

Evaluation of Continuous Sampling Plan indexed through Maximum Allowable Average Outgoing Quality

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Article History:

Received: 21-09-2024

Revised: 15-11-2024

Accepted: 25-11-2024

Abstract:

This paper introduces a result oriented designing and selection procedure for Continuous Sampling Plan type of CSP-1 indexed through Maximum Allowable Average Outgoing Quality (MAAOQ) for final products. Expressions for performance measures such as Operating Characteristic function and MAAOQ functions are derived with using Polya distribution. It is a preferred quality index for engineers and quality controllers, when the sample size is large sufficient.

Keywords: Acceptable Quality Level, Average Outgoing Quality Level, Maximum Allowable Average Outgoing Quality, Maximum Allowable Percent Defective.

Introduction

Continuous sampling plans (CSPs) are used in continuous production processes where no separate lots are formed for inspection. They are generally used on some type of conveyor but are applicable to any continuous type operation where it is not desirable to accumulate the product into lots for purposes of inspection. Dodge (1943) has introduced the concept of CSP and provided mathematical rationale and rules of operation for CSP-1. Extensions of CSP-1 which have received considerable attention and use are devised by Dodge and Torrey (1951) and labeled as CSP-2 and CSP-3. As indicated in their paper,

both plans grew out of suggestions given by inspection personnel engaged in applying continuous sampling.

CSPs have an acceptance number 'c', which is greater than zero, while inspection. CSP-1 has two parameters 'i' and 'f'. 'i' is a clearance number and 'f' is a sampling frequency. In CSP-2 an additional parameter 'k' is introduced which is a minimum number of consecutive sampled units. That is, it allows for sampling to continue with the occurrence of an occasional defect provided that a defect does not occur too frequently. CSP-3 introduces a simple and effective refinement of CSP-2 designed to provide extra protection against the case of spotty quality, i.e., the clustering of excessive defectiveness. The additional four consecutive units are inspected upon finding a defect in sampling on the CSP-2 basis. If the given conditions are not up to the satisfactory level, the 100% inspection phase is invoked immediately. Evaluation of CSP-2 and CSP-3 is similar to that of CSP-1. Only slight variations in some symbols and definitions are presented. The other conditions are the same as for CSP-1. Chung-Ho Chen (2004) developed the AOQL for lot by lot CSPs, which is one of the indices to measure the performance of the CSP-1.

Continuous sampling plans are useful when the formation of lots for sampling inspection is impracticable. The application of continuous sampling plan is continuous flow of units from which the production process and units are offered for inspection item by item in the order of their production. To improve the quality of any product and services, it is customary to modernize the quality practices. An efficient quality improvement program can be instrumental in increasing productivity at reduced cost. As a result of increasing customer quality requirements and development of new product technology, many existing quality assurance practices and techniques need to be modified. The need for statistical and analytical techniques in quality assurance is rapidly increasing owing to stiff competition in industry towards product quality management (PQM).

Suresh and Ramkumar (1996) have proposed a new procedure for the selection of a Single Sampling Plan (SSP) in terms of the AOQL and MAAOQ. The AOQL is defined as the poorest average quality that the consumer will receive in the long run, when defective items are replaced by non-defective items. Mandelson (1962) has explained the desirability of developing a system of sampling plans indexed through the MAPD and shown that $p^* = c/n$. Suresh and Nirmala (2015) have presented construction and selection of various CSPs indexed through quality decision regions and studied comparison of certain type of CSPs and their operating procedures. Many works on designing of continuous sampling plans are available in the literature, which are proposed by various authors.

Glossary of Symbols:

p	=	probability of a unit produced by the process being non-	conforming
q	=	$1 - p$	
i	=	clearance number	
f	=	sampling frequency	
F	=	Average Fraction Inspected	
P^*	=	MAPD = Maximum Allowable Percent Defective	

p_m = $AOQL$ = Average Outgoing Quality Level

$AOQ(p)$ = Average Outgoing Quality when the process non-conformance Probability is p

p_{MAOQ} = $MAAOQ$ = Maximum Allowable Average Outgoing Quality

$P_a(p)$ = probability of acceptance during sampling phase when the process non-conformance probability is p . Technical terms are defined as in ANSI (1987) standards.

Operating Function of CSP – 1 Plans:

The operating procedure of a CSP -1 plan is,

1. At the outset, the inspect 100 % of the units consecutively as produced for the characteristic of interest until i units in succession are found to be conforming.
2. When i consecutive conforming units are found, inspect every f unit.
3. If a nonconformity is found during sampling revert to 100% inspection and return to step one.

For the CSP -1 plan with parameters i and f , the operating characteristic OC function is,

$$P_a(p) = q^i [f(1-q^i) + q^i]^{-1}$$

Maximum Allowable Percent Defective (MAPD)

The point on the OC curve at which the descent is steepest is called point of inflection. The proportion nonconforming corresponding to the point of inflection of OC curve is interpreted as the maximum allowable percent defective.

Maximum Allowable Average Outgoing Quality (MAAOQ)

The MAAOQ of a sampling plan is designated as the Average Outgoing Quality (AOQ) at the MAPD.

$$AOQ = p P_a(p)$$

Then MAAOQ = AOQ at $p = p_*$ which can be rewritten as,

$$MAAOQ = p_* * P_a(p_*)$$

One of the desirable properties of an OC curve is that the decrease of $P_a(p)$ should be lower for smaller values of p and steeper for higher values of p , which provides better overall discrimination. Since p corresponds to the inflection point of an OC curve, it implies that

$$\frac{d^2 P_a(p)}{dp^2} < 0, \quad \text{for } p < p_*$$

$$\frac{d^2 P_a(p)}{dp^2} > 0, \quad \text{for } p > p_*$$

$$\frac{d^2 P_a(p)}{dp^2} = 0, \quad \text{for } p = p_*$$

Designing plans for given MAPD

The proportion nonconforming corresponding to the inflection point of the OC curve, denoted by p^* and interpreted as the Maximum Allowable Percent Defective (MAPD) by Mayer (1967) is also used as the quality standard along with some other conditions for the selection of the sampling plans. The relative slope of the OC curve at this point, denoted as h^* is also used to fix the discrimination of the OC curve of any sampling plan. The desirability of developing a set of sampling plans indexed by p^* has been explained by Mandelson (1962) and Soundararajan (1971). While choosing a plan for given p^* , one has to specify the measure of discrimination $K = p_T / p^*$, where p_T is the point at which the tangent line at the inflection point of the OC curve cuts the p -axis or h^* , the relative slope of the OC curve at p^* . Suresh and Srivenkataramana (1996) have designed procedure for the selection of single sampling plan using producer and consumer quality levels. Suresh (1993) has studied various sampling plans with the quality levels along with their relative slopes.

According to Suresh and Ramkumar (1996) the Maximum Allowable Average Outgoing Quality is the outgoing quality defined with p which is a favoured quality index for engineers and it protects the interests of the consumer. Considering the simplicity, practicability and consumer protection offered, the MAAOQ has major practical advantages in acceptance sampling compared with AOQL, which can be considered as a measure for selection of plan parameters. Dodge and Romig (1959) have proposed procedure for the selection of Single Sampling Plan indexed through AOQL by minimizing the Average Total Inspection. Soundararajan (1981) has suggested procedure for the selection of Single Sampling Plan in terms of AQL and AOQL.

Connectivity of AOQL, MAPD and MAAOQ in CSPs plan

The collection of average outgoing quality limit (AOQL), Maximum allowable present defective (MAPD) and maximum allowable average outgoing quality (MAAOQ) for all CSPs plans are consider. It is found that MAAOQ is more precious than AOQL. For example, Table are presented for designing of all types of CSPs plans, for given consideration. Here MAAOQ value is very less compared to AOQL value. It implies that through MAAOQ designing, additional strengthening is ensured to the producer from the risk of rejecting the good quality items measure up to AOQL designing, which is perceptible through figure 1.

The procedure for designing a CSP with quality standards p^* and MAAOQ, where p^* is the quality standard to the Y-axis from the inflection point of the AOQ curve-2. The MAAOQ of a CSP is defined as the AOQ at the MAPD. Using MAPD as a standard, and the MAAOQ as an average outgoing quality, the parameters for CSP are determined.

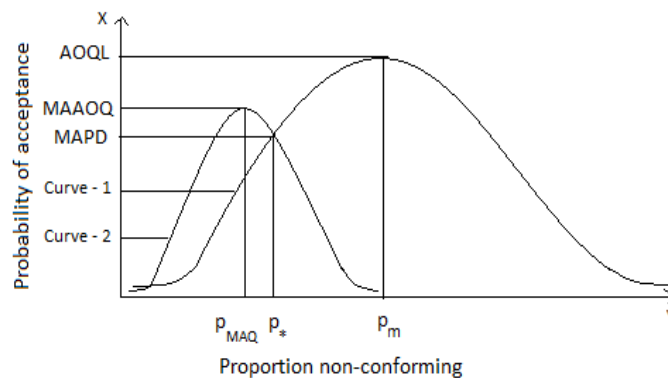


Figure: 1: The AOQL and MAAOQ curves for CSP - 1 plan

Figure 1 in attendance the Average Outgoing Quality (AOQ) curves along with AOQL curve and MAAOQ curve. It can be observed that two curves are in desirable shape with engorge at higher value of AOQL in curve-1 and a sudden drop at higher value of MAAOQ in curve-2. It point towards that the MAAOQ practice doubly protects the security of the consumer in terms of incoming and outgoing quality and protects the producer's interest against acceptable quality and safeguards the consumer against poor quality.

$$PY_{N,m,r,c}(x) = \frac{\binom{r+x-1}{c} \binom{N-r}{c} + n-x-1}{\frac{N}{c} + n - 1} C n - x, x = 0, 1, 2, \dots, n.$$

Where N,n,r and c are natural numbers.

Thus, the addition of prior information about the process the past benefits the producer with minimising the risk for discarded items of good quality. However, the CSP satisfies the interests of the consumer by accepting poor quality items with probabilities not exceeding the assigned risk of non-conformities. On the other hand, the nature of the AOQ curve-2 of the CSP provides relatively more protection to the consumer against the items of poor quality.

The AQLs for the plans are also provided for a fixed producer risk ($\alpha = 0.05$). The Technical terms are defined as in ANSI (1987) standards.

Selection Procedure

The CSP-1 plan starts with 100% inspection of the units produced (phase1) and proceeds to sampling inspection (phase-ii) only when 'i' consecutive units are found conforming in phase-1. During the sampling inspection, only a fraction 'f' of the units are selected at random and inspected. A return to 100% inspection phase is made whenever a nonconforming unit is found during the sampling inspection phase.

For specified MAAOQ and MAPD

Table 1 is used to construct the plans when MAPD and MAAOQ are specified. For any given values of MAPD (p_*) and MAAOQ (p_{MAOQ}), find the value in Table 1 under the column 2 which is approximately equal to the calculated value. Then the corresponding value of c and f are noted. From this one can determine the parameters c and f for the continuous sampling plan-1.

Table 1: Values of AOQL, AQL, LQL, MAPD and MAAOQ for specified values of ‘i’, c = 0.01 and ‘f = 0.05’ for Continuous Sampling Plan (CSP-1)

AOQL (%)	AQL i	LQL	MAPD	MAAOQ = 0.01
0.02	8248	0.0652	0.0602	8242.895
0.0225	6205	0.0802	0.0752	6199.867
0.025	4730	0.0973	0.0923	4724.835
0.0275	3590	0.119	0.114	3584.795
0.03	2619	0.153	0.148	2613.729
MAAOQ = 0.025				
0.035	7785	0.089	0.084	7779.862
0.04	5586	0.112	0.107	5580.821
0.045	4226	0.137	0.132	4220.776
0.05	3298	0.163	0.158	3292.729
0.055	2622	0.194	0.189	2616.672
0.06	2106	0.226	0.221	2100.613
0.065	1696	0.266	0.261	1690.538
0.07	1353	0.317	0.312	1347.441
0.075	1045	0.383	0.378	1039.314
MAAOQ = 0.065				
3080	122	0.227	0.222	3196.551
2361	69	0.272	0.267	2424.461
1878	45	0.319	0.314	1917.367
1530	32	0.37	0.365	1556.265
1267	24	0.424	0.419	1285.157
1060	19	0.484	0.479	1073.037
894	15	0.548	0.543	902.909
756	12	0.619	0.614	761.767
640	10	0.702	0.697	643.601
538	8	0.794	0.789	539.417
445	6	0.896	0.891	444.213
352	5	1.08	1.075	349.845

For specified AOQL and MAPD

Table 1 is used to construct continuous sampling plan-1 for given MAPD and MAAOQ quality levels. For any given values of the I, c and f one can find the performance measure MAAOQ and various ratios $R_1 = \text{MAAOQ} / \text{MAPD}$, $R_2 = \text{AOQL} / \text{MAPD}$ and $R_3 = \text{AQL} / \text{MAPD}$.

Numerical Examples

- Given $\text{MAPD} = 0.0602$, and $\text{MAAOQ} = 8242.895$ compute the ratio R_1 which is $R_1 = \text{MAAOQ} / \text{MAPD} = 8242.895 / 0.0602 = 136925.2$, $R_2 = \text{AOQL} / \text{MAPD} = 0.46387 / 0.0602 = 0.33223$ and $R_3 = \text{AQL} / \text{MAPD} = 8248 / 0.0602 = 137010$ which is associated with, $f = 1/2$. Thus, $f = 1/2$ are the

parameters selected for continuous sampling plan CSP – 1 for a given MAPD of 0.0602 defective and MAAOQ of 8242.895 acceptance.

Construction of Tables

The expression for the OC function of continuous sampling plan-1 is given by,

$$Pa(p) = q^i [f(1-q)^i + q^i]^{-1}$$

$$R_1 = MAAOQ / MAPD, R_2 = AOQL / MAPD \text{ \& } R_3 = AQL / MAPD$$

The incoming quality MAPD column of Table 1 is constructed by equation 1 is equating the second order derivative of operating characteristic function. The values of AQL, AOQL, MAAOQ, MAPD, R₁, R₂ and R₃ are anticipated.

Table 2: Certain Characteristic values for R₁, R₂ and R₃ Continuous Sampling Plan (CSP-1)

i	f	AOQL	AQL	MAPD	MAAOQ	R1	R2	R3
2	1/2	0.02	8248	0.0602	8242.895	136925.2	0.33223	137010
2	1/3	0.0225	6205	0.0752	6199.867	82445.04	0.2992	82513.3
2	1/4	0.025	4730	0.0923	4724.835	51189.98	0.27086	51245.94
3	1/5	0.0275	3590	0.114	3584.795	31445.57	0.24123	31491.23
3	1/8	0.03	2619	0.148	2613.729	17660.33	0.2027	17695.95
3	1/9	0.035	7785	0.084	7779.862	92617.4	0.41667	92678.57
3	2/3	0.04	5586	0.107	5580.821	52157.21	0.37383	52205.61
4	2/4	0.045	4226	0.132	4220.776	31975.58	0.34091	32015.15
4	2/5	0.05	3298	0.158	3292.729	20840.06	0.31646	20873.42
4	2/6	0.055	2622	0.189	2616.672	13844.83	0.29101	13873.02
4	2/7	0.06	2106	0.221	2100.613	9505.036	0.27149	9529.412
4	2/8	0.065	1696	0.261	1690.538	6477.157	0.24904	6498.084
4	2/9	0.07	1353	0.312	1347.441	4318.721	0.22436	4336.538
4	3/6	0.075	1045	0.378	1039.314	2749.508	0.19841	2764.55
5	3/7	3080	122	0.222	3196.551	14398.88	13873.9	549.5495
5	3/8	2361	69	0.267	2424.461	9080.378	8842.7	258.427
5	3/9	1878	45	0.314	1917.367	6106.264	5980.89	143.3121
5	4/5	1530	32	0.365	1556.265	4263.74	4191.78	87.67123
5	4/6	1267	24	0.419	1285.157	3067.2	3023.87	57.27924
5	4/7	1060	19	0.479	1073.037	2240.161	2212.94	39.66597
5	4/8	894	15	0.543	902.909	1662.816	1646.41	27.62431
5	4/9	756	12	0.614	761.767	1240.663	1231.27	19.54397
5	5/6	640	10	0.697	643.601	923.3874	918.221	14.3472
5	5/7	538	8	0.789	539.417	683.6717	681.876	10.13942
5	5/8	445	6	0.891	444.213	498.5556	499.439	6.734007
5	5/9	352	5	1.075	349.845	325.4372	327.442	4.651163

Conclusion

The present work mainly emphasizes that the selection of sampling system with this procedure is more advantageous to the producer and consumer than conventional methods. This method facilitates user friendly attitude for engineers who are working on the floor and the system that adopt readymade tables, which are provided for selecting plans based on specific input/output parameters. This design of sampling system is constructed for application towards industrial shop-floor situations for manufacturing of product/process, which are tailor made to the industrial situations. The main advantages of this technique are less handling of units during inspection, more Economical owing to fewer inspections thereby simplifying recruiting, training and supervising.

The present development would be valuable addition to the literature and a useful device for quality control practitioners. Conversions of parameters are also provided for convenience. Tables are provided which are tailor-made, handy and afford ready-made uses to the industrial shop floor situations. These tables provide higher probability of acceptable product selection and further satisfy the producers internally making quality production feasible. Therefore, the proposed plan can be used for the inspection of the product quality to save the cost, time and efforts. The future research work shall concentrate at various categories of continuous sampling plans namely CSP-2, CSP-3, CSP-4, CSP- 5, CSP-M, CSP-F, CSP-T, and CSP-R with Palya distributions and fuzzy logic analysis.

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