost of waste transferred from segregation stations to incineration stations per unit weight
$\tilde{tc}_{jn}^{anb}$ – Transportation cost of waste transferred from segregation stations to ADS per unit weight
$\tilde{tc}_{jo}^{lf}$ – Transportation cost of waste transferred from segregation stations to landfill sites per unit weight

Revenue produce from different stations of waste management are denoted as follows:

$\tilde{R}_k^{mrf}$ – Revenue generated from MRS per unit weight
$\tilde{R}_l^{comp}$ – Revenue generated from composting stations per unit weight
$\tilde{R}_m^{inc}$ – Revenue generated from incineration stations per unit weight
$\tilde{R}_n^{anb}$ – Revenue generated from ADS per unit weight
$\tilde{R}_o^{lf}$ – Revenue generated from landfill sites per unit weight

Fractions of waste transferred from segregation station to different stations are denoted as below:

α^{mrf} – Fraction of recoverable waste transfer to MR stations.
α^{comp} – Fraction of compostable waste transfer to composting stations.
α^{inc} – Fraction of dry waste transfer to incineration stations.

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$\alpha^{anb}-$ Fraction of waste transfer to anaerobic digestion stations.
$\alpha^{lf} = 1-(\alpha^{mrf}+\alpha^{comp}+\alpha^{inc}+\alpha^{anb})$- Fraction of untreated waste transfer to the landfill.

Emission coeff. for GHG effect

$coeff_j^{seg}-$ GHG emission coeff. from SS per unit weight per day
$coeff_k^{mrf}-$ GHG emission coeff. from MRS per unit weight per day
$coeff_l^{comp}-$ GHG emission coeff. from composting stations per unit weight per day
$coeff_m^{inc}-$ GHG emission coeff. from incineration stations per unit weight per day
$coeff_n^{anb}-$ GHG emission coeff. from ADS per unit weigh per dayt
$coeff_o^{lf}-$ GHG emission coeff. from landfill sites per unit weight per day
$\widetilde{\mathbf{W_i}}-$ Total generated waste at source i per day
Note: Some parameters denoted with (˜) tilde sign is to be considered as fuzzy in nature.

DEFINING OBJECTIVE FUNCTIONS

The following three objective functions are defined to optimize:

- **OP1:**Minimization of total cost which includes transportation cost, maintenance cost, processing cost or other costs related to different stations of solid waste management system.
- **OP2:**Minimization of GHG emissions like carbon and methane from different waste management stations.
- **OP3:**Minimization of final waste disposal at landfill sites to approach the aim of zero waste.

Mathematical expressions for the defined objectives is as follows:

$$Min\ OP1 = Fixed\ cost + Processing\ cost + Transportation\ cost - Revenue$$

where

$$Fixed\ cost(FC) = \sum_{j=1}^{J} c_j^{seg} x_j^{seg} + \sum_{k=1}^{K} c_k^{mrf} x_j^{mrf} + \sum_{l=1}^{L} c_l^{comp} x_l^{comp} + \sum_{m=1}^{M} c_m^{inc} x_m^{inc} + \sum_{n=1}^{N} c_n^{anb} x_n^{anb} + \sum_{o=1}^{O} c_o^{lf} x_o^{lf}$$

$$Processing\ cost(PC) = pc_j^{seg} \sum_{j=1}^{J}\sum_{i=1}^{I} x_{ij}^{seg} + pc_k^{mrf} \sum_{k=1}^{K}\sum_{j=1}^{J} x_{ij}^{mrf} + pc_l^{comp} \sum_{l=1}^{L}\sum_{j=1}^{J} x_{ij}^{comp} + pc_m^{inc} \sum_{m=1}^{M}\sum_{j=1}^{J} x_{ij}^{inc} + pc_n^{anb} \sum_{n=1}^{N}\sum_{j=1}^{J} x_{ij}^{anb} + pc_o^{lf} \sum_{o=1}^{O}\sum_{j=1}^{J} x_{ij}^{lf}$$

$$Transportation\ cost(TC) = \sum_{i=1}^{I}\sum_{j=1}^{J} tc_{ij}^{seg} x_{ij}^{seg} + \sum_{j=1}^{J}\sum_{k=1}^{K} tc_{jk}^{mrf} x_{jk}^{mrf} + \sum_{j=1}^{J}\sum_{l=1}^{L} tc_{jl}^{comp} x_{jl}^{comp} + \sum_{j=1}^{J}\sum_{m=1}^{M} tc_{jm}^{inc} x_{jm}^{inc} + \sum_{j=1}^{J}\sum_{n=1}^{N} tc_{jn}^{anb} x_{jn}^{anb} + \sum_{j=1}^{J}\sum_{o=1}^{O} tc_{jo}^{lf} x_{jo}^{lf}$$

Advances in Nonlinear Variational Inequalities
ISSN: 1092-910X
Vol 26 No. 4 (2023)
Received: 24-08-2023 Revised: 22-10-2023 Accepted: 30-10-2023

$$Revenue(R) = R_k^{mrf}\sum_{j=1}^{J}\sum_{k=1}^{K}x_{jk}^{mrf} + R_l^{mrf}\sum_{j=1}^{J}\sum_{l=1}^{L}x_{jl}^{comp} + R_m^{mrf}\sum_{j=1}^{J}\sum_{m=1}^{M}x_{jm}^{inc} + R_n^{mrf}\sum_{j=1}^{J}\sum_{n=1}^{N}x_{jn}^{anb} + R_o^{mrf}\sum_{j=1}^{J}\sum_{o=1}^{O}x_{jo}^{lf}$$

$$Min\ OP2 = coeff_j^{seg}\sum_{j=1}^{J}\sum_{i=1}^{I}x_{ij}^{seg} + coeff_k^{mrf}\sum_{k=1}^{K}\sum_{j=1}^{J}x_{jk}^{mrf} + coeff_l^{comp}\sum_{l=1}^{L}\sum_{j=1}^{J}x_{jl}^{comp} + coeff_m^{inc}\sum_{m=1}^{M}\sum_{j=1}^{J}x_{jm}^{inc} + coeff_n^{anb}\sum_{n=1}^{N}\sum_{j=1}^{J}x_{jn}^{anb} + coeff_o^{lf}\sum_{o=1}^{O}\sum_{j=1}^{J}x_{jo}^{lf}$$

$$Min\ OP3 = \sum_{j=1}^{J}\sum_{o=1}^{O}x_{jo}^{lf}$$

subject to the constraints

$$\sum_{j=1}^{J}x_{ij}^{seg} = W_i, \quad i = 1 \text{ to } I$$

$$\sum_{k=1}^{K}\sum_{j=1}^{J}x_{jk}^{mrf} = \sum_{i=1}^{I}\sum_{j=1}^{J}\alpha^{mrf}x_{ij}^{seg}$$

$$\sum_{l=1}^{L}\sum_{j=1}^{J}x_{jl}^{comp} = \sum_{i=1}^{I}\sum_{j=1}^{J}\alpha^{comp}x_{ij}^{seg}$$

$$\sum_{m=1}^{M}\sum_{j=1}^{J}x_{jm}^{inc} = \sum_{i=1}^{I}\sum_{j=1}^{J}\alpha^{inc}x_{ij}^{seg}$$

$$\sum_{n=1}^{N}\sum_{j=1}^{J}x_{jn}^{anb} = \sum_{i=1}^{I}\sum_{j=1}^{J}\alpha^{anb}x_{ij}^{seg}$$

$$\sum_{o=1}^{O}\sum_{j=1}^{J}x_{jo}^{lf} = \sum_{i=1}^{I}\sum_{j=1}^{J}\alpha^{lf}x_{ij}^{seg}$$

Note: $\alpha^{lf} = 1 - (\alpha^{mrf} + \alpha^{comp} + \alpha^{inc} + \alpha^{anb})$

$$\sum_{i=0}^{I}x_{ij}^{seg} \le p_j x_j^{seg}, \quad j = 1\ to\ J$$

$$\sum_{j=0}^{J}x_{jk}^{mrf} \le p_k x_k^{mrf}, \quad k = 1\ to\ K$$

$$\sum_{j=0}^{J}x_{jl}^{comp} \le p_l x_l^{comp}, \quad l = 1\ to\ L$$

$$\sum_{j=0}^{J}x_{jm}^{inc} \le p_m x_m^{inc}, \quad m = 1\ to\ M$$

$$\sum_{j=0}^{J}x_{jn}^{anb} \le p_n x_n^{anb}, \quad n = 1\ to\ N$$

$$\sum_{j=0}^{J}x_{jo}^{lf} \le p_o x_o^{lf}, \quad o = 1\ to\ O$$

$$where \quad x_{ij}^{seg} \ge 0,\ x_{jk}^{mrf} \ge 0,\ x_{jl}^{comp} \ge 0,\ x_{jm}^{inc} \ge 0,\ x_{jn}^{anb} \ge 0,\ x_{jo}^{lf} \ge 0.$$

$$and \quad x_j^{seg},\ x_k^{mrf},\ x_l^{comp},\ x_m^{inc},\ x_n^{anb},\ x_o^{lf}\ \ are\ \ 0\ \ or\ \ 1.$$

Advances in Nonlinear Variational Inequalities
ISSN: 1092-910X
Vol 26 No. 4 (2023)
Received: 24-08-2023 Revised: 22-10-2023 Accepted: 30-10-2023

NUMERICAL ILLUSTRATION

The following considerations are according to the present situation of Dinanagar, study area. After a detailed discussion with the municipal manager and other workers of municipal corporation of Dinanagar(MCD), the collected data is summarized in the tables III,IV,V according to which the various parameters like different costs, GHG emission coefficient, and waste generation at sources and so on are decided. The objectives of this study are defined and shown in table II.

TABLE II. Objectives defined for optimization

Objectives	functions	Target	Current
Minimize OP1	Total cost	₹20000(with revenue)	4 lac per month
Minimize OP2	GHG emissions	Reduce to some extent	Not available
Minimize OP3	landfill disposal	500 kg per day	more than 2.5 ton per day

Consider the following:
$i-$ Generators of solid waste, $i = 1$ to $I, Take\ I = 3$
$j-$ Segregation stations(SS), $j = 1$ to $J, Take\ J = 1$
$k-$ Material recovery stations(MRS), $k = 1$ to $K, Take\ K = 3$
$l-$ Composting stations(CS), $l = 1$ to $L, Take\ L = 3$
$m-$ Incineration stations(IS), $m = 1$ to $M, Take\ M = 0$
$n-$ Anaerobic digestion stations(ADS), $n = 1$ to $N, Take\ N = 1$
$o-$ Landfill sites(LF) , $o = 1$ to $O, Take\ O = 1$

TABLE III. Parametric values

Parameters	per capita per day
Cost of WM	₹7 - 8
GHG emissions	$3.16 Kg\ CO_2 - eq$
Waste generation	200 gm

TABLE IV. Amount of waste treated at different stations

Stations	% of waste	waste per day(in tons)
Compost	50 -60	2.5 - 3
MRF	20 -30	1 - 1.8
ANB	15 - 20	0.75 - 1.2
Landfill	5 - 6	0.25 - 0.3

TABLE V. Cost (in ₹1000s) related to different stations for MSWM in Dinanagar

Stations	Fixed cost	Transportation	Processing
Segregation	18	27	27
MRF	40	60	60
Compost	20	30	30
ANB	10	15	15
LF	2	3	3

Note: Total budget for MSWM is 50 lac per year(Source: MCD). 10 % of total budget is reserved by MCD

TABLE VI. Green House Gas emission values

Stations	GHG*(per ton)	waste treated(in tons)*	waste (in Kg)	GHG emission(per Kg)
MRF	4	60	1800	0.12
Compost	25700	40	2000	1285
ANB	46200	340	1000	135.68
Landfill	12900	600	200	4.3

Note: * [48] to estimate GHG (in $Kg\ CO_2 - eq$)

Mathematical formulation of MSWM under consideration is as follows:

$$
\begin{aligned}
Min\ \ OP1 =& FC + PC + TC - R \\
& where \\
FC =& c_1^{seg}x_1^{seg} + c_1^{mrf}x_1^{mrf} + c_2^{mrf}x_2^{mrf} + c_3^{mrf}x_3^{mrf} + c_1^{comp}x_1^{comp} + c_2^{comp}x_2^{comp} + c_3^{comp}x_3^{comp} + \\
& c_1^{anb}x_1^{anb} + c_1^{lf}x_1^{lf} \\
PC =& pc_1^{seg}(x_{11}^{seg} + x_{21}^{seg} + x_{31}^{seg}) + pc_1^{mrf}x_{11}^{mrf} + pc_2^{mrf}x_{12}^{mrf} + pc_3^{mrf}x_{13}^{mrf} + pc_1^{comp}x_{11}^{comp} + pc_2^{comp}x_{12}^{comp} + \\
& pc_3^{comp}x_{13}^{comp} + pc_1^{anb}x_{11}^{anb} + pc_1^{lf}x_{11}^{lf} \\
TC =& tc_{11}^{seg}x_{11}^{seg} + tc_{21}^{seg}x_{21}^{seg} + tc_{31}^{seg}x_{31}^{seg} + tc_{11}^{mrf}x_{11}^{mrf} + tc_{12}^{mrf}x_{12}^{mrf} + tc_{13}^{mrf}x_{13}^{mrf} + tc_{11}^{comp}x_{11}^{comp} + tc_{12}^{comp}x_{12}^{comp} + \\
& tc_{13}^{comp}x_{13}^{comp} + tc_{11}^{anb}x_{11}^{anb} + tc_{11}^{lf}x_{11}^{lf} \\
R =& R_1^{mrf}x_{11}^{mrf} + R_2^{mrf}x_{12}^{mrf} + R_3^{mrf}x_{13}^{mrf} + R_1^{comp}x_{11}^{comp} + R_2^{comp}x_{12}^{comp} + R_3^{comp}x_{13}^{comp} + R_1^{anb}x_{11}^{anb} + R_1^{lf}x_{11}^{lf} \\
Min\ \ OP2 =& coeff_1^{seg}(x_{11}^{seg} + x_{21}^{seg} + x_{31}^{seg}) + coeff_1^{mrf}x_{11}^{mrf} + coeff_2^{mrf}x_{12}^{mrf} + coeff_3^{mrf}x_{13}^{mrf} + coeff_1^{comp}x_{11}^{comp} + \\
& coeff_2^{comp}x_{12}^{comp} + coeff_3^{comp}x_{13}^{comp} + coeff_1^{anb}x_{11}^{anb} + coeff_1^{lf}x_{11}^{lf} \\
Min\ \ OP3 =& x_{11}
\end{aligned}
$$

$subject\ to\ the\ constraints$

$$
\begin{aligned}
x_{11}^{seg} =& W_1;\ x_{21}^{seg} = W_2;\ x_{31}^{seg} = W_3 \\
x_{11}^{mrf} + x_{12}^{mrf} + x_{13}^{mrf} =& \alpha_{mrf}(x_{11}^{seg} + x_{21}^{seg} + x_{31}^{seg}) \\
x_{11}^{comp} + x_{12}^{comp} + x_{13}^{comp} =& \alpha_{comp}(x_{11}^{seg} + x_{21}^{seg} + x_{31}^{seg}) \\
x_{11}^{anb} =& \alpha_{anb}(x_{11}^{seg} + x_{21}^{seg} + x_{31}^{seg}) \\
x_{11}^{lf} =& \alpha_{lf}(x_{11}^{seg} + x_{21}^{seg} + x_{31}^{seg}) \\
x_{11}^{seg} + x_{21}^{seg} + x_{31}^{seg} \leq& p_1^{seg}x_1^{seg} \\
x_{11}^{mrf} \leq& p_1^{mrf}x_1^{mrf};\ x_{12}^{mrf} \leq p_2^{mrf}x_2^{mrf};\ x_{13}^{mrf} \leq p_3^{mrf}x_3^{mrf} \\
x_{11}^{comp} \leq& p_1^{comp}x_1^{comp};\ x_{12}^{comp} \leq p_2^{comp}x_2^{comp};\ x_{13}^{comp} \leq p_3^{comp}x_3^{comp} \\
x_{11}^{anb} \leq& p_1^{anb}x_1^{anb};\ x_{11}^{lf} \leq p_1^{lf}x_1^{lf}
\end{aligned}
$$

Advances in Nonlinear Variational Inequalities
ISSN: 1092-910X
Vol 26 No. 4 (2023)
Received: 24-08-2023 Revised: 22-10-2023 Accepted: 30-10-2023

TABLE VII. Score function and Accuracy function values of fuzzy parameters

S.No	Notations	Fuzzy value(FV)	$t = FV * 0.9$	$i = FV * 0.4$	$f = FV * 0.2$	$t_1 = t - 0.2 * t$	$t_2 = t$	$t_3 = t + 0.1 * t$	$i_1 = i - 0.9 * i$	$i_2 = i$	$i_3 = i + 0.1 * i$
1	pc_1^{seg}	0.17400	0.15660	0.06960	0.03480	0.12528	0.15660	0.17226	0.00696	0.06960	0.07656
2	pc_1^{mrf}	1.21000	1.08900	0.48400	0.24200	0.87120	1.08900	1.19790	0.04840	0.48400	0.53240
3	pc_2^{mrf}	0.96700	0.87030	0.38680	0.19340	0.69624	0.87030	0.95733	0.03868	0.38680	0.42548
4	pc_3^{mrf}	1.80750	1.62675	0.72300	0.36150	1.30140	1.62675	1.78943	0.07230	0.72300	0.79530
5	pc_1^{comp}	0.55200	0.49680	0.22080	0.11040	0.39744	0.49680	0.54648	0.02208	0.22080	0.24288
6	pc_2^{comp}	0.46900	0.42210	0.18760	0.09380	0.33768	0.42210	0.46431	0.01876	0.18760	0.20636
7	pc_3^{comp}	0.42900	0.38610	0.17160	0.08580	0.30888	0.38610	0.42471	0.01716	0.17160	0.18876
8	pc_1^{anb}	0.48300	0.43470	0.19320	0.09660	0.34776	0.43470	0.47817	0.01932	0.19320	0.21252
9	pc_1^{lf}	0.48500	0.43650	0.19400	0.09700	0.34920	0.43650	0.48015	0.01940	0.19400	0.21340
10	tc_{11}^{seg}	0.15220	0.13698	0.06088	0.03044	0.10958	0.13698	0.15068	0.00609	0.06088	0.06697
11	tc_{21}^{seg}	0.15760	0.14184	0.06304	0.03152	0.11347	0.14184	0.15602	0.00630	0.06304	0.06934
12	tc_{31}^{seg}	0.20190	0.18171	0.08076	0.04038	0.14537	0.18171	0.19988	0.00808	0.08076	0.08884
13	tc_{11}^{mrf}	0.96750	0.87075	0.38700	0.19350	0.69660	0.87075	0.95783	0.03870	0.38700	0.42570
14	tc_{12}^{mrf}	0.96670	0.87003	0.38668	0.19334	0.69602	0.87003	0.95703	0.03867	0.38668	0.42535
15	tc_{13}^{mrf}	1.20880	1.08792	0.48352	0.24176	0.87034	1.08792	1.19671	0.04835	0.48352	0.53187
16	tc_{11}^{comp}	0.46140	0.41526	0.18456	0.09228	0.33221	0.41526	0.45679	0.01846	0.18456	0.20302
17	tc_{12}^{mrf}	0.58540	0.52686	0.23416	0.11708	0.42149	0.52686	0.57955	0.02342	0.23416	0.25758
18	tc_{13}^{mrf}	0.43060	0.38754	0.17224	0.08612	0.31003	0.38754	0.42629	0.01722	0.17224	0.18946
19	tc_{11}^{anb}	0.48300	0.43470	0.19320	0.09660	0.34776	0.43470	0.47817	0.01932	0.19320	0.21252
20	tc_{11}^{lf}	0.48500	0.43650	0.19400	0.09700	0.34920	0.43650	0.48015	0.01940	0.19400	0.21340
21	R_1^{mrf}	7.50000	6.75000	3.00000	1.50000	5.40000	6.75000	7.42500	0.30000	3.00000	3.30000
22	R_2^{mrf}	6.67000	6.00300	2.66800	1.33400	4.80240	6.00300	6.60330	0.26680	2.66800	2.93480
23	R_3^{mrf}	7.50000	6.75000	3.00000	1.50000	5.40000	6.75000	7.42500	0.30000	3.00000	3.30000
24	R_1^{anb}	7.00000	6.30000	2.80000	1.40000	5.04000	6.30000	6.93000	0.28000	2.80000	3.08000
25	R_1^{lf}	5.00000	4.50000	2.00000	1.00000	3.60000	4.50000	4.95000	0.20000	2.00000	2.20000
26	W_1	2115.0000	1903.5000	846.0000	423.0000	1522.8000	1903.50000	2093.85000	84.60000	846.00000	930.60000
27	W_2	1637.0000	1473.3000	654.8000	327.4000	1178.6400	1473.30000	1620.63000	65.48000	654.80000	720.28000
28	W_3	1436.0000	1292.4000	574.4000	287.2000	1033.9200	1292.40000	1421.64000	57.44000	574.40000	631.84000

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Advances in Nonlinear Variational Inequalities
ISSN: 1092-910X
Vol 26 No. 4 (2023)
Received: 24-08-2023 Revised: 22-10-2023 Accepted: 30-10-2023

TABLE VIII. Score function and Accuracy function values of fuzzy parameters(continuation of tableVII)

S.No	Notations	Fuzzy value(FV)	$f_1 = f - 0.9 * f$	$f_2 = f$	$f_3 = f + 0.1 * f$	Sc(t)	Sc(i)	Sc(f)	Acc(FV)
1	pc_1^{seg}	0.17400	0.00348	0.03480	0.03828	0.34974	0.10672	0.05336	0.16994
2	pc_1^{mrf}	1.21000	0.02420	0.24200	0.26620	2.43210	0.74213	0.37107	1.18177
3	pc_2^{mrf}	0.96700	0.01934	0.19340	0.21274	1.94367	0.59309	0.29655	0.94444
4	pc_3^{mrf}	1.80750	0.03615	0.36150	0.39765	3.63308	1.10860	0.55430	1.76533
5	pc_1^{comp}	0.55200	0.01104	0.11040	0.12144	1.10952	0.33856	0.16928	0.53912
6	pc_2^{comp}	0.46900	0.00938	0.09380	0.10318	0.94269	0.28765	0.14383	0.45806
7	pc_3^{comp}	0.42900	0.00858	0.08580	0.09438	0.86229	0.26312	0.13156	0.41899
8	pc_1^{anb}	0.48300	0.00966	0.09660	0.10626	0.97083	0.29624	0.14812	0.47173
9	pc_1^{lf}	0.48500	0.00970	0.09700	0.10670	0.97485	0.29747	0.14873	0.47368
10	tc_{11}^{seg}	0.15220	0.00304	0.03044	0.03348	0.30592	0.09335	0.04667	0.14865
11	tc_{21}^{seg}	0.15760	0.00315	0.03152	0.03467	0.31678	0.09666	0.04833	0.15392
12	tc_{31}^{seg}	0.20190	0.00404	0.04038	0.04442	0.40582	0.12383	0.06192	0.19719
13	tc_{11}^{mrf}	0.96750	0.01935	0.19350	0.21285	1.94468	0.59340	0.29670	0.94493
14	tc_{12}^{mrf}	0.96670	0.01933	0.19334	0.21267	1.94307	0.59291	0.29645	0.94414
15	tc_{13}^{mrf}	1.20880	0.02418	0.24176	0.26594	2.42969	0.74140	0.37070	1.18059
16	tc_{11}^{comp}	0.46140	0.00923	0.09228	0.10151	0.92741	0.28299	0.14150	0.45063
17	tc_{12}^{comp}	0.58540	0.01171	0.11708	0.12879	1.17665	0.35905	0.17952	0.57174
18	tc_{13}^{comp}	0.43060	0.00861	0.08612	0.09473	0.86551	0.26410	0.13205	0.42055
19	tc_{11}^{anb}	0.48300	0.00966	0.09660	0.10626	0.97083	0.29624	0.14812	0.47173
20	tc_{11}^{lf}	0.48500	0.00970	0.09700	0.10670	0.97485	0.29747	0.14873	0.47368
21	R_1^{mrf}	7.50000	0.15000	1.50000	1.65000	15.07500	4.60000	2.30000	7.32500
22	R_2^{mrf}	6.67000	0.13340	1.33400	1.46740	13.40670	4.09093	2.04547	6.51437
23	R_3^{mrf}	7.50000	0.15000	1.50000	1.65000	15.07500	4.60000	2.30000	7.32500
24	R_1^{anb}	7.00000	0.14000	1.40000	1.54000	14.07000	4.29333	2.14667	6.83667
25	R_1^{lf}	5.00000	0.10000	1.00000	1.10000	10.05000	3.06667	1.53333	4.88333
26	W_1	2115.0000	42.30000	423.00000	465.30000	4251.15000	1297.20000	648.60000	2065.65000
27	W_2	1637.0000	32.74000	327.40000	360.14000	3290.37000	1004.02667	502.01333	1598.80333
28	W_3	1436.0000	28.72000	287.20000	315.92000	2886.36000	880.74667	440.37333	1402.49333

Advances in Nonlinear Variational Inequalities
ISSN: 1092-910X
Vol 26 No. 4 (2023)
Received: 24-08-2023 Revised: 22-10-2023 Accepted: 30-10-2023

Mathematical model formulation using parametric values is as given below:

$$\begin{aligned}
Min\ \ OP1 =& 0.166x_1^{seg} + 1.21x_1^{mrf} + 0.805x_2^{mrf} + 0.404x_3^{mrf} + 0.46x_1^{comp} + 0.88x_2^{comp} + 0.644x_3^{comp} \\
&+ 0.323x_1^{anb} + 0.325x_1^{lf} + 0.174(x_{11}^{seg} + x_{21}^{seg} + x_{31}^{seg}) + 1.21x_{11}^{mrf} + 0.967x_{12}^{mrf} + 1.0875x_{13}^{mrf} \\
&+ 0.552x_{11}^{comp} + 0.469x_{12}^{comp} + 0.429x_{13}^{comp} + 0.483x_{11}^{anb} + 0.485x_{11}^{lf} + 0.1522x_{11}^{seg} + 0.1576x_{21}^{seg} \\
&+ 0.2019x_{31}^{seg} + 0.9675x_{11}^{mrf} + 0.9667x_{12}^{mrf} + 1.2088x_{13}^{mrf} + 0.464x_{11}^{comp} + 0.5854x_{12}^{comp} \\
&+ 0.4306x_{13}^{comp} + 0.483x_{11}^{anb} + 0.485x_{11}^{lf} - (7.5x_{11}^{mrf} + 6.67x_{12}^{mrf} + 7.5x_{13}^{mrf} + 0x_{11}^{comp} \\
&+ 0x_{12}^{comp} + 0x_{13}^{comp} + 7x_{11}^{anb} + 5x_{11}^{lf}) \\
Min\ \ OP2 =& 0(x_{11}^{seg} + x_{21}^{seg} + x_{31}^{seg}) + 0.0267x_{11}^{mrf} + 0.04x_{12}^{mrf} + 0.0533x_{13}^{mrf} + 449.75x_{11}^{comp} + \\
& 353.375x_{12}^{comp} + 481.875x_{13}^{comp} + 135.88x_{11}^{anb} + 4.3x_{11}^{lf} \\
Min\ \ OP3 =& x_{11}^{lf}
\end{aligned}$$

subject to the constraints

$$\begin{aligned}
x_{11}^{seg} =& 2115;\ x_{21}^{seg} = 1637;\ x_{31}^{seg} = 1436 \\
x_{11}^{mrf} + x_{12}^{mrf} + x_{13}^{mrf} =& \frac{9}{25}(x_{11}^{seg} + x_{21}^{seg} + x_{31}^{seg}) \\
x_{11}^{comp} + x_{12}^{comp} + x_{13}^{comp} =& \frac{1}{5}(x_{11}^{seg} + x_{21}^{seg} + x_{31}^{seg}) \\
x_{11}^{anb} =& \frac{2}{5}(x_{11}^{seg} + x_{21}^{seg} + x_{31}^{seg}) \\
x_{11}^{lf} =& \frac{1}{25}(x_{11}^{seg} + x_{21}^{seg} + x_{31}^{seg}) \\
x_{11}^{seg} + x_{21}^{seg} + x_{31}^{seg} \leq& 5000x_1^{seg} \\
x_{11}^{mrf} \leq& 400x_1^{mrf};\ x_{12}^{mrf} \leq 600x_2^{mrf};\ x_{13}^{mrf} \leq 800x_3^{mrf} \\
x_{11}^{comp} \leq& 700x_1^{comp};\ x_{12}^{comp} \leq 550x_2^{comp};\ x_{13}^{comp} \leq 750x_3^{comp} \\
x_{11}^{anb} \leq& 1000x_1^{anb};\ x_{11}^{lf} \leq 200x_1^{lf}
\end{aligned}$$

Mathematical model formulation using defuzzified parametric values is as given below:

The defuzzified values of parameters having fuzzy nature are shown in table VII and VIII. The degree of confirmation based on the previous knowledge of municipal manager and others is decided as (0.9, 0.4, 0.2) for truthiness,

Advances in Nonlinear Variational Inequalities
ISSN: 1092-910X
Vol 26 No. 4 (2023)
Received: 24-08-2023 Revised: 22-10-2023 Accepted: 30-10-2023

indeterminacy, & falsity memberships of spherical fuzzy numbers respectively.

$$\begin{aligned}
Min\ \ OP1 =& 0.166x_1^{seg} + 1.21x_1^{mrf} + 0.805x_2^{mrf} + 0.404x_3^{mrf} + 0.46x_1^{comp} + 0.88x_2^{comp} + 0.644x_3^{comp} \\
& + 0.323x_1^{anb} + 0.325x_1^{lf} + 0.16994(x_{11}^{seg} + x_{21}^{seg} + x_{31}^{seg}) + 1.18177x_{11}^{mrf} + 0.94444x_{12}^{mrf} \\
& + 1.7653x_{13}^{mrf} + 0.53912x_{11}^{comp} + 0.45806x_{12}^{comp} + 0.41899x_{13}^{comp} + 0.47173x_{11}^{anb} + 0.47368x_{11}^{lf} \\
& + 0.14865x_{11}^{seg} + 0.15392x_{21}^{seg} + 0.19719x_{31}^{seg} + 0.94493x_{11}^{mrf} + 0.94414x_{12}^{mrf} + 1.18059x_{13}^{mrf} \\
& + 0.45063x_{11}^{comp} + 0.57174x_{12}^{comp} + 0.42055x_{13}^{comp} + 0.47173x_{11}^{anb} + 0.47368x_{11}^{lf} \\
& - (7.32500x_{11}^{mrf} + 6.51437x_{12}^{mrf} + 7.32500x_{13}^{mrf} + 0x_{11}^{comp} + 0x_{12}^{comp} + 0x_{13}^{comp} \\
& + 6.83667x_{11}^{anb} + 4.88333x_{11}^{lf}) \\
Min\ \ OP2 =& 0(x_{11}^{seg} + x_{21}^{seg} + x_{31}^{seg}) + 0.0267x_{11}^{mrf} + 0.04x_{12}^{mrf} + 0.0533x_{13}^{mrf} + 449.75x_{11}^{comp} + \\
& 353.375x_{12}^{comp} + 481.875x_{13}^{comp} + 135.88x_{11}^{anb} + 4.3x_{11}^{lf} \\
Min\ \ OP3 =& x_{11}^{lf}
\end{aligned}$$

subject to the constraints

$$\begin{aligned}
x_{11}^{seg} =& 2065.65;\ x_{21}^{seg} = 1598.80333;\ x_{31}^{seg} = 1402.49333 \\
x_{11}^{mrf} + x_{12}^{mrf} + x_{13}^{mrf} =& \frac{9}{25}(x_{11}^{seg} + x_{21}^{seg} + x_{31}^{seg}) \\
x_{11}^{comp} + x_{12}^{comp} + x_{13}^{comp} =& \frac{1}{5}(x_{11}^{seg} + x_{21}^{seg} + x_{31}^{seg}) \\
x_{11}^{anb} =& \frac{2}{5}(x_{11}^{seg} + x_{21}^{seg} + x_{31}^{seg}) \\
x_{11}^{lf} =& \frac{1}{25}(x_{11}^{seg} + x_{21}^{seg} + x_{31}^{seg}) \\
x_{11}^{seg} + x_{21}^{seg} + x_{31}^{seg} \leq& 5000x_1^{seg} \\
x_{11}^{mrf} \leq& 400x_1^{mrf};\ x_{12}^{mrf} \leq 600x_2^{mrf};\ x_{13}^{mrf} \leq 800x_3^{mrf} \\
x_{11}^{comp} \leq& 700x_1^{comp};\ x_{12}^{comp} \leq 550x_2^{comp};\ x_{13}^{comp} \leq 750x_3^{comp} \\
x_{11}^{anb} \leq& 1000x_1^{anb};\ x_{11}^{lf} \leq 200x_1^{lf}
\end{aligned}$$

RESULTS & DISCUSSION

The optimal values of defined objectives are shown in table IX. As per records provided by data operator of MCD,

TABLE IX. Objective functions value after optimization

Objectives(units)	functions	without fuzzy	with fuzzy	fuzzy+TLBO
Min OP1(in ₹)	Total cost	15382	14555	12538
Min OP2(in $Kg\ CO_2 - eq$)	GHG emissions	1142000	1115300	1002600
Min OP3(in Kg/day)	landfill disposal	207.5200	202.6779	180.3245

the problem is defined. It is worth mentioning that some of the data is to be assumed due to unavailability of data and information. The study determines the current state of waste management in the study region, Dinanagar, Punjab, India. Finally, constructed and solved the mathematical model of solid waste management in the study area. The findings of comparing the suggested model to the current framework show that the new model provides better solutions in terms of sustainability.

CONCLUSION

In this study, a SFLPP model with TLBO was developed for supporting the municipal solid waste management under fuzzy environment. Spherical fuzzy set's ability to capture imprecise and contradictory information results in a substantial contribution to decision-making issues. Thus, we introduce SFLPP in a spherical fuzzy environment in this article, which entails maximization of truthiness and minimization of indeterminacy and falsity membership functions. In present era TLBO is gaining the popularity of being less complex and only two algorithmic parameters based algorithm. Due to this it become flexible to inculcate with other optimization techniques in the form of hybridization or modification, so that standard TLBO can be enhanced to perform well with fast convergence towards the local/global optima as compare to other optimization algorithms. In addition, a numerical example of MSWM is provided to demonstrate the applicability of the suggested SFLPP solution approach. The obtained results reflects that the proposed model has the capability to handle uncertainties involved at various stages of waste management. Comparison with current practices in study area demonstrated the advances of general solutions in the aspects of minimization of cost, GHG emission and landfill disposal. Some amendments and parameters estimation in this model could further increase the applicability to many other problems having fuzziness. The SFLPP can also used to solve other real-world problems, involving parameters contains uncertainty. The application of SFLPP to real-world problems, such as transportation, supplier selection, supply chain, inventory control, and portfolio management, is also an open door for academics.

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ISSN: 1092-910X
Vol 26 No. 4 (2023)
Received: 24-08-2023 Revised: 22-10-2023 Accepted: 30-10-2023

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