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Micro Pre Separation Axioms in Circuit of Light System on Four Stroke Engines

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Abstract:

The basic objective of this research work is to introduce and investigate the properties of micro-T_0, T_1, T_2 spaces, micro semi-T_0, T_1, T_2 and micro pre-T_0, T_1, T_2 spaces using the concept of micro open, micro semi-open and micro pre-open sets and comparing them to the circuit system of light in four stroke engines.

Introduction: Levine's introduction of generalized closed sets in 1970 [7], providing a foundational framework for subsequent developments. In 1975, Maheshwari and Prasad [9] gave the idea about the s-normal spaces and in 1990, Ashish Kar and Bhattacharyya [1] introduced the concept of weak separation axioms. In 1995, Chattpadhyay [3] investigated some properties of pre-T_0, pre-T_1 and pre-T_2 spaces in topological spaces. In 2013, Lellis Thivagar [6] defined the new concept of Nano topology based on generalized closed sets. In 2019, Sathishmohan et al., [10] established the concepts of nano pre-T_0, nano pre-T_1 and nano pre-T_2 spaces and obtain many interesting results. In 2019, Chandrasekar [2] introduced the concept of Micro topology which is a simple extension of nano topology with focus on micro pre-open and micro semi-open sets. In 2020, Hariwan Z. Ibrahim [4] introduced the separation axioms on micro topology with particular focus on micro open sets. In 2022, Wei Liu et al., [12] established the importance of battery in electric vehicles and at 2024, Liu. H et al., [8] studied the control of angle formation based on circuit diagrams. This motivates us to introduce and study some new concepts of micro-T_0, T_1, T_2 spaces, micro semi-T_0, T_1, T_2 spaces and micro pre- T_0, T_1, T_2 spaces combining with the circuit of the four stroke engines.

Conclusions: In this paper, we introduced the notions of micro semi-T_0, T_1, T_2 and micro pre-T_0, T_1, T_2 by employing the concepts of separation axioms elucidating various associated properties. Based on the theorems and results, I have concluded that circuit of the light system in vehicles and the study of my spaces are more closely related to each other. Our intent is to further

elaborate on those findings in forthcoming research endeavors, with a particular focus on exploring practical applications.

Keywords: Micro-**{**T_0, T_1, T_2}, Micro semi-**{**T_0, T_1, T_2}, Micro pre-**{**T_0, T_1, T_2}

1. Introduction

Levine's introduction of generalized closed sets in 1970 [7], providing a foundational framework for subsequent developments. In 1975, Maheshwari and Prasad [9] gave the idea about the s-normal spaces and in 1990, Ashish Kar and Bhattacharyya [1] introduced the concept of weak separation axioms. In 1995, Chattpadhyay [3] investigated some properties of pre-T_0, pre-T_1 and pre-T_2 spaces in topological spaces. In 2013, Lellis Thivagar [6] defined the new concept of Nano topology based on generalized closed sets. In 2019, Sathishmohan et al., [10] established the concepts of nano pre-T_0, nano pre-T_1 and nano pre-T_2 spaces and obtain many interesting results. In 2019, Chandrasekar [2] introduced the concept of Micro topology which is a simple extension of nano topology with focus on micro pre-open and micro semi-open sets. In 2020, Hariwan Z. Ibrahim [4] introduced the separation axioms on micro topology with particular focus on micro open sets. In 2022, Wei Liu et al., [12] established the importance of battery in electric vehicles and at 2024, Liu. H et al., [8] studied the control of angle formation based on circuit diagrams. This motivates us to introduce and study some new concepts of micro-T_0, T_1, T_2 spaces, micro semi-T_0, T_1, T_2 spaces and micro pre- T_0, T_1, T_2 spaces combining with the circuit of the four stroke engines.

2. Preliminaries

Definition 2.1. [6] Let U be the universe, R be an equivalence relation on U and $\tau_R(X) = \{U, \emptyset, L_R(X), U_R(X), B_R(X)\}$ where $X \subseteq U$. $\tau_R(X)$ satisfies the following axioms:

(1) U and $\emptyset \in \tau_R(X)$
(2) The union of elements of any sub collection of $\tau_R(X)$ is in $\tau_R(X)$.
(3) The intersection of the elements of any finite sub collection of $\tau_R(X)$ is in $\tau_R(X)$.

That is, $\tau_R(X)$ forms a topology on U is called the nano topology on U with respect to X. $\{U, \tau_R(X)\}$ is called the nano topological sapce.

Definition 2.2. [2] Let $\{U, \tau_R(X)\}$ is a nano topological space here $\mu_R(X) = \{\{N \cup (N' \cap \mu)\} : N, N' \in \tau_R(X)\}$ and called it Micro topology of $\tau_R(X)$ by μ where $\mu \notin \tau_R(X)$.

Definition 2.3. [2] The Micro topology $\mu_R(X)$ satisfies the following axioms.

(1) U and $\emptyset \in \mu_R(X)$.
(2) The union of elements of any sub collection of $\mu_R(X)$ is in $\mu_R(X)$.
(3) The intersection of the elements of any finite sub collection of $\mu_R(X)$ is in $\mu_R(X)$.

Then $\mu_R(X)$ is called Micro topology on U with respect to X. The triplet $(U, \tau_R(X), \mu_R(X))$ is called Micro topological space and the elements of $\mu_R(X)$ are called Micro open sets and the complement of a Micro open set is called a Micro closed set.

Definition 2.4. [2] The Micro closure of a set A is denoted by Mic-cl(A) and is defined as Mic-cl(A) = ∩{B : B is Micro closed and A ⊆ B}. The Micro interior of a set A is denoted by Mic-int(A) and is defined as Mic-int(A) = ∪{B : B is Micro open and A ⊇ B}.

Definition 2.5. [11] The union of all micro pre-open sets which are contained in A is called the micro pre-interior of A and is denoted by Mic-Pint(A) or Mic-PA$_*$. As the union of micro pre-open sets is micro pre-open, Mic-PA$_*$ is micro pre-open always. Micro pre-open is denoted by Mic-PO(U) and micro pre-closed is denoted by Mic-PF(U).

Definition 2.6. [11] The intersection of micro pre-closed sets containing a set A is called the micro pre-closure of A and is denoted by Mic-Pcl(A) or Mic-PA*.

Definition 2.7. [2] Let (U, $\tau_R(X)$, $\mu_R(X)$) be a micro topological space and A ⊆ U. Then A is called micro pre-open if A ⊆ Mic-int(Mic-cl(A)).

Definition 2.8. [4] Let (U, $\tau_R(X)$, $\mu_R(X)$) be a micro topological space. Then, U is said to be:

(1) Micro T_0 if for each pair of distinct points x, y in U, there exists a Micro open set L such that either x ∈ L and y ∉ L or x ∉ L and y ∈ L.
(2) Micro T_1 if for each pair of distinct points x, y in U, there exists two Micro open sets L and K such that either x ∈ L and y ∉ L or y ∈ K and x ∉ K.
(3) Micro T_1 if for each pair of distinct points x, y in U, there exists two Micro open sets L and K containing x and y respectively.

3. Micro semi-T_0 and pre-T_0 spaces

In this section, we introduce and study the concepts of micro-T_0, micro semi-T_0 and micro pre-T_0 in micro topological space and obtain some of their basic results.

Definition 3.1. A space U is called micro semi-T_0 (or Mic-ST_0) space, for each pair of distinct points x, y ∈ U, there exists a micro semi-open set H such that x ∈ H and y ∉ H.

Definition 3.2. A space U is called micro pre-T_0 (or Mic-PT_0) space, for each pair of distinct points x, y ∈ U, there exists a micro pre-open set H such that x ∈ H and y ∉ H.

We prove some relations and properties of micro pre-T_0 in the following.

Theorem 3.3. Let (U, $\tau_R(X)$, $\mu_R(X)$) be a micro topological space, then for every Mic-T_0 space is micro pre-T_0 (resp. micro semi-T_0) spaces.

Proof: Let U be micro-T_0 space, x and y be two distinct points of U, as U is micro-T_0 space there exists micro open set H such that x ∈ H and y ∉ H, since every micro open set is micro pre (resp. micro semi)-open and hence H is micro pre (resp. micro semi)-open set such that x ∈ H and y ∉ H ⇒ U is micro pre-T_0 (resp. micro semi-T_0) space.

But the converse of the theorem need not be true in general.

Example 3.4. Let U = {a, b, c, d}, U/R = {{a, b}, {c}, {d}}, X = {a}, $\tau_R(X)$ = {U, ϕ, {a, b}}, μ = {b, c, d} and $\mu_R(X)$ = {U, ϕ, {b}, {a, b}, {b, c, d}}. Mic-SO(U) = { U, ϕ, {b}, {a, b}, {b, c}, {b, d}, {a, b, c}, {b, c, d}, {a, b, d}} and Mic-PO(U) = { U, ϕ, {b}, {a, b}, {b, c}, {b, d}, {a, b, c}, {b, c, d}, {a, b, d}}. Let x = {c} and y = {d}, there exist a micro (semi-open and pre-open) sets either containing

the point x or y, but there is no micro open set either containing the point x or y. Therefore, it is Mic-ST_0 and Mic-PT_0 spaces but not a Mic-T_0 space.

Theorem 3.5. A space U is micro pre-T_0 iff Mic-P($\{x\}^*$) ≠ Mic-P($\{y\}^*$) for every pair of distinct points x, y ∈ U.

Proof: Let x and y be any two distinct points of micro pre-T_0 space U. We show that Mic-P(x^*) ≠ Mic-P($\{y\}^*$). By hypothesis, suppose X ∈ Mic-PO(U) such that x ∈ X and y ∉ X. Hence y ∈ U − X and U − X is micro pre-closed. Therefore Mic-P(y^*) ⊂ U − X. Hence y ∈ Mic-P(y^*) but x ∉ Mic-P(y^*) as x ∈ U − X. Hence Mic-P($\{x\}^*$) ≠ Mic-P($\{y\}^*$).

Conversely, suppose for any x, y ∈ U with x ≠ y, Mic-P($\{x\}^*$) ≠ Mic-P($\{y\}^*$). Now without loss of generality, let z ∈ U such that x ∈ Mic-P($\{x\}^*$) but z ∉ Mic-P($\{y\}^*$). Now, we claim that x ∉ Mic-P($\{y\}^*$). For if x ∈ Mic-P($\{y\}^*$) then x ⊂ Mic-P($\{y\}^*$) which implies that Mic-P($\{x\}^*$) ⊂ Mic-P($\{y\}^*$). Thus, z ∈ Mic-P($\{x\}^*$) and z ∈ Mic-P($\{y\}^*$). This is a contradiction. Therefore, x ∈ ∈ Mic-P($\{y\}^*$). Hence U − Mic-P($\{y\}^*$) is a micro pre-open set containing x but not y. It gives that U is micro pre-T_0 space.

Result 3.6. If P ∈ Mic-PO(X) and Q ∈ Mic-PO(U) then Q ∈ Mic-PO(X).

Theorem 3.7. A space U is micro pre-T_0 iff for each x ∈ U, there exists a micro pre-open set X of U containing x such that the subspace X is micro pre-T_0.

Proof: If U is micro pre-T_0, take U as X. Then, X is a micro pre-open set containing x such that the subspace X is micro pre-T_0 for every x ∈ U.

Conversely, suppose that x_1, x_2 be any two distinct points of U. By hypothesis, there exist X_j ∈ Mic-PO(U) such that x_j ∈ X_j and the subspace X_j is micro pre-T_0, for j = 1, 2. If x_2 ∉ X_1 then the proof is completed. If x_2 ∈ X_1 is micro pre-T_0, there exists W_1 ∈ Mic-PO(X_1) such that x_1 ∈ w_1 and x_2 ∉ w_1 or there exists W_2 ∈ Mic-PO(X_1) such that x_2 ∈ w_2 and x_1 ∉ w_2. Since X_1 ∈

Mic-PO(U), it follows that from Result 3.6., w_j ∈ Mic-PO(U) for j = 1, 2. This means that the space U is micro pre-T_0.

4. Micro semi-T_1 and pre-T_1 spaces

In this section, we introduce and study the concepts of micro-T_1, micro semi-T_1 and micro pre-T_1 in micro topological spaces and obtained some of their basic results.

Definition 4.1. A space U is called micro semi-T_1 (or Mic-ST_1) space, for each pair of distinct points x, y ∈ U, there exists a micro semi-open H and I such that x ∈ H, y ∉ H and y ∈ I, x ∉ I.

Definition 4.2. A space U is called micro pre-T_1 (or Mic-PT_1) space, for each pair of distinct points x, y ∈ U, there exists a micro pre-open H and I such that x ∈ H, y ∉ H and y ∈ I, x ∉ I.

Theorem 4.3. Every micro-T_1 space is micro pre-T_1 (resp. micro semi-T_1) space.

Proof: Let U be micro-T_1 space and let x ≠ y in U. Then there exists distinct micro-open sets H and I such that x ∈ H, y ∉ H and x ∉ I, y ∈ I. As every micro open set is micro pre (resp. micro semi)-open sets and hence H and I is micro pre (resp. micro semi)-open sets such that x ∈ H, y ∉ H and x ∉ I, y ∈ I ⇒ U is micro pre-T_1 (resp. micro semi-T_1) space.

But the converse of the theorem need not be true in general.

Example 4.4. Let U = {a, b, c, d}, U/R = {{a, b}, {c}, {d}}, X = {a}, $\tau_R(X)$ = {U, ϕ, {a, b}}, μ = {a, c, d} and $\mu_R(X)$ = {U, ϕ, {a}, {a, b}, {a, c, d}}. Mic-SO(U) = { U, ϕ, {a}, {a, b}, {a, d}, {a, c}, {a, b, c}, {a, c, d}, {a, b, d}} and Mic-PO(U) = { U, ϕ, {a}, {a, b}, {a, d}, {a, c}, {a, b, c}, {a, c, d}, {a, b, d}}. Let x = {c} and y = {d}, there exist a distinct micro pre (resp. micro semi)-open sets in which one set containing x but not y and another set containing y but not x, hence it is Mic-ST_1 and Mic-PT_1 space, but there is no distinct micro open sets in which one set containing x but not y and another set containing y but not x, hence it is not Mic-T_1 space.

Theorem 4.5. A space U is micro pre-T_1 space iff for any point x ∈ U, the singleton set {x} is micro pre-closed set.

Proof: Let every singleton set {x} of U be micro pre-closed. Therefore U − {x} is micro pre-open. Now we show that U is micro pre-T_1 space. Let x, y ∈ U with x ≠ y. Then {x} and {y} are micro pre-closed sets. Therefore U − {x} is a micro pre-open set containing y but not x and U − {y} is a micro pre-open set containing x but not y. Thus, U is micro pre-T_1 space.

Conversely, let U be a micro pre-T_1 space. Assume that x ∈ U be an arbitrary point. Now, we show that {x} is micro pre-closed or U − {x} is micro pre-open. Let z ∈ U − {x} then clearly z ≠ x. Now, U is micro pre-T_1 and z is a point different from x so there exists a micro pre-open set G_z such that z ∈ G_z but x ≠ G_z ⊂ U − {x}. Therefore U − {x} = $\bigcup\{G_z/z \neq x\}$. So U − {x} being the union of micro pre-open sets is micro pre-open. Hence {x} is a micro pre-closed set.

Theorem 4.6. A space U is micro pre-T_1 iff for each point x ∈ U, there exists a micro pre-open set X of U containing x such that the subspace X is micro pre-T_1.

Proof: Similar to Theorem 3.7.

Theorem 4.7. Let (U, $\tau_R(X)$, $\mu_R(X)$) be a micro topological space, then

(1) Every micro semi-T_1 space is micro semi-T_0 space.
(2) Every micro pre-T_1 space is micro pre-T_0 space.

Proof:

(1) Let U be micro semi-T_1 space and let x ≠ y in U. Then there exists distinct micro semi-open sets H and I such that x ∈ H, y ∉ H, y ∈ I, x ∉ I. So there exist a micro smei-open set either containing x or y. Hence U is micro semi-T_0 space.
(2) Let U be micro pre-T_1 space and let x ≠ y in U. Then there exists distinct micro pre-open sets H and I such that x ∈ H, y ∉ H, y ∈ I, x ∉ I. So there exist a micro pre-open set either containing x or y. Hence U is micro pre-T_0 space.

Converse of the above theorem need not be true in general.

Example 4.8. Let U = {a, b, c, d}, U/R = {{a, b}, {c}, {d}}, X = {a}, $\tau_R(X)$ = {U, ϕ, {a, b}}, μ = {a, c, d} and $\mu_R(X)$ = {U, ϕ, {a}, {a, b}, {a, c, d}}. Mic-SO(U) = { U, ϕ, {a}, {a, b}, {a, d}, {a, c}, {a, b, c}, {a, c, d}, {a, b, d}} and Mic-PO(U) = { U, ϕ, {a}, {a, b}, {a, d}, {a, c}, {a, b, c}, {a, c, d}, {a, b, d}}. Let x = {a} and y = {b}, there exist a micro semi-open set containing either x or y but there is no

distinct micro semi-open set H and I where x ∈ H, y ∉ H, y ∈ I, x ∉ I. Hence U is Mic-ST_0 space but not Mic-ST_1 space. Similarly, there exist a micro pre-open set containing either x or y but there is no distinct micro pre-open set H and I where x ∈ H, y ∉ H, y ∈ I, x ∉ I. Hence U is Mic-PT_0 space but not Mic-PT_1 space.

5. Micro semi-T_2 and pre-T_2 spaces

In this section, we introduce and study the concepts of micro-T_2, Microsemi-T_2 and micro pre-T_2 in micro topological spaces and obtained some of the reults.

Definition 5.1. A space U is called micro semi-T_2 (or Mic-ST_2) space, for each pair of distinct points x, y ∈ U, there exists disjoint micro semi-open sets H and I such that x ∈ H and y ∈ I. **Definition 5.2.** A space U is called micro pre-T_2 (or Mic-PT_2) space, for each pair of distinct points x, y ∈ U, there exists disjoint micro pre-open sets H and I such that x ∈ H and y ∈ I.

Theorem 5.3. Every micro-T_2 space is micro pre-T_2 (resp. micro semi-T_2) space.

Proof: Let U be micro-T_2 space and let x ≠ y in U. Then there exists disjoint micro-open sets H and I such that x ∈ H and y ∈ I. As every micro-open set is micro pre (resp. micro semi)-open. Hence H and I are disjoint micro pre (resp. micro semi)-open sets such that x ∈ H and y ∈ I.

Converse of the above theorem need not be true which is shown in the following example.

Example 5.4. Let U = {a, b, c, d}, U/R = {{a, b}, {c}, {d}}, X = {a}, $\tau_R(X)$ = {U, ϕ, {a, b}}, μ = {a} and $\mu_R(X)$ = {U, ϕ, {c}, {a, b}, {a, b, c}}. Mic-SO(U) = { U, ϕ, {c}, {a, b}, {c, d}, {a, b, c}, {a, b, d}} and Mic-PO(U) = { U, ϕ, {a}, {b}, {c}, {a, b}, {b, c}, {a, c}, {a, b, c}, {b, c, d}, {a, c, d}}. Let x = {b} and y = {c}, there exist a disjoint micro pre-open and micro semi-open sets H and I such that x ∈ H and y ∈ I, hence it is micro pre-T_2 and micro semi-T_2 spaces, but there is no disjoint micro open sets H and I such that x ∈ H and y ∈ I, hence it is not micro-T_2 space.

Theorem 5.5. Let (U, $\tau_R(X)$, $\mu_R(X)$) be a micro topological space, then

(1) Every micro semi-T_2 space is micro semi-T_0 space.
(2) Every micro pre-T_2 space is micro pre-T_0 space.

Proof:

(1) Let U be micro semi-T_2 space and x and y be two distinct points of U, as U is micro semi-T_2 space there exists disjoint micro semi-open sets H and I such that x ∈ H and y ∈ I, so the micro semi-open sets either contain x or y. Therefore, it is micro semi-T_0 space.
(2) Let U be micro pre-T_2 space and x and y be two distinct points of U, as U is micro pre-T_2 space there exists disjoint micro pre-open sets H and I such that x ∈ H and y ∈ I, so the micro pre-open sets either contain x or y. Therefore, it is micro pre-T_0 space.

The converse of the above theorem need not be true which is shown in following example.

Example 5.6.

Let U = {a, b, c, d}, U/R = {{a, b}, {c}, {d}}, X = {a}, $\tau_R(X)$ = {U, ϕ, {a, b}}, μ = {a, c, d} and $\mu_R(X)$ = {U, ϕ, {a}, {a, b}, {a, c, d}}. Mic-SO(U) = { U, ϕ, {a}, {a, b}, {a, d}, {a, c}, {a, b, c}, {a, c, d}, {a, b, d}} and Mic-PO(U) = { U, ϕ, {a}, {a, b}, {a, d}, {a, c}, {a, b, c}, {a, c, d}, {a, b, d}}.Let x =

{a} and y = {b}, there exist a micro pre-open and micro semi-open sets either containing x or y hence it is micro pre-T_0, but there is no disjoint micro pre-open and micro semi-open sets containing x and y separately, so it is not a micro semi-T_2 and micro pre-T_2 spaces.

Theorem 5.7. Let (U, $\tau_R(X)$, $\mu_R(X)$) be a micro topological space, then

(1) Every micro semi-T_2 space is micro semi-T_1 space.
(2) Every micro pre-T_2 space is micro pre-T_1 space.

Proof:

(1) Let U be micro semi-T_2 space. Then by Definition 5.1 $x \neq y$, there exists two disjoint micro semi-open sets H and I such that $x \in H$ and $y \in I$ and $H \cap I = \phi$ which implies $x \notin I$ and $y \notin H$. Hence U is micro semi-T_1 space.
(2) Let U be micro pre-T_2 space. Then by Definition 5.1 $x \neq y$, there exists two disjoint micro pre-open sets H and I such that $x \in H$ and $y \in I$ and $H \cap I = \phi$ which implies $x \notin I$ and $y \notin H$. Hence U is micro pre-T_1 space.

Converse of the above theorem need not be true in general.

Example 5.8. Let U = {a, b, c, d}, U/R = {{a, b}, {c}, {d}}, X = {a}, $\tau_R(X)$ = {U, ϕ, {a, b}}, μ = {a, c, d} and $\mu_R(X)$ = {U, ϕ, {a}, {a, b}, {a, c, d}}. Mic-SO(U) = { U, ϕ, {a}, {a, b}, {a, d}, {a, c}, {a, b, c}, {a, c, d}, {a, b, d}} and Mic-PO(U) = { U, ϕ, {a}, {a, b}, {a, d}, {a, c}, {a, b, c}, {a, c, d}, {a, b, d}}. Let x = {b} and y = {c}, there exist a distinct micro semi-open sets H and I where H = {a, b} and I = {a, c} where $x \in H$, $y \notin H$ and $y \in I$, $x \notin I$. Hence U is Mic-ST_1 space, but there are no disjoint micro semi-open sets where $x \in H \cap y \in I = \phi$, hence U is not a Mic-$ST_2$ space. Similarly, there exist a distinct micro pre-open sets H and I where H = {a, b} and I = {a, c} where $x \in H$, $y \notin H$ and $y \in I$, $x \notin I$. Hence U is Mic-PT_1 space, but there are no disjoint micro pre-open sets where $x \in H \cap y \in I = \phi$, Hence U is not a Mic-$PT_2$ space.

Theorem 5.9. Let (U, $\tau_R(X)$, $\mu_R(X)$) be a micro topological space, then the following statements are equivalent:

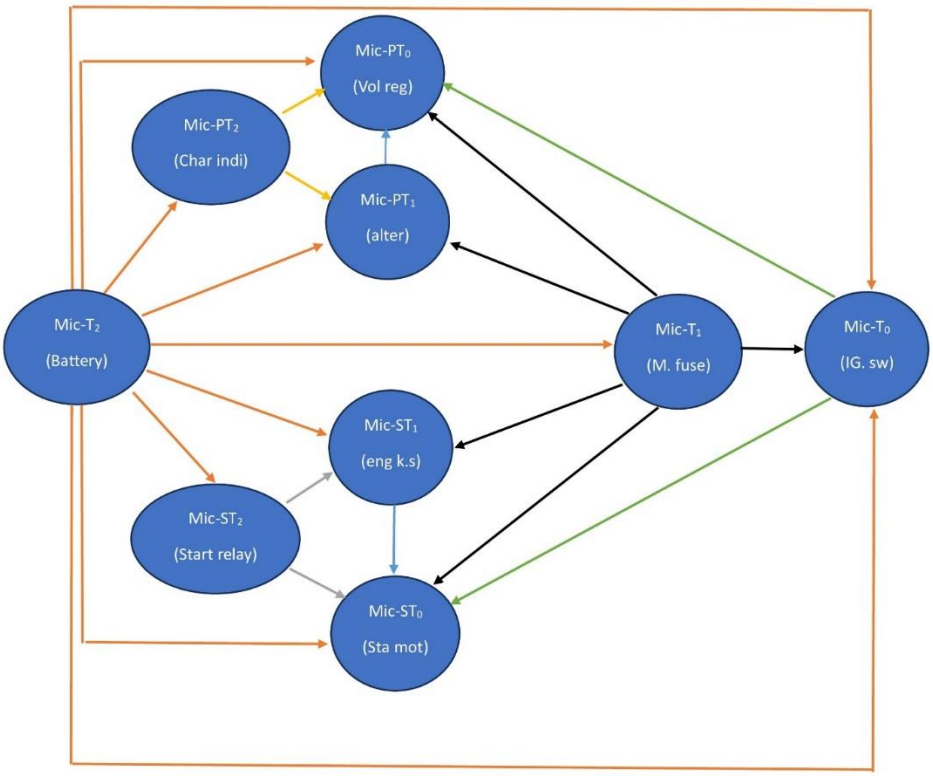
(1) U is a Mic-PT_2 space.
(2) Let $x \in U$, for each $x \neq y$, there exists a micro pre-open set H containing x such that $y \notin$ Mic-PH^*.
(3) For each $x \in U$, $\bigcap${Mic-PH^* : H $\in$ micro pre-open and $x \in H$} = {x}.

Proof: (1) $\Rightarrow$ (2): Since U is Mic-PT_2 space, then there exist disjoint micro pre-open sets H and I containing x and y respectively. So, H $\subseteq$ U/I. Therefore, Mic-PH^* $\subseteq$ U/I. So, $y \notin$ Mic-PH^*.

(2) $\Rightarrow$ (3) : If possible for some $x \neq y$, we have $y \in$ Mic-PH^* for every micro pre-open set H containing x, which then contradicts (2).

(3) $\Rightarrow$ (1) : Let $x, y \in U$ and $x \neq y$. Then, there exists a micro pre-open set H containing x such that $y \notin$ Mic-PH^*. Let I = U/Mic-PH^*, then $y \in I$, $x \in H$ and $H \cap I = \phi$. Thus, U is a Mic-PT_2 space.

6. Vehicle light system via micro pre-separation axiom



6.1. Harnessing battery power

The battery is the second-most important thing in a vehicle after the engine. The battery supplies electricity to all the parts that need electricity. Similarly, the Mic-T_2 space acts like a battery, which is connected to all the spaces. Due to the results of every micro-open set is micro semi-open and micro pre-open set and $T_2 \to T_1 \to T_0$ hence Mic-T_2 is connected to all the spaces. When the vehicles ignition switch is off, the electricity is passed from the battery (Mic-T_2) to the starter motor (Mic-ST_0), and the

engine is started, likewise, on the down side of the diagram, the battery (Mic-T_2) is connected to the starter relay (Mic-ST_2), engine kill switch (Mic-ST_1), and starter motor (Mic-ST_0). After the vehicle is started, the electricity passes through the alternator (Mic-PT_1). Likewise, in this diagram, on the up side, the charging indicator (Mic-PT_2) is connected to the battery (Mic-T_2) and voltage regulator (Mic-PT_0). So on both sides, either one side will only be running either by starter motor or alternator; similarly, there is no relation between the micro semi-open sets and the micro pre-open sets because there exist an independent cases for both micro semi-open and micro pre-open sets.

6.2. Alternator gets charged through charger

The alternator and starter motor are connected to the battery. Similarly, the space Mic-T_2 is connected with Mic-PT_1 and Mic-ST_0 spaces due to the results of every Mic-T_2 space being Mic-PT_0 and Mic-ST_0 spaces, so I have taken the alternator as Mic-PT_1. The starter motor helps to start the vehicle; once the vehicle is started, the charger indicator gets charged, and

the electricity passes through the alternator to the other parts. So before starting the vehicle, the lower part of the diagram is active, and after the vehicle starts, the upper part of the diagram is active. Considering all the sets, Micro pre-open sets are stronger than the micro open sets and the micro semi-open sets. So, first Micro open sets are related to micro semi-open sets and parallelly micro open sets are related to micro pre-open sets. The upper part of the diagram consists of micro pre-open sets, and the lower part of the diagram consists of micro semi-open set.

6.3. Regulating the volts

The voltage regulator is connected to the battery (Mic-T_2), the alternator (Mic-PT_1) and the charging indicator (Mic-PT_2). Due to the results that every Mic-T_2 space is Mic-PT_2 space and every Mic-PT_1 space is Mic-PT_0 space, I have taken the voltage regulator as Mic-PT_1 space. When the voltage regulator stops working, high or low volts will be sent to the main fuse box, resulting in fuse damage. If the main fuse is working properly, we can say that the voltage regulator works perfectly where the converse need not be true, which is the same as the situation where every Mic-T_1 space is Mic-PT_0 space, but the converse need not be true. Basically, Mic-PT_0 can be attained more easily than the Mic-T_1 space, because every micro open set is micro pre-open set.

6.4. Function of ignition switch

An ignition switch is used to turn on and off the lights. The ignition switch is connected with the battery, voltage regulator, main fuse and starter motor. Similarly, due to the results that every Mic-T_2 space is Mic-T_0, every Mic-T_0 space is Mic-PT_0 and Mic-ST_0 space, and every Mic-T_1 is Mic-T_0, I have taken Mic-T_0 as the ignition switch. If the lights and sensors work properly, then the fuses will be in good condition and likewise, if Mic-T_1 space is satisfied then Mic-T_0 space will also be satisfied due to the fact that every Mic-T_1 space is Mic-T_0 space.

6.5. Uses of starter relay and starter motor

The battery is connected to the stater relay, engine kill switch and starter motor. Similarly, the results of every Mic-T_2 space is Mic-ST_1, Mic-ST_2 and Mic-ST_0 spaces, so I have taken Mic-ST_2 as the starter relay, Mic-ST_1 as the engine kill switch and Mic-ST_0 as the starter motor. The starter relay and main fuse are connected to the engine kill switch to start the engine. The electricity passes from the battery to the starter relay, which checks the power supply, then the electricity passes to the engine kill switch to start the starter motor; likewise, Mic-ST_2 is connected to Mic-ST_1 and Mic-ST_1 is connected to Mic-ST_0. If the starter motor runs, then there would be no problem with the starter relay. Suppose if high or low electricity is passed to the starter relay, the relay gets damaged and is not able to start the starter motor; likewise, if Mic-ST_2 space is satisfied, then Mic-ST_0 and Mic-ST_1 space will also be satisfied due to the fact of every Mic-ST_2 space is Mic-ST_1 and Mic-ST_0 space.

6.6. Fuses of the vehicle

For each part of the vehicle, there will be a fuse in the main fuse box. The use of the fuse is suppose the electricity is highly or lowly passed, first it passes through the fuse, where the fuse will get damaged and save the parts. All the electrical parts in the vehicle have a fuse in the main fuse box; if any part is not working, the fuse is the first thing to be checked. Likewise, every micro open set is a micro semi-open and micro pre-open set, if any damage occurs to the fuse, then the parts of that fuse will not be working and the connection will be cut off. Similarly, the parts are connected to the main fuse with the help of the results, like every Mic-T_1 space is Mic-PT_0, Mic-ST_0 and Mic-T_0 spaces. If the micro open sets are not in the micro pre-open sets or micro semi-open sets, the marking will be cut-off like the connection cut-off. So I have taken Mic-T_1 as the main fuse box.

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