Advances in Nonlinear Variational Inequalities
ISSN: 1092-910X
Vol 26 No. 3 (2023)

Randic Skew - Hermitian Matrix and Randic Skew - Hermitian Energy of Mixed Middle Graphs

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Article History:

Received: *22-05-2023*

Revised: *26-07-2023*

Accepted: *25-08-2023*

Abstract:

Let 〖SH(M〗_di) be the matrix of Skew-Hermitian adjacency and let M_dibe an asymmetrical middle graph. A latterly weighed Skew-Hermitian connection matric can be obtained as we proceed assign a Randic the amount as to each curve and boundary in 〖SH(M〗_di). In light of here, we as a species describe the Randic Skew-Hermitian matrix using these Skew-Hermitian adjacency matrix features. 〖R^*〗_SH (M_di)=(〖r^*〗_SH)_xy within a asymmetrical graph M_di Whereas (〖r^*〗_SH)_xy=((-i)/√(d_x d_y)) (i=√(-1) if (v_x,v_y)) is an arc of M_di, (〖r^*〗_SH)_xy=(i/√(d_x d_y))(i=√(-1) if (v_x,v_y)) is an arc of M_di, (〖r^*〗_SH)_xy=((-1)/√(d_x d_y)) if (v_x,v_y) is an undirected edge of M_di, and (〖r^*〗_SH)_xy=0 if of M_di. The main purpose of this study is to calculate the Randic Skew-Hermitian matrix of a asymmetrical middle graph's characteristic polynomial. Moreover, we provide boundaries on the applicable asymmetrical middle graph's Randic Skew-Hermitian energy. Finally, we provide some findings regarding the asymmetrical middle graphs' Randic energy with skew-Hermitian .

Keywords: Skew-Hermitian Randic Matrix, asymmetrical Graph, Middle Graph, Energy, Randic Skew- Hermitian Energy, asymmetrical middle Graph.

1. Introduction

Only simple graphs devoid of loops and multi edges are taken into consideration in this paper. A graph M_{di} is derived from an undirected middle graph, in which case it is considered mixed. $M^*{}_{di}$ by positioning a subset of its edges. We denote $M^*{}_{di}$ the fundamental representation of M_{di} . It is obvious that a mixed middle graph discovers both the extreme cases of all edges undirected and all edges orientated. concludes both scenarios for every edge orientation. Take M as a mixed graph with a collection of vertices. it is shown by $V*(M_{di}) = \{v_1, v_2, \dots \dots \dots, v_\rho\}$ and a collection of edges $E*(M_{di})$ regarding $(v_x, v_y) \in V*(M_{di})$, We designate v_x, v_y $(or\ v_x \leftrightarrow v_y)$ as an undirected edge connecting two vertices of M_{di}. Indicate by v_x, v_y $(or\ v_x \to v_y)$ a directed edge (or arc) from v_x to v_y. Furthermore, let E^*M_{di} represent the entire a collection of undirected edges and $E^{**}M_{di}$ represent the entire collection of directed arcs. $E(M_{di})$ is undoubtedly the product of E^*M_{di} and $E^{**}M_{di}$. Considering a asymmetrical graph a asymmetrical tree (or asymmetrical bipartite graph) occurs when the asymmetrical graph's underlying graph is a tree. A vertex in (M_{di}) has the same order, dimensions, quantity, and degree as its vertex in M^*_{di}, generally speaking. When referring to terms and notations that are not explained here, we use Bondy and Murty [1].

Suppose that G* is a basic graph with vertex set $\{v_1, v_2, v_3 \ldots\ldots v_n\}$ {The n × n symmetric square matrix $S = S(G^*) = (s_{xy})$ is the adjacency matrix of a simple graph S of rank n. If v_x, v_y is an edge of G*, then (S_{xy}) = 1, otherwise $(S_{xy}) = 0$. The degree of vertex (v_x) is indicated by $d^*_x = d^*(v_x) = d_{G^*}(v_x)$ $(x = 1,2,\ldots\ldots n)$. Furthermore, if $v_i \in V(M_i)$ for a asymmetrical graph (M_i), we additionally denote $(d_x) = d(v_x) = d_{M^*}(V_X)$. $\varepsilon_S(G^*) \sum_{k=1}^{n} |\lambda_k|$. Their eigenvalues of $S(G^*)$are $\lambda_1, \lambda_2, \lambda_3, \ldots\ldots, \lambda_k$ is the definition of their the graph G*'s energy (handle the surveying Li, Shi, and Gutman [3] and Zhang, Li, and Gutman [2] for more information). First proposed by Randic [4] in 1975, the molecular structure-descriptor is shown to be the total of $\frac{1}{\sqrt{(d^*_x d^*_y)}}$ across every edge xy in G ∗. Currently, $R^* = R^*(G^*) = \sum_{x,y \in E(G^*)} \frac{1}{\sqrt{(d^*_x d^*_y)}}$. Their Randic index is which is called this. In [5-7], mathematical properties, innumerable chemical applications, and the mathematical chemistry of the Randic index were described. Motivated by this, we define a asymmetrical middle graph M_{di} Randic Skew – Hermitian matrix.

TESTIMONIUM

For the purpose of studying this paper, the following are necessary.

2. Skew- Hermitian Randic matrix

Given an asymmetrical graph (G^*) on its vertex set $\{v_1, v_2, \ldots, v_\rho\}$, the p × p matrix is the Randic Skew-Hermitian matrix an G.

$$R^*_{SH}(G^*) = \begin{cases} \frac{1}{\sqrt{d_x d_y}}, & \text{if } v_x \leftrightarrow v_y, \\ \frac{-i}{\sqrt{d_x d_y}}, & \text{if } v_x \rightarrow v_y, \\ \frac{i}{\sqrt{d_x d_y}}, & \text{if } v_y \rightarrow v_x, \\ 0 \quad , & \text{otherwise.} \end{cases}$$

3. $R^*_{SH}(G^*)$ **energy**

$R^*_{SH}(G^*)$expressed with the representation of $E_{R^*_{SH}}(G^*)$, is that the energy of $R^*_{SH}(G^*)$ Furthermore explained is the unique collection of scalar values that is the sum of the absolute significance of the real numbers of $R^*_{SH}(G^*)$.

$$\text{i.e } E_{R^*_{SH}}(G^*) = \sum_{k=1}^{n} |\lambda_k|.$$

4. Asymmetrical graph

An arbitrary graph with asymmetric elements A graph with the formula is made up of a collection as a point (X), a graph in which the set of nodes is joined together and all of the connections are unidirectional (Y), and directed edges, which are ordered pairs (or arcs). $M_i = (X, Y, A)$

Example 4.1

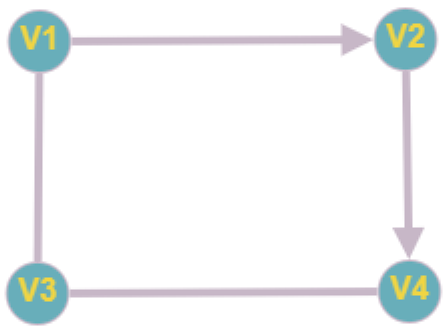

Figure 1: Asymmetrical Graph

5. Middle Graph

When two vertices of a graph (G^*) are adjacent, it means that they are either adjacent edges of $X(G^*) \cup Y(G^*)$, or one vertex of (G^*) and the other is an edge incident with it. This is known as the Middle graph $M(G^*)$.

Example 5.1

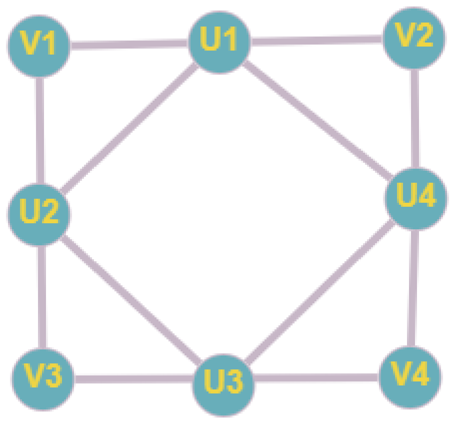

Figure 2: Middle Graph

6. Asymmetrical Middle Graph

The asymmetrical middle graph, or $M_{di}(G^*)$ of a graph (G^*), is the graph whose vertex set is $X(G^*) \cup Y(G^*)$, and two vertices are adjacent if and only if they are either adjacent edges of (G^*) or one vertex of (G^*) and the other is an edge incident with it. A graph $M_i(G^*)$ with the formula $G^* = (X, Y, A)$ contains a set of directed edges (or arcs) and a set of vertices (X), as well as a set of (undirected) edges (Y). An $M_d(G^*)$ graph has asymmetrical components. This graph is known as the asymmetrical middle graph. The symbol for it is $M_{di}(G^*)$.

Example 6.1

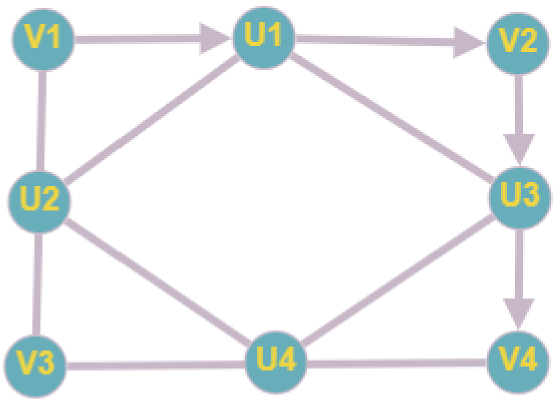

Figure 3: Asymmetrical Middle Graph

7. Randic Skew – Hermitian Characteristic Polynomial of an asymmetrical Graph

The characteristic polynomial of an asymmetrical graph M_i, Randic Skew-Hermitian matrix, To start, allow us to present certain essential definitions.

Let M_i become an asymmetrical graph of order p and let $R^*_{SH}(M_i)$ be its Randic Skew-Hermitian matrix.

Determine for each $v_{xy} \in M_i$, indicate the $R^*_{SH}(M_i)$ characteristic polynomial of $R^*_{SH}(M_i)$ of M_i .

$$P_{R^*_{SH}}(M_i, z) = -z^p - n_1 z^{p-1} - n_2 z^{p-2} - \cdots - n_p$$

8. Fundamental Properties

Firstly, we present asymmetrical middle graph instances along with their Randic Skew - Hermitian energies and Randic Skew – Hermitian Randic adjacency matrices.

8.1 Illustration

Randic Skew – Hermitian matrix of an asymmetrical middle connected graph

An adjacency matrix is a square matrix that has a finite graphs representation. To show if there is directed path between two vertices, values are used in the Skew-Hermitian Randic matrix. A vertex matrix or connection matrix are other common names for the adjacency matrix. If we consider (Md_i) to be the mixed graph of order 8×8, Skew – Hermitian Randic Mixed Middle cycle adjacency matrix as

Example 8.2.1

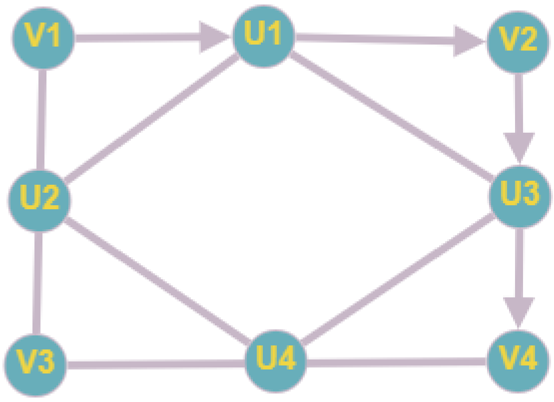

Figure 4: Asymmetrical Middle Connected Graph

$$R^*_{SH}((M_{di})) = \begin{cases} \dfrac{1}{\sqrt{d_x d_y}}, & \text{if } v_x \leftrightarrow v_y, \\ \dfrac{-i}{\sqrt{d_x d_y}}, & \text{if } v_x \rightarrow v_y, \\ \dfrac{i}{\sqrt{d_x d_y}}, & \text{if } v_y \rightarrow v_x, \\ 0, & \text{otherwise.} \end{cases}$$

$$(R^*{}_{SH}(M_{di}))K_8 = \begin{bmatrix} 0 & \frac{-i}{\sqrt{8}} & 0 & \frac{1}{\sqrt{8}} & 0 & 0 & 0 & 0 \\ \frac{i}{\sqrt{8}} & 0 & \frac{-i}{\sqrt{8}} & \frac{1}{\sqrt{16}} & 0 & \frac{1}{\sqrt{16}} & 0 & 0 \\ 0 & \frac{i}{\sqrt{8}} & 0 & 0 & 0 & \frac{-i}{\sqrt{8}} & 0 & 0 \\ \frac{1}{\sqrt{8}} & \frac{1}{\sqrt{16}} & 0 & 0 & \frac{1}{\sqrt{8}} & 0 & 0 & \frac{1}{\sqrt{16}} \\ 0 & 0 & 0 & \frac{1}{\sqrt{8}} & 0 & 0 & 0 & \frac{1}{\sqrt{8}} \\ 0 & \frac{1}{\sqrt{16}} & \frac{i}{\sqrt{8}} & 0 & 0 & 0 & \frac{-i}{\sqrt{8}} & \frac{1}{\sqrt{16}} \\ 0 & 0 & 0 & 0 & 0 & \frac{i}{\sqrt{8}} & 0 & \frac{1}{\sqrt{8}} \\ 0 & 0 & 0 & \frac{1}{\sqrt{16}} & \frac{1}{\sqrt{8}} & \frac{1}{\sqrt{16}} & \frac{1}{\sqrt{8}} & 0 \end{bmatrix}$$

Randic Skew – Hermitian energy of an asymmetrical Middle Connected graph

The total of the absolutes of the eigenvalues of the Skew-Hermitian Randic adjacency matrix of an asymmetrical middle cycle graph is its Randic Skew-Hermitian energy. The characteristics polynomial of is denoted by $P_{R^*_{SH}}(M_{di}, z)$

$$P_{R^*_{SH}}(M_{di}, z) = -z^p - n_1 z^{p-1} - n_2 z^{p-2} - \cdots - n_p$$

$$\left(P_{R^*_{SH}}(M_{di}, z)\right)K_8 = 1z^8 + 0z^7 - 1.2500z^6 - 0.0000z^5 + 0.4375z^4 + 0.0000z^3 - 0.0430z^2 - 0.0000Z + 0$$

In general, it implies that

$$\det\left(X^*I - R^*{}_{SH}(M_{di})\right) = \det\left(-(R^*{}_{SH}(M_{di}) - X^*I)\right) = (-1)\dim V \det(R^*{}_{SH}(M_{di}) - X^*I),$$

whatever whether $\left(R^*{}_{SH}(M_{di})\right)$'s entries are complex or real integers. It will be possible to find the characteristic polynomial by ignoring (−1) dimV.

The eigenvalues of $(R^*{}_{SH}(M_{di}))K_8$ are

$K(\lambda_1) = -0.8563, K(\lambda_2) = -0.5905, K(\lambda_3) = -0.4100, K(\lambda_4) = 0.0000,$

$K(\lambda_5) = 0.0000, K(\lambda_6) = 0.4100, K(\lambda_7) = 0.5905, K(\lambda_8) = 0.8563.$

The energy of a Randic Skew – Hermitian energy of the asymmetrical middle cycle graph

$$E(R^*{}_{SH}(M_{di})C_6 = 3.7136$$

The entire sum of the absolutes of the eigenvalues of the Randic Skew-Hermitian adjacency matrix of a cycle mixed middle graph is the Randic Skew-Hermitian energy of a cycle asymmetrical middle graph. We are taking a complicated number and its absolute value.

8.2 Illustration

Randic Skew – Hermitian matrix of an asymmetrical middle Dense graph.

Example 8.2.1

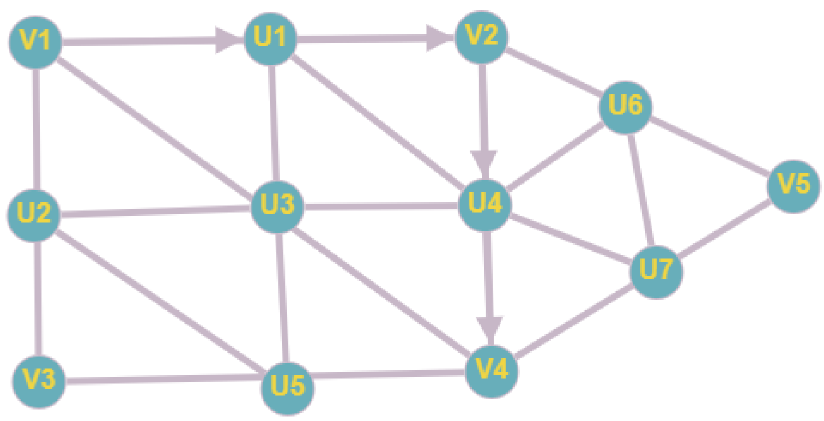

Figure 5: Asymmetrical Middle Dense Graph

Randic Skew – Hermitian matrix of an asymmetrical Middle Dense Graph

$$(R^{*}{}_{SH}(M_{di}))D_{12} = \begin{bmatrix} 0 & \frac{-i}{\sqrt{12}} & 0 & \frac{1}{\sqrt{12}} & 0 & \frac{1}{\sqrt{18}} & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{i}{\sqrt{12}} & 0 & \frac{-i}{\sqrt{12}} & 0 & 0 & \frac{1}{\sqrt{24}} & 0 & \frac{1}{\sqrt{24}} & 0 & 0 & 0 & 0 \\ 0 & \frac{i}{\sqrt{12}} & 0 & 0 & 0 & 0 & 0 & \frac{-i}{\sqrt{18}} & 0 & 0 & \frac{1}{\sqrt{12}} & 0 \\ \frac{1}{\sqrt{12}} & 0 & 0 & 0 & \frac{1}{\sqrt{8}} & \frac{1}{\sqrt{24}} & 0 & 0 & 0 & \frac{1}{\sqrt{16}} & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{\sqrt{8}} & 0 & 0 & 0 & 0 & 0 & \frac{1}{\sqrt{8}} & 0 & 0 \\ \frac{1}{\sqrt{18}} & \frac{1}{\sqrt{24}} & 0 & \frac{1}{\sqrt{24}} & 0 & 0 & \frac{1}{\sqrt{24}} & \frac{1}{\sqrt{36}} & 0 & \frac{1}{\sqrt{24}} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{\sqrt{24}} & 0 & \frac{i}{\sqrt{24}} & 0 & \frac{1}{\sqrt{16}} & 0 & \frac{1}{\sqrt{16}} \\ 0 & \frac{1}{\sqrt{24}} & \frac{i}{\sqrt{18}} & 0 & 0 & \frac{1}{\sqrt{36}} & \frac{-i}{\sqrt{24}} & 0 & 0 & 0 & \frac{1}{\sqrt{24}} & \frac{1}{\sqrt{24}} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{\sqrt{8}} & \frac{1}{\sqrt{8}} \\ 0 & 0 & 0 & \frac{1}{\sqrt{16}} & \frac{1}{\sqrt{8}} & \frac{1}{\sqrt{24}} & \frac{1}{\sqrt{16}} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{\sqrt{12}} & 0 & 0 & 0 & 0 & \frac{1}{\sqrt{24}} & \frac{1}{\sqrt{8}} & 0 & 0 & \frac{1}{\sqrt{16}} \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{\sqrt{16}} & \frac{1}{\sqrt{24}} & \frac{1}{\sqrt{8}} & 0 & \frac{1}{\sqrt{16}} & 0 \end{bmatrix}$$

Characteristic polynomial of an asymmetric Middle Path graph with a Randic Skew and Hermitian matrix is denoted by $\left(P_{R^{*}_{SH}}(M_{di}, z)\right)$

$$\left(P_{R^{*}_{SH}}(M_{di}, z)\right)D_{12} = 1.0000z^{12} + 0.0000z^{11} - 1.5556z^{10} - 0.2014z^{9} + 0.8314z^{8} + 0.1853z^{7} - 0.1753z^{6} - 0.0473z^{5} + 0.0133z^{4} + 0.0040z^{3} - 0.0002z^{2} - 0.0001z + 0.0000$$

The eigenvalues of $(R^{*}{}_{SH}(M_{di}))P_5$ are $p(\lambda_1) = -0.6961, p(\lambda_2) = -0.5531,$

$$p(\lambda_3) = -0.4845 , p(\lambda_4) = -0.4485 , p(\lambda_5) = -0.2651, \; p(\lambda_6) = -0.2047, p(\lambda_7) = 0.0093, p(\lambda_8) = 0.1603, p(\lambda_9) = 0.3453, p(\lambda_{10}) = 0.4600, p(\lambda_{11}) = 0.7767, p(\lambda_{12}) = 0.9010$$

$$E(R^{*}{}_{SH}(M_{di})D_{12} = 5.3046$$

8.3 Illustration

Randic Skew – Hermitian matrix of an asymmetrical Middle Tree Graph

Example 8.3.1

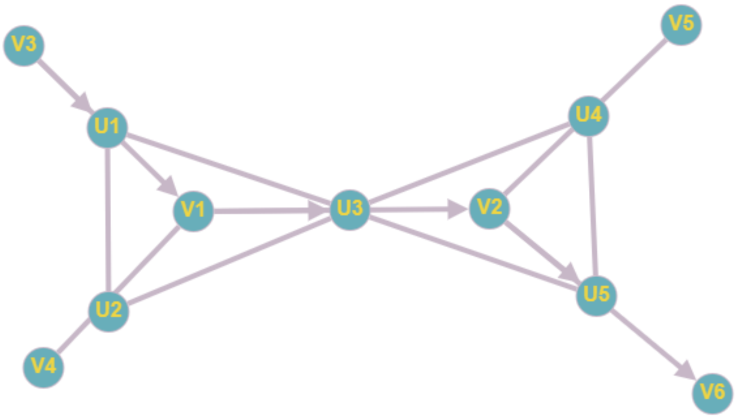

Figure 6: Asymmetrical Middle Tree Graph

Randic Skew – Hermitian matrix of an asymmetrical Middle Tree graph

$$\left(R^*{}_{SH}(M_{di})\right)T_{11} = \begin{bmatrix} 0 & \frac{i}{\sqrt{12}} & 0 & \frac{1}{\sqrt{12}} & 0 & \frac{-i}{\sqrt{18}} & 0 & 0 & 0 & 0 & 0 \\ \frac{-i}{\sqrt{12}} & 0 & 0 & \frac{1}{\sqrt{16}} & \frac{i}{\sqrt{4}} & \frac{1}{\sqrt{24}} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{i}{\sqrt{18}} & 0 & \frac{1}{\sqrt{12}} & 0 & \frac{-i}{\sqrt{12}} & 0 \\ \frac{1}{\sqrt{12}} & \frac{1}{\sqrt{16}} & 0 & 0 & 0 & \frac{1}{\sqrt{24}} & \frac{1}{\sqrt{4}} & 0 & 0 & 0 & 0 \\ 0 & \frac{-i}{\sqrt{4}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{i}{\sqrt{18}} & \frac{1}{\sqrt{24}} & \frac{-i}{\sqrt{18}} & \frac{1}{\sqrt{24}} & 0 & 0 & 0 & \frac{1}{\sqrt{24}} & 0 & \frac{1}{\sqrt{24}} & 0 \\ 0 & 0 & 0 & \frac{1}{\sqrt{4}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{\sqrt{12}} & 0 & 0 & \frac{1}{\sqrt{24}} & 0 & 0 & \frac{1}{\sqrt{4}} & \frac{1}{\sqrt{16}} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{\sqrt{4}} & 0 & 0 & 0 \\ 0 & 0 & \frac{i}{\sqrt{12}} & 0 & 0 & \frac{1}{\sqrt{24}} & 0 & \frac{1}{\sqrt{16}} & 0 & 0 & \frac{-i}{\sqrt{4}} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{i}{\sqrt{4}} & 0 \end{bmatrix}$$

Characteristic polynomial of an asymmetric Middle Tree graph with a Randic Skew and Hermitian matrix is denoted by $\left(P_{R^*_{SH}}(M_{di}, z)\right)T_{11}$

$\left(P_{R^*_{SH}}(M_{di}, z)\right)T_{11}$

$= 1z^{11} + 0z^{10} - 1.7361z^9 + 0.0139z^8 + 1.0745z^7 - 0.0240z^6 - 0.2840z^5 + 0.0116z^4 + 0.0297z^3 - 0.0014z^2 - 0.0007z + 0.000$

As the eigenvalues of $(R^*_{SH}(M_{di}))T_{11}$ are $T(\lambda_1) = -0.8074$, $T(\lambda_2) = -0.7309$

$$T(\lambda_3) = -0.5962, T(\lambda_4) = -0.4416, T(\lambda_5) = -0.1513, T(\lambda_6) = 0.0000,$$

$$W(\lambda_7) = 0.2329, W(\lambda_8) = 0.4416, W(\lambda_9) = 0.4947, T(\lambda_{10}) = 0.7309$$

$$T(\lambda_{11}) = 0.8274$$

$$E(R^*_{SH}(M_{di})W_{10} = 3.9166$$

Boundaries for the Randic Skew – Hermitian energy of an asymmetrical Graphs

We shall provide some limitations regarding the Randic Skew – Hermitian energy of asymmetrical graphs. Initially, we'll be listing a few characteristics of an asymmetrical graph Randic Skew - Hermitian matrix.

Lemma

Let M_{di} be an order $\rho \geq 1$ asymmetrical middle graph.

1. $E_{RSH}(M_{di}) = 0$ if and only if $M_{di} \cong \overline{K}_\rho$.
2. If $M_{di} = M_{di1} \cup M_{di2} \cup \cdots \cup M_{diP}$,

then $E_{RSH}(M_{di}) = E_{RSH}(M_{di1}) + E_{RSH}(M_{di2}) + \cdots + E(M_{diP})$.

From Lemma, We can derive the subsequent theorem.

If M_{di} become an asymmetrical middle graph with a set of vertices

$V(M_{di}) = \{V_1, V_2, \dots, V_\rho\}$.

And d_k is the extent of V_k, k = 1, 2, …, ρ. Usually (M_{di}) and $R_{SH}(M_{di})$ become the Skew – Hermitian adjacency matrices and the Randic Skew – Hermitian matrix of M_{di} respectively.

If M_{di} has isolated vertices, then $\det SH(M_{di}) = -\det R_{SH}(M_{di}) = 0$. If (M_{di}) has no independent vertices then

$$\det R_{SH}(M_{di}) = \frac{(-1)^n}{d_1 d_2, \dots, d_n} \det SH(M_{di}).$$

Proof

If $\mathbf{M_{di}}$ has **n** independent vertices, then $\mathbf{M_{di}} = \mathbf{M_{di}}' \cup \overline{\mathbf{K}}_{\mathbf{n}}$, where $\mathbf{M_{di}}'$ has no independent vertices. By Lemma, we have $\mathbf{Sp_{RSH}}(\mathbf{M_{di}}) = \mathbf{Sp_{RSH}}(\mathbf{M_{di}}') \cup \{\mathbf{0}, \mathbf{n\ times}\}$ and a similar relationship exists for adjacency spectrum of Skew-Hermitian of $\mathbf{M_{di}}$. That is, $\mathbf{SHM_{di}}$ and $\mathbf{RSHM_{di}}$ have zero eigenvalues, signifying that zero is the sum of their determinants.

Might $(\mathbf{M_{di}})$ has not independent vertices, then $\mathbf{R_{SH}M_{di}} = \left(-\mathbf{D}(\mathbf{M_{di}U})^{\frac{-1}{2}}\mathbf{SH}(\mathbf{M_{di}})\mathbf{D}(\mathbf{M_{di}U})^{\frac{-1}{2}}\right)$

is appropriate, where $\mathrm{D}(\mathrm{M_{di}U})$ denotes the diagonal vertex degree matrix.

The squares$\mathrm{R_{SH}M_{di}}$ and $\left(-\mathrm{D}(\mathrm{M_{di}U})^{\frac{-1}{2}}\mathrm{SH}(\mathrm{M_{di}})\mathrm{D}(\mathrm{M_{di}U})^{\frac{-1}{2}}\right)$ possess equivalent eigenvalues since they are comparable in addition. As we've

$$\left(-\mathrm{D}(\mathrm{M_{di}U})^{\frac{-1}{2}}\,\mathrm{R_{SH}M_{di}}\,\mathrm{D}(\mathrm{M_{di}U})^{\frac{-1}{2}}\right) = -(\mathrm{D}(\mathrm{M_{di}U})^{-1}\mathrm{SH}(\mathrm{M_{di}}))$$

Hence, $\det \mathrm{R_{SH}M_{di}} = (\det[-\mathrm{D}(\mathrm{M_{di}U})^{-1}\mathrm{SH}(\mathrm{M_{di}})]) = -(\det\mathrm{D}(\mathrm{M_{di}U})^{-1}\det\mathrm{SH}(\mathrm{M_{di}}).)$

$$\text{Thus, } \det \mathrm{R_{SH}M_{di}} = \frac{(-1)^{n}}{d_1\, d_2, \dots, d_n}\det\mathrm{SH}(\mathrm{M_{di}}).$$

The proof has become complete.

The procedure used to acquire the following theorem can also be obtained using the theorem.

Theorem 1

Let $(\mathrm{M_{di}})$ is an asymmetrical graph with vertex set $V*(\mathrm{M_{di}}) = \{V_1, V_2, \dots, V_\rho\}$, as well as the assymmetrical graph $(\mathrm{M_{di}U})$ is r usual, then $E_{RSH}(\mathrm{M_{di}}) = \frac{-1}{r} E_{SH}(\mathrm{M_{di}})$.

Furthermore, if $r = 0$, subsequently $E_{RSH}(\mathrm{M_{di}}) = 0$.

Proof

Let r = 0, subsequently $(\mathrm{M_{di}})$ represents the graph with no edges. At that point $\mathrm{R_{SH}M_{di}}$ has exactly zero entries, i.e., $\mathrm{R_{SH}M_{di}} = 0$. Likewise, $\mathrm{SH}(\mathrm{M_{di}}) = 0$.

Considering that there is a matrix with zero entries in each row.of the event when every eigenvalue of a matrix is zero, the matrix is said to be nilpotent.

$\mathrm{ER_{SH}M_{di}} = \mathrm{ESH}(\mathrm{M_{di}}) = 0$.

If $r > 0$, That means $(\mathrm{M_{di}})$ is regular of degree $r > 0$, then $d1 = d2 = \cdots = dn = r$, where d_k is the degree of V_y, $y = 1,2, \dots, \rho$. Hence, $(r_{sh})_{xy} = \frac{-i}{r}$;$(r_{sh})_{yx} = \frac{i}{r}$ if (V_x, V_y) is an arc of $(\mathrm{M_{di}})$, $(r_{sh})_{xy} = (r_{sh})_{yx} = \frac{-1}{r}$ if (V_x, V_y) is an unfocused edge of $(\mathrm{M_{di}})$, and $(r_{sh})_{xy} = 0$ in any case.

This suggests that $\mathrm{R_{SH}M_{di}} = \frac{-1}{r}\mathrm{SH}(\mathrm{M_{di}})$. Therefore, $\mu_i = \frac{-1}{r}(\lambda_i)$.

In this case, μ_i is the value of eigenvalue $R_{SH}M_{di}$, additionally λ_i is the value of eigenvalue SH (M_{di}) for $i = 1, 2, \dots, n$.

Then, based on the definitions, this theorem is implied of $ER_{SH}M_{di}$ and ESH (M_{di}).

We are able to determine the Randic Skew-Hermitian energy's lower and upper bounds. We have to first apply the next theorem. As well as in the future, I_ρ represents the arrange ρ unit matrix.

Theorem 2

Let (M_{di}) be a assymmetrical graph of arrange ρ and $\alpha_1 \geq \alpha_2 \geq \cdots \geq \alpha_\rho$

become the skew – Hermitian Randic spectrum an $R_{SH}M_{di}$.

Following that $|\alpha_1| = |\alpha_2| = \cdots = |\alpha_\rho|$ if and only if a constant is present $C^* = |\alpha_i|^2$ for all i such that

$$R_{SH}{}^2M_{di} = C^*I^*{}_\rho$$

Proof

Let $\alpha_1 \geq \alpha_2 \geq \cdots \geq \alpha_\rho$become the Skew – Hermitian Randic spectrum an $R_{SH}M_{di}$.

Next, a unitary matrix U exists in such a way that

$$U * R_{SH}M_{di}U{=}U * R_{SH}M_{di} * U = \text{diag}\{\alpha_1, \dots, \alpha_\rho\}.$$

So,

$$|\alpha_1| = |\alpha_2| = \cdots |\alpha_\rho|$$

$$\Leftrightarrow U * R_{SH}M_{di} * R_{SH}M_{di}U = C^*I^*{}_\rho$$

$$\Leftrightarrow U(U * R_{SH}M_{di} * R_{SH}M_{di}U) * U = C^*UU *$$

$$\Leftrightarrow R_{SH}M_{di} * R_{SH}M_{di} = C^*I^*{}_\rho$$

$$\Leftrightarrow R^2{}_{SH}M_{di} = C^*I^*{}_\rho,$$

where C^* is unchanging and $C^* = |\alpha_i *|^2$ for everyone i.

Now that the proof is finished.

Theorem 3

Let (M_{di}) become an assymetrical graph of arrange ρ and $\alpha_1 \geq \alpha_2 \geq \cdots \geq \alpha_\rho$ become the Randic Skew-Hermitian spectrum of $R_{SH}M_{di}$. Let (M_{di}) U become the underlying graphs of the (M_{di}), $p = |\det R_{SH}M_{di}|$. Then

$$\sqrt{\left(2R_{-1}(M_{di}U) + \rho(\rho-1)p^{\frac{2}{\rho}}\right)} \leq E_{R_{SH}}(M_{di}) \leq \sqrt{\left(2\rho R_{-1}(M_{di}U)\right)}$$

equalities holding in the upper and lower bounds solely in the event that a constant exists. $C^* = |\alpha_{i*}|^2$ for everyone i in a way that $R^2{}_{SH}M_{di} = C^*I^*{}_\rho$.

Proof

Let $\{\alpha_1, \alpha_2, \cdots, \alpha_\rho\}$ become the Skew–Hermitian Randic spectrum an (M_{di}), Whereas $\alpha_1 \geq \alpha_2 \geq \cdots \geq \alpha_\rho$. Since

$$\sum_{j=1}^{\rho} {\alpha_\rho}^2 = tr\left(R_{SH}^2(M_{di})\right)$$

$$= \sum_{j=1}^{\rho} \sum_{k=1}^{\rho} (r_{sh})_{jk} (r_{sh})_{kj}$$

$$= \sum_{j=1}^{\rho} \sum_{k=1}^{\rho} (r_{sh})_{jk} \overline{r_{sh}}_{jk}$$

$$= \sum_{j=1}^{\rho} \sum_{k=1}^{\rho} \left|(r_{sh})_{jk}\right|^2 = 2R_{-1}(M_{di}U),$$

where $R_{-1}(M_{di}U) = \sum_{v_j v_k \in E(M_{di})} \frac{-1}{d_j d_k}$ (unarranged).

Cauchy-Schwarz inequality applied, we have

$$E_{R_{SH}}(M_{di}) = \sum_{j=1}^{\rho} |\alpha_j| \leq \sqrt{\sum_{j=1}^{\rho} |\alpha_j|^2} \cdot \sqrt{\rho} = \sqrt{2_\rho R_{-1}(M_{di}U)}$$

On the other hand,

$$\left|E_{R_{SH}}(M_{di})\right|^2 = \left(\sum_{j=1}^{\rho} |\alpha_j|^2 = \sum_{j=1}^{\rho} |\alpha_j|^2 + \sum_{1 \leq i \neq j \leq \rho} |\alpha_i||\alpha_j|\right)$$

We can obtain that by applying an arithmetic geometric average inequality.

$$\left|E_{R_{SH}}(M_{di})\right|^2 = \sum_{j=1}^{\rho} |\alpha_j|^2 + \sum_{1 \leq i \neq j \leq \rho} |\alpha_i||\alpha_j| \geq 2R_{-1}(M_{di}) + n(n-1)\rho^{\frac{2}{n}}$$

We know that the equalities hold at the upper and lower bounds if and only if we apply their Cauchy-Schwarz inequality and the geometric average inequality in arithmetic. $|\alpha_1| = |\alpha_2| = \cdots |\alpha_\rho|$ that is, and there exists a constant $C^* = |\alpha_i|^2$ for all i such that $\left(R_{SH}^2(M_{di})\right) = C^* {I^*}_\rho$. The proof is so finished.

Conclusion

The clarifications of the Randic Skew-Hermitian characteristic polynomial and the Randic Skew-Hermitian energy of a asymmetrical middle graphs $({R^*}_{SH}(M_{di})$ are provided in this work together with the Randic Skew-Hermitian matrix of a asymmetrical middle graphs M_{di}. We provide limitations on the Randic Skew-Hermitian energy, Randic the Skew-Hermitian spectrum, and a general Randic's indexes (with $\alpha = -1$) of an asymmetrical middle graphs $({R^*}_{SH}(M_{di})$with respect to its order.

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