Advances in Nonlinear Variational Inequalities
ISSN: 1092-910X
Vol 28 No. 3s (2025)

A Note on Linear Operator Defined by Lambda Function for Certain Subclass of Uniformly Convex Functions

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Article History:

Received: *26-09-2024*

Revised: *19-11-2024*

Accepted: *28-11-2024*

Abstract:

In this manuscript, we initiate a novel Categories of consistently curved functions with unconstructive coefficients defined by lambda operator. We obtain the coefficient limits, tremendous points and radii of Celestial resemblance as well as roundness for functions belonging to the class TS$(\tau, \mho, \varrho, \varpi, \lambda)$. Moreover, incomplete sums u_k of functions $v(\varkappa)$ in the class $S(\tau, \mho, \varrho, \varpi, \lambda)$ are careful with pointed lower bounds for the ratios of real part of $v(\varkappa)$ to $v_k(\varkappa)$ and $v'(\varkappa)$ to $v_k'(\varkappa)$are determined and also discussed neighbourhood results for this group.

Keywords: Analytic, Coefficient bounds, Partial sums

1. Introduction

Let Adenote the class of all functions $v(\varkappa)$ of the form

$$v(\varkappa) = \varkappa + \sum_{\eta=2}^{\infty} e_\eta \varkappa^\eta \qquad (1.1)$$

In the open unit disc $E = \{\varkappa \in \mathbb{C}: |\varkappa| < 1\}$.

Let S be the subclass of$A$ consists of univalent functions and satisfy the following usual normalization condition$v(0) = v'(0) - 1$ =0.

We denote by S the subclass of A consisting of functions$v(\varkappa)$which are allunivalent in E.

A function $v \in A$ is a starlike function of the order $\mho$, $0 \leq \mho < 1$ if it satisfy

$$\Re\left\{\frac{\varkappa v'(\varkappa)}{v(\varkappa)}\right\} > \mho, \quad (\varkappa \in E). \qquad (1.2)$$

We denote this class with $S^*(\mho)$.

A function $v \in A$ is a convex function of the order $\mho$, $0 \leq \mho < 1$, if it satisfy

$$\Re\left\{1+\frac{\varkappa v''(\varkappa)}{v'(\varkappa)}\right\} > \text{Ѿ}, \quad (\varkappa \in E). \qquad (1.3)$$

We denote this class with $K(\text{Ѿ})$.

Let T denote the class of functions analytic in E that are of the form

$$v(\varkappa) = \varkappa - \sum_{\eta=2}^{\infty} e_\eta \varkappa^\eta, \quad \left(e_\eta \geq 0, \varkappa \in E\right). \qquad (1.4)$$

And let $T^*(\text{Ѿ}) = T \cap S^*(\text{Ѿ}), C(\text{Ѿ}) = T \cap K(\text{Ѿ})$. This class $T^*(\text{Ѿ})$ and allied classes possess some interesting properties and have been extensively studied by Silverman [11] and others.

A function $v \in A$ is said to be in the class of uniformlyconvex functions of order γ and type ϱ, denoted by $UCV(\gamma,\varrho)$, if

$$\Re\left\{1+\frac{\varkappa v''(\varkappa)}{v'(\varkappa)}-\gamma\right\} > \varrho\left|\frac{\varkappa v''(\varkappa)}{v'(\varkappa)}\right|. \qquad (1.5)$$

Where $\varrho \geq 0,\ \gamma \in [-1,1) and\ \varrho+\gamma \geq 0$ and it is said to be in the class correspoondinng class denoted by $SP(\gamma,\varrho)$, if

$$\Re\left\{\frac{\varkappa\, v'(\varkappa)}{v(\varkappa)}-\gamma\right\} > \varrho\left|\frac{\varkappa\, v'(\varkappa)}{v(\varkappa)}-1\right|. \qquad (1.6)$$

Where $\varrho \geq 0,\ \gamma \in [-1,1) and\ \varrho+\gamma \geq 0$. Indeed it follows from (1.5) and (1.6) that $v \in UCV(\gamma,\varrho) \Leftrightarrow zv' \in SP(\gamma,\varrho)$. (1.7)

For $\varrho = 0$, we get respectively, the classes $K(\gamma) and\ S^*(\gamma)$. The function of the class $UCV(0,1) \equiv UCV$ are called uniformlyconvex functions, were introduced and studied by Goodman with geometric interpretation in [4]. The class $SP(0,1) \equiv SP$ is defined by Ronning [9].

The classes $UCV(1,\varrho) \equiv UCV(\varrho) and\ SP(1,\varrho) \equiv SP(\varrho)$ are investigated by Ronning in [8]. For $\gamma = 0$, the classes $UCV(0,\varrho) \equiv \varrho - UCV\ and\ SP(0,\varrho) \equiv \varrho - SP$ are defined respectively, by Kanas and Wisniowska in [5, 6].

Further, Ahuja et al. [1], Bharathi et al. [2], Murugusundarmoorthy and Magesh [7] and Thirupathi Reddy and Venkateswarlu[15] have studied and investigated interesting properties for the classes $UCV(\gamma,\varrho)$ and $SP(\gamma,\varrho)$.

For $v \in A$ given by (1.1) and $t(\varkappa)$ given by

$$t(\varkappa) = \varkappa + \sum_{\eta=2}^{\infty} b_\eta \varkappa^\eta \qquad (1.8)$$

Their convolution (or Hadamard product), denoted by $(v * t)$, is defined as

$$(v * t)(\varkappa) = \varkappa + \sum_{\eta=2}^{\infty} e_\eta b_\eta \varkappa^\eta = (t * v)(\varkappa), \quad (\varkappa \in E). \qquad (1.9)$$

Note that $v * t \in A$.

Let usrecall lambda function [14] specified by

$$\lambda(\varkappa, \lambda) = \sum_{\eta=2}^{\infty} b_\eta \frac{\varkappa^\eta}{(2\eta+1)^\eta}$$

Where $\varkappa \in E, \lambda \in \mathbb{C}, when\ |\varkappa| < 1, \Re(\lambda) > 1, when\ |\varkappa| = 1$ and let $\lambda^{(-1)}(\varkappa, \lambda)$be defined such that

$$\lambda(\varkappa, \lambda) * \lambda^{(-1)}(\varkappa, \lambda) = \frac{1}{(1-\varkappa)^{\varpi+1}}, \qquad \varpi > -1.$$

We now define $\left(\varkappa\, \lambda^{(-1)}(\varkappa, \lambda)\right)$ as the following

$$(\varkappa\, \lambda(\varkappa, \lambda)) * (\varkappa\, \lambda^{(-1)}(\varkappa, \lambda)) = \frac{\varkappa}{(1-\varkappa)^{\varpi+1}}$$

$$(\varkappa\, \lambda(\varkappa, \lambda))\left(\varkappa\, \lambda^{(-1)}(\varkappa, \lambda)\right) = \varkappa + \sum_{\eta=2}^{\infty} \frac{(\varpi+1)_{\eta-1}}{(\eta-1)!}\varkappa^\eta, \qquad \varpi > -1$$

And obtain the following linear operator

$$\mathscr{J}_{\varpi,\lambda} v(\varkappa) = \left(\varkappa\, \lambda^{(-1)}(\varkappa, \lambda)\right) * v(\varkappa) \quad where\ v \in A, \varkappa \in E \quad and$$

$$\left(\varkappa\, \lambda^{(-1)}(\varkappa, \lambda)\right) = \varkappa + \sum_{\eta=2}^{\infty} \frac{(\varpi+1)_{\eta-1}(2\eta-1)^{\lambda}}{(\eta-1)!}\varkappa^\eta.$$

A simple computation gives us

$$\mathscr{J}_{\varpi,\lambda} v(\varkappa) = \varkappa + \sum_{\eta=2}^{\infty} \phi(\varpi, \lambda, \eta) e_\eta \varkappa^\eta \tag{1.10}$$

$$where\ \phi(\varpi, \lambda, \eta) = \frac{(\varpi+1)_{\eta-1}(2\eta-1)^{\lambda}}{(\eta-1)!}, \tag{1.11}$$

Where $(v)_\eta$ is the Pochhammer symbol defined in terms of the Gamma function by

$$(v)_\eta = \frac{\Gamma(\varpi+\eta)}{\Gamma(\varpi)} = \begin{cases} 1, & if\ \eta = 0 \\ \varpi(\varpi+1)\ldots(\varpi+\eta-1), & if\ \eta \in \mathbb{N} \end{cases}.$$

Now, by making use of the linear operator$\mathscr{J}_{\varpi,\lambda} v$, we define a newsubclass of functions belonging to theclass A.

Definition 1.1. For $-1 \le \varpi < 1,\ 0 \le \tau < 1$ and $\varrho \ge 0$,we let $S(\tau, \varpi, \varrho, \varpi, \lambda)$ be the subclass of A Composed offunctions of the type (1.1) and rewarding theregular principle

$$\Re\left\{\frac{\varkappa\left(\mathscr{J}_{\varpi,\lambda} v(\varkappa)\right)' + \tau\varkappa^2\left(\mathscr{J}_{\varpi,\lambda} v(\varkappa)\right)''}{(1-\tau)\mathscr{J}_{\varpi,\lambda} v(\varkappa) + \tau\varkappa\left(\mathscr{J}_{\varpi,\lambda} v(\varkappa)\right)'} - \varpi\right\} \ge \varrho\left|\frac{\varkappa\left(\mathscr{J}_{\varpi,s} v(\varkappa)\right)' + \tau\varkappa^2\left(\lambda v(\varkappa)\right)''}{(1-\tau)\mathscr{J}_{\varpi,\lambda} v(\varkappa) + \tau\varkappa\left(\mathscr{J}_{\varpi,\lambda} v(\varkappa)\right)'} - 1\right|,$$

for $\varkappa \in E$.

By correctly concentrating the principles of τ, ϖ and λ, the class $S(\tau, \mho, \varrho, \varpi, \lambda)$ can be shrinks to the class deliberate former by Ronning[8, 9].

2. Constraints on coefficients

In this segment, we acquire a obligatory and competent ailment for function$v(\varkappa)$ is in the classes $S(\tau, \mho, \varrho, \varpi, \lambda)$ and $TS(\tau, \mho, \varrho, \varpi, \lambda)$.

Theorem 2.3.1. The function$v$ specified by(1.1) is in the class$S(\tau, \mho, \varrho, \varpi, \lambda)$if

$$\sum_{\eta=2}^{\infty}[1+\tau(\eta-1)]\,[\eta(1+\varrho)-(\mho+\varrho)]\phi(\varpi,\lambda,\eta)\left|e_\eta\right| \leq 1-\mho, \tag{2.1}$$

where$-1 \leq \mho < 1,\ \ 0 \leq \tau \leq 1$ *and* $\varrho \geq 0$.

Proof. It be adequate to show that

$$\varrho\left|\frac{\varkappa\left(\mathscr{J}_{\varpi,\lambda}v(\varkappa)\right)'+\tau\varkappa^2\left(\mathscr{J}_{\varpi,\lambda}v(\varkappa)\right)''}{(1-\tau)\mathscr{J}_{\varpi,\lambda}v(\varkappa)+\tau\varkappa\left(\mathscr{J}_{\varpi,\lambda}v(\varkappa)\right)'}-1\right|-\Re\left\{\frac{\varkappa\left(\mathscr{J}_{\varpi,\lambda}v(\varkappa)\right)'+\tau\varkappa^2\left(\mathscr{J}_{\varpi,\lambda}v(\varkappa)\right)''}{(1-\tau)\mathscr{J}_{\varpi,s}v(\varkappa)+\tau\varkappa\left(\mathscr{J}_{\varpi,\lambda}v(\varkappa)\right)'}-1\right\}$$
$$\leq \quad 1-\mho.$$

We have

$$\varrho\left|\frac{\varkappa\left(\mathscr{J}_{\varpi,\lambda}v(\varkappa)\right)'+\tau\varkappa^2\left(\mathscr{J}_{\varpi,\lambda}v(\varkappa)\right)''}{(1-\tau)\mathscr{J}_{\varpi,\lambda}v(\varkappa)+\tau\varkappa\left(\mathscr{J}_{\mu,\lambda}v(\varkappa)\right)'}-1\right|-\Re\left\{\frac{\varkappa\left(\mathscr{J}_{\varpi,\lambda}v(\varkappa)\right)'+\tau\varkappa^2\left(\mathscr{J}_{\varpi,\lambda}v(\varkappa)\right)''}{(1-\tau)\mathscr{J}_{\varpi,\lambda}v(\varkappa)+\tau\varkappa\left(\mathscr{J}_{\varpi,\lambda}v(\varkappa)\right)'}-1\right\}$$
$$\leq \quad (1+\varrho)\left|\frac{\varkappa\left(\mathscr{J}_{\varpi,\lambda}v(\varkappa)\right)'+\tau\varkappa^2\left(\mathscr{J}_{\varpi,\lambda}v(\varkappa)\right)''}{(1-\tau)\mathscr{J}_{\varpi,s}v(\varkappa)+\tau\varkappa\left(\mathscr{J}_{\varpi,\lambda}v(\varkappa)\right)'}-1\right|$$
$$\leq \quad \frac{(1+\varrho)\sum_{\eta=2}^{\infty}(\eta-1)\,[1+\tau(\eta-1)]\phi(\mu,\lambda,\eta)\left|e_\eta\right|}{1-\sum_{\eta=2}^{\infty}[1+\tau(\eta-1)]\,\phi(\mu,\lambda,\eta)\left|e_\eta\right|}.$$

This last communication is constrained beyond by $(1-\mho)$ by

$$\sum_{\eta=2}^{\infty}[1+\tau(\eta-1)]\,[\eta(1+\varrho)-(\mho+\varrho)]\phi(\varpi,\lambda,\eta)\left|e_\eta\right| \leq 1-\mho.$$

Theorem2.2. Prerequisite and fulfilling$v(\varkappa)$of the type (1.4) to be in the class$TS(\tau, \mho, \varrho, \varpi, \lambda), -1 \leq \mho < 1, 0 \leq \tau \leq 1, \varrho \geq 0$is that

$$\sum_{\eta=2}^{\infty}[1+\tau(\eta-1)]\,[\eta(1+\varrho)-(\mho+\varrho)]\phi(\varpi,\lambda,\eta)\left|e_\eta\right| \leq 1-\mho. \tag{2.2}$$

Proof. In interpretation of Theorem 2.1, only the obligation needs to be established.

If $v \in TS(\tau, \mho, \varrho, \varpi, \lambda)$ and $\varkappa$ is real then

$$\frac{1-\sum_{\eta=2}^{\infty}\eta\,[1+\tau(\eta-1)]\phi(\varpi,\lambda,\eta)e_\eta\,\varkappa^{\eta-1}}{1-\sum_{\eta=2}^{\infty}[1+\tau(\eta-1)]\,\phi(\mu,s,\eta)e_\eta\,\varkappa^{\eta-1}}-\mho$$
$$\geq\quad \varrho\left|\frac{\sum_{\eta=2}^{\infty}(\eta-1)\,[1+\tau(\eta-1)]\phi(\varpi,\lambda,\eta)|e_\eta|}{1-\sum_{\eta=2}^{\infty}[1+\tau(\eta-1)]\,\phi(\varpi,\lambda,\eta)|e_\eta|}\right|.$$

Letting $\varkappa \to 1$ the length of the real axis, we acquire the preferred dissimilarity

$$\sum_{\eta=2}^{\infty}[1+\tau(\eta-1)]\,[\eta(1+\varrho)-(\mho+\varrho)]\phi(\varpi,\lambda,\eta)|e_\eta|\leq 1-\mho.$$

Theorem 2.3. Let $v(\varkappa)$ be specified by (1.4) and $t(\varkappa)=\varkappa-\sum_{\eta=2}^{\infty}b_\eta\,\varkappa^\eta$ be in the class $TS(\tau,\mho,\varrho,\varpi,\lambda)$. Then the function $\varkappa(\varkappa)$ specified by

$$\varkappa(\varkappa)=(1-\zeta)v(\varkappa)+\zeta\,t(\varkappa)=\varkappa-\sum_{\eta=2}^{\infty}c_\eta\,\varkappa^\eta,$$

where $c_\eta=(1-\zeta)e_\eta+\zeta b_\eta, 0\leq\zeta<1$ is also in the class $TS(\tau,\mho,\varrho,\varpi,\lambda)$.

Proof. Let the function

$$v_j=\varkappa-\sum_{\eta=2}^{\infty}e_{\eta,j}\,\varkappa^\eta,\ e_{\eta,j}\geq\ 0,\ j=1,2, \tag{2.3}$$

be in the class $TS(\tau,\mho,\varrho,\varpi,\lambda)$. It is ample to show that the function $t(\varkappa)$ specified by

$$t(\varkappa)=\zeta v_1(\varkappa)+(1-\zeta)v_2(\varkappa),\quad o\leq\zeta\leq 1,$$

$\in TS(\tau,\mho,\varrho,\varpi,\lambda)$. Since

$$t(\varkappa)=\varkappa-\sum_{\eta=2}^{\infty}\left[\zeta e_{\eta,1}+(1-\zeta)e_{\eta,2}\right]\varkappa^\eta,$$

an simple calculation through the assist of Theorem 2.2, provides

$$\sum_{\eta=2}^{\infty}[1+\tau(\eta-1)]\,[\eta(\varrho+1)-(\mho+\varrho)]\phi(\varpi,\lambda,\eta)\zeta e_{\eta,1}$$
$$+\sum_{\eta=2}^{\infty}[1+\tau(\eta-1)]\,[\eta(\varrho+1)-(\mho+\varrho)]\phi(\varpi,\lambda,\eta)(1-\zeta)e_{\eta,2}$$
$$\leq\quad \zeta(1-\mho)+(1-\zeta)(1-\mho)$$
$$\leq\quad 1-\mho,$$

$$\Longrightarrow t\in TS(\tau,\mho,\varrho,\varpi,\lambda).$$

Hence $TS(\tau,\mho,\varrho,\varpi,\lambda)$ is convex.

3. Extreme points

The evidence of Theorem 3.1, surveys on appearance parallel to the evidence of the theorem on tremendous points provided in Silverman [11].

Theorem 3.1. Let $v_1(\varkappa) = \varkappa$ and

$$v_\eta(\varkappa) = \varkappa - \frac{1-\mho}{[1+\tau(\eta-1)][\eta(\varrho+1)-(\mho+\varrho)]\phi(\varpi,\lambda,\eta)}\varkappa^\eta, \qquad (3.1)$$

for $\eta = 2,3,\cdots$.Then $v(\varkappa) \in TS(\tau,\mho,\varrho,\varpi,\lambda) \Leftrightarrow v(\varkappa)$ can be expressed in the type $v(\varkappa) = \sum_{\eta=2}^{\infty}\zeta_\eta v_\eta(\varkappa)$,where $\zeta_\eta \geq 0$ and $\sum_{\eta=1}^{\infty}\zeta_\eta = 1$.

4. Closure theorem

Theorem 4.1. Let the function $v_j(\varkappa), j = 1,2,\cdots,p$ specified by (2.3) be in the classes $TS(\tau,\mho_j,\varrho,\varpi,\lambda)$, $j = 1,2,\cdots,p$, respectively. Then the function specified by $\varkappa(\varkappa) = \varkappa - \frac{1}{p}\sum_{\eta=2}^{\infty}\left(\sum_{j=1}^{p} e_{\eta,j}\right)\varkappa^\eta$ is in the class $TS(\tau,\mho,\varrho,\varpi,\lambda)$,where $\mho = \min_{1\leq j\leq p}\{\mho_j\}$,where $-1 \leq \mho_j \leq 1$.

Proof. Since $v_j(\varkappa) \in TS(\tau,\mho_j,\varrho,\varpi,\lambda)$, $j = 1,2,\cdots,p$, by smearing Theorem 2.2 to the equation (2.3) we scrtinise that

$$\begin{aligned}
&\sum_{\eta=2}^{\infty}[1+\tau(\eta-1)]\,[\eta(\varrho+1)-(\mho+\varrho)]\phi(\varpi,\lambda,\eta)\left(\frac{1}{p}\sum_{j=1}^{p}e_{\eta,j}\right)\\
=\;& \frac{1}{p}\sum_{j=1}^{p}e_{\eta,j}\left(\sum_{\eta=2}^{\infty}[1+\tau(\eta-1)]\,[\eta(\varrho+1)-(\mho+\varrho)]\phi(\varpi,\lambda,\eta)e_{\eta,j}\right)\\
\leq\;& \frac{1}{p}\sum_{j=1}^{p}(1-\mho_j)\\
\leq\;& 1-\mho
\end{aligned}$$

which in examination of Theorem 2.2, once more implies that $\varkappa(\varkappa) \in TS(\tau,\mho,\varrho,\varpi,\lambda)$ and so the evidence is inclusive.

Theorem 4.2. *Let* $v \in TS(\tau,\mho,\varrho,\varpi,\lambda)$.Then

1. v is functionstarshape of order w, $0 \leq w < 1$,in the disc $|\varkappa| < r_1$ *i.e.*, $\Re\left\{\frac{\varkappa v'(\varkappa)}{v(\varkappa)}\right\} > w$, $|\varkappa| < r_1$,where

$$r_1 = \inf_{\eta\geq 2}\left\{\left(\frac{1-w}{\eta-w}\right)\frac{[1+\tau(\eta-1)][\eta(\varrho+1)-(\mho+\varrho)]\phi(\varpi,\lambda,\eta)}{1-\mho}\right\}^{\frac{1}{\eta-1}}.$$

2. *v is convex of order w*, $0 \leq w < 1$,in the disc $|\varkappa| < r_2$ *i.e.*, $\Re\left\{1+\frac{\varkappa v''(\varkappa)}{v'(\varkappa)}\right\} > w$, $|\varkappa| < r_2$,where

$r_2 = \inf_{\eta \geq 2}\left\{\left(\frac{1-w}{\eta-w}\right)\frac{[1+\tau(\eta-1)][\eta(\varrho+1)-(\varpi+\varrho)]\phi(\omega,\lambda,\eta)}{1-\varpi}\right\}^{\frac{1}{\eta}}$.Each of these consequences are sharp for the extremal function$v(\varkappa)$provided by (3.1).

Proof.(1). Provided $v \in A$ and v is function starshape of order w,we have

$$\left|\frac{\varkappa v'(\varkappa)}{v(\varkappa)} - 1\right| < 1 - w. \qquad (4.1)$$

For the LHS (4.1), we have

$$\left|\frac{\varkappa v'(\varkappa)}{v(\varkappa)} - 1\right| \leq \frac{\sum_{\eta=2}^{\infty}(\eta-1)\, e_\eta |\varkappa|^{\eta-1}}{1-\sum_{\eta=2}^{\infty} e_\eta\, |\varkappa|^{\eta-1}}.$$

The last communication is less than $1 - w$ if

$$\sum_{\eta=2}^{\infty}\frac{\eta-w}{1-w}e_\eta|\varkappa|^{\eta-1} < 1.$$

Using the fact that $v \in TS(\tau, \varpi, \varrho, \omega, \lambda) \Leftrightarrow$

$$\sum_{\eta=2}^{\infty}\frac{[1+\tau(\eta-1)][\eta(\varrho+1)-(\varpi+\varrho)]\phi(\omega,\lambda,\eta)}{1-\varpi}e_\eta < 1.$$

We can say (4.1) is true if

$$\frac{\eta-\mathrm{w}}{1-w}|\varkappa|^{\eta-1} < \frac{[1+\tau(\eta-1)][\eta(\varrho+1)-(\varpi+\varrho)]\phi(\omega,\lambda,\eta)}{1-\varpi}$$

or equivalently,

$$|\varkappa|^{\eta-1} < \frac{(1-w)[1+\tau(\eta-1)][\eta(\varrho+1)-(\varpi+\varrho)]\phi(\omega,\lambda,\eta)}{(\eta-w)(1-\varpi)}$$

which yields the functionstarshapeness of the family.

5. Partial Sums

Succeeding the former endeavors by Silverman [12] and Silvia [13] on PS of regular functions. We deliberate in this section PS of functions in this class$S(\tau, \varpi, \varrho, \omega, \lambda)$ and acquire pointed lesser boundaries for the ratios of actual fraction of $v(\varkappa)$ to$v_\ell(\varkappa)$ and $v'(\varkappa)$ to $v'_\ell(\varkappa)$.

Theorem 5.1. Let$v(\varkappa) \in S(\tau, \varpi, \varrho, \omega, \lambda)$be provided by (1.1) and indicate the PS$v_1(\varkappa)$and $v_\ell$by

$$v_1(\varkappa) = \varkappa \text{ and } v_{\not{e}}(\varkappa) = \varkappa + \sum_{\eta=2}^{\not{e}} e_\eta \varkappa^\eta, \qquad (\not{e} \in \mathbb{N}\backslash\{1\}). \quad (5.1)$$

Suppose that $\sum_{\eta=2}^{\infty} l_\eta |e_\eta| \leq 1,$

where $l_\eta = \dfrac{[1+\tau(\eta-1)][\eta(1+\varrho)-(\varpi+\varrho)]\phi(\omega,\lambda,\eta)}{1-\varpi}.$ (5.2)

Then $v \in S(\tau, \varpi, \varrho, \omega, \lambda)$.

Further more, $\Re\left[\dfrac{v(\varkappa)}{v_{\not{e}}(\varkappa)}\right] > 1 - \dfrac{1}{l_{\not{e}+1}}, \qquad (\varkappa \in E, \quad \not{e} \in \mathbb{N})$ (5.3)

and $\Re\left[\dfrac{v_{\not{e}}(\varkappa)}{v(\varkappa)}\right] > \dfrac{l_{\not{e}+1}}{1+l_{\not{e}+1}}.$ (5.4)

Proof. For the Proportions l_η provided by (5.2) it is not tough to authenticate that

$$l_{\eta+1} > l_\eta > 1. \quad (5.5)$$

Therefore we have $\sum_{\eta=2}^{\not{e}} |e_\eta| + l_{\not{e}+1} \sum_{\eta=\not{e}+1}^{\infty} |e_\eta| \leq \sum_{\eta=2}^{\infty} l_\eta |e_\eta| \leq 1$ (5.6)

by using the postulate (5.2). By setting

$$t_1(\varkappa) = l_{\not{e}+1}\left[\frac{v(\varkappa)}{v_{\not{e}}(\varkappa)} - \left(1 - \frac{1}{l_{\not{e}+1}}\right)\right] = 1 + \frac{l_{\not{e}+1}\sum_{\eta=\not{e}+1}^{\infty} e_\eta \varkappa^{\eta-1}}{1+\sum_{\eta=2}^{\not{e}} e_\eta \varkappa^{\eta-1}} \quad (5.7)$$

and smearing (5.6), we find that

$$\left|\frac{t_1(\varkappa)-1}{t_1(\varkappa)+1}\right| \leq \frac{l_{\not{e}+1}\sum_{\eta=y+1}^{\infty}|e_\eta|}{2-2\sum_{\eta=2}^{\not{e}}|e_\eta| - l_{\not{e}+1}\sum_{\eta=\not{e}+1}^{\infty}|e_\eta|} \leq 1 \quad (5.8)$$

Justifying the need is all that is required. (5.3) of Theorem 5.1. To understand that

$$v(\varkappa) = \varkappa + \frac{\varkappa^{\not{e}+1}}{l_{\not{e}+1}} \quad (5.9)$$

provides sharp consequence, we scrtinise that for $\varkappa = re^{\frac{i\pi}{\not{e}}}$ that

$$\frac{v(\varkappa)}{v_{\not{e}}(\varkappa)} = 1 + \frac{\varkappa^{\not{e}}}{l_{\not{e}+1}} \to 1 - \frac{1}{l_{\not{e}+1}} \text{ as } \varkappa \to 1^-.$$

Similarly, if we take

$$\begin{aligned} t_2(\varkappa) &= \left(1+l_{\not e+1}\right)\left(\frac{v_{\not e}(\varkappa)}{v(\varkappa)}-\frac{l_{\not e+1}}{1+l_{\not e+1}}\right) \\ &= 1-\frac{\left(1+l_{\eta+1}\right)\sum_{\eta=\not e+1}^{\infty} e_\eta\ \varkappa^{\eta-1}}{1+\sum_{\eta=2}^{\infty} e_\eta\ \varkappa^{\eta-1}} \end{aligned} \quad (5.10)$$

and constructing use of (5.6), we can comprehend that

$$\left|\frac{t_2(\varkappa)-1}{t_2(\varkappa)+1}\right| \le \frac{\left(1+l_{\not e+1}\right)\sum_{\eta=\not e+1}^{\infty}|e_\eta|}{2-2\sum_{\eta=2}^{\not e}|e_\eta|-\left(1-l_{\not e+1}\right)\sum_{\eta=\not e+1}^{\infty}|e_\eta|}$$

which results is directly to the Pronouncement(5.4) of the principle 5.1.

For each, the bound is sharp in (5.4) $\not e \in \mathbb{N}$ with the external function$v(\varkappa)$ provided by (5.9). The evidence of the Theorem 5.1, is consequently absolute.

Theorem 5.2. If$v(\varkappa)$of the type (1.1) fulfils the ailment (2.1)then

$$\Re\left[\frac{v'(\varkappa)}{v'_{\not e}(\varkappa)}\right] \ge 1-\frac{\not e+1}{l_{\not e+1}}. \quad (5.11)$$

Proof. By setting

$$\begin{aligned} t(\varkappa) &= l_{\not e+1}\left[\frac{v'(\varkappa)}{v'_{\not e}(\varkappa)}\right]-\left(1-\frac{\not e+1}{l_{\not e+1}}\right) \\ &= \frac{1+\frac{l_{\not e+1}}{\not e+1}\sum_{\eta=\not e+1}^{\infty}\eta\, e_\eta\ \varkappa^{\eta-1}+\sum_{\eta=2}^{\infty}\eta\, e_\eta\, \varkappa^{\eta-1}}{1+\sum_{\eta=2}^{\infty}\eta\, e_\eta\, \varkappa^{\eta-1}} \\ &= 1+\frac{\frac{l_{\not e+1}}{\not e+1}\sum_{\eta=\not e+1}^{\infty}\eta\, e_\eta\ \varkappa^{\eta-1}}{1+\sum_{\eta=2}^{\infty}\eta\, e_\eta\, \varkappa^{\eta-1}} \end{aligned}$$

$$\left|\frac{t(\varkappa)-1}{t(\varkappa)+1}\right| \le \frac{\frac{l_{\not e+1}}{\not e+1}\sum_{\eta=\not e+1}^{\infty}\eta|e_\eta|}{2-2\sum_{\eta=2}^{\not e}\eta|e_\eta|-\frac{l_{\not e+1}}{\not e+1}\sum_{\eta=\not e+1}^{\infty}\eta|e_\eta|}. \quad (5.12)$$

$$\text{Now } \left|\frac{t(\varkappa)-1}{t(\varkappa)+1}\right| \le 1 \text{ if } \sum_{\eta=2}^{\not e}\eta|e_\eta|+\frac{l_{\not e+1}}{\not e+1}\eta|e_\eta| \le 1. \quad (5.13)$$

Since the L.H.S of (5.13)is enclosed over by $\sum_{\eta=2}^{\not e} l_\eta|e_\eta|$ if

$$\sum_{\eta=2}^{\not e}\left(l_\eta-\eta\right)|e_\eta|+\sum_{\eta=\not e+1}^{\infty} l_\eta-\frac{l_{\not e+1}}{\not e+1}\eta|e_\eta| \ge \text{o}. \quad (5.14)$$

The impact on the extreme function is severe.

$$v(\varsigma) = \varsigma + \frac{\varsigma^{\not e+1}}{l_{\not e+1}}.$$

Theorem 5.3. If$v(\varsigma)$of the type *(*1.1*)* fulfils the ailment *(*2.1*)* then

$$\Re\left[\frac{v'_{\phi}(\varkappa)}{v'(\varkappa)}\right] \geq \frac{l_{\phi+1}}{\phi + 1 + l_{\phi+1}}. \tag{5.15}$$

Proof. By setting

$$\begin{aligned} t(\varkappa) &= [\phi + 1 + l_{\phi+1}]\left[\frac{v'_{\phi}(\varkappa)}{v'(\varkappa)} - \frac{l_{\phi+1}}{\phi + 1 + l_{\phi+1}}\right] \\ &= 1 - \frac{\left(1 + \frac{l_{\phi+1}}{\phi+1}\right)\sum_{\eta=\phi+1}^{\infty} \eta\, e_\eta\, \varkappa^{\eta-1}}{1 + \sum_{\eta=2}^{\phi} \eta\, e_\eta\, \varkappa^{\eta-1}} \end{aligned}$$

and making use of (5.14), we construe that

$$\left|\frac{t(\varkappa) - 1}{t(\varkappa) + 1}\right| \leq \frac{\left(1 + \frac{l_{\phi+1}}{\phi+1}\right)\sum_{\eta=\phi+1}^{\infty} \eta\, |e_\eta|}{2 - 2\sum_{\eta=2}^{\phi} \eta\, |e_\eta| - \left(1 + \frac{l_{\phi+1}}{\phi+1}\right)\sum_{\eta=\phi+1}^{\infty} \eta\, |e_\eta|} \leq 1,$$

which indications us without delay to the allegation of the Theorem 5.3.

6. Neighborhood Consequences

Neighborhood for the class $S^{\xi}(\tau, \mho, \varrho, \varpi, \lambda)$:

In this segment, we establish the neighborhoods for the group $S^{\xi}(\tau, \mho, \varrho, \varpi, \lambda)$which we describe as follow:

Definition 6.1. A function $v \in A$ is said to be in the class $S^{\xi}(\tau, \mho, \varrho, \varpi, \lambda)$ if there exist a function

$t \in S(\tau, \mho, \varrho, \varpi, \lambda)$ such that

$$\left|\frac{v(\varkappa)}{t(\varkappa)} - 1\right| < 1 - \mho, \quad (\varkappa \in E, 0 \leq \mho < 1). \tag{6.1}$$

For any function $v(\varkappa) \in A,\ \varkappa \in E \ and\ w \geq 0$, we specified

$$N_{\eta,w}(v) = \left\{t \in S: t(\varkappa) = \varkappa + \sum_{\eta=2}^{\infty} b_\eta \varkappa^\eta \ \ and \ \ \sum_{\eta=2}^{\infty} \eta\left|e_\eta - b_\eta\right| \leq\ w\right\}. \tag{6.2}$$

Theorem 6.2. If$t \in S(\tau, \mho, \varrho, \varpi, \lambda)$ and

$$\xi = 1 - \frac{w(1 - \mho)}{2[(1 - \mho) - (1 + \tau)(2 + \varrho - \mho)\phi(\varpi, \lambda, 2)]} \tag{6.3}$$

then $N_{\eta,w}(t) \subset S^{\xi}(\tau, \mho, \varrho, \varpi, \lambda)$.

Proof. Suppose $v \in N_{\eta,w}(t)$. Then from (6.2)

$$\sum_{\eta=2}^{\infty} n\left|e_\eta - b_\eta\right| \leq w \qquad (6.4)$$

the coefficient inequality is produced by.

$$\sum_{\eta=2}^{\infty}\left|e_\eta - b_\eta\right| \leq \frac{w}{2}, \ \ (\eta \in \mathbb{N}). \qquad (6.5)$$

Next, since $t \in S(\tau, \varpi, \varrho, \omega, \lambda)$, we have

$$\sum_{\eta=2}^{\infty} b_\eta \leq \frac{(1+\tau)(2+\varrho-\varpi)\phi(\omega,\lambda,2)}{1-\varpi}. \qquad (6.6)$$

So that

$$\begin{aligned}\left|\frac{v(\varkappa)}{t(\varkappa)} - 1\right| &< \frac{\sum_{\eta=2}^{\infty}\left|e_\eta - b_\eta\right|}{1-\sum_{\eta=2}^{\infty} b_\eta} \\ &= \frac{w(1-\varpi)}{2[(1-\varpi)-(1+\tau)(2+\varrho-\varpi)\phi(\omega,\lambda,2)]} \\ &= 1-\xi\end{aligned}$$

Providedξ is provided by (6.3). Hence proved.

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