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Einstein Operators of Fuzzy Neutrosophic Soft Matrix of Type A

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Abstract:

In this article, we define two new operators Einstein sum and Einstein product of fuzzy neutrosophic soft matrices and investigated the algebraic properties of fuzzy neutrosophic soft matrices using the max-min operators and then constructed the scalar multiplication of fuzzy neutrosophic soft matrices and investigate their algebraic properties.

Keywords: Fuzzy neutrosophic soft matrices, Einstein sum, Einstein multiplication, Comparable fuzzy neutrosophic soft matrices, Einstein scalar multiplication.

1. Introduction

Theories based on uncertainties or undetermined data have resulted in the invention of many new methodologies, but these models persist as complications to tackle. Zadeh [27] examined the membership degree to present the brilliant idea of Fuzzy Set ($\mathscr{FS}$). The idea of an intuitionistic fuzzy set ($I\ \mathscr{FS}$) was then introduced by Atanassov [2], who advanced on the fuzzy set concept by outlining the non-membership degree. Treating unknown information was erroneous. To begin with such a context, an engaging mathematical notion Neutrosophic set ($\mathbb{N}S$) was provided by Smarandache [19], an influential structure for the generalisation of $\mathscr{FS}$ and $I\ \mathscr{FS}$. Murugadas et al. [11] used novel modal and composition operators to improve the decomposition notion in Neutrosophic fuzzy matrices. Modelling ambiguity and uncertainty was addressed by a new theory soft set that was raised by Molodtsov [10]. Maji et al. [5] have investigated multiple features on fuzzy soft sets with respect to the theory of union, intersection, complement, etc. In addition, Maji et al. [6, 7] generated the soft sets to the $I\ \mathscr{FS}$ and $\mathbb{N}S$. Matrix theory is an essential idea in mathematics theory with many applications in the fields of engineering and science. Matrix theory is an indispensable idea in mathematics theory with many applications in the fields of science and engineering. Thomason [23] pioneered the notion of fuzzy matrices for illustrating fuzzy relations based on fuzzy sets utilising max-min composition to investigate the convergence powers of fuzzy matrices. Kim and Roush [3] proposed a systematic metaphor of fuzzy matrices to Boolean matrices. A matrix with max-min composition was presented by Ragab and Emam [12]. The theory of a soft matrix was initially brought up by Manoj Bora et al. [9] in their intuitionistic study of its relevance to medical diagnostics. Arockiarani and Sumathi [1] indicated several matrix models incorporating some of the definitions of the concept of a soft matrix in the framework of neutrosophic theory and then used their approach to tackle a decision-making problem. By examining various properties of the defined operators, Shyamal and Pal [16] defined the operators $\oplus$ and $\odot$. They also examined some results that have been observed with these operators in utilisation. With its algebraic features, Sriram and Boobalan [21] extend these operations to intuitionistic fuzzy matrices. Sophia and Jayapriya [20] used the operators $\oplus$, $\circledast$, and $\ominus$ to explain multiple varieties of neutrosophic fuzzy matrices and their traits. On fuzzy matrices, the algebraic characteristics of the Hamacher sum and Hamacher product operators have been identified and examined by Silambarasan and Sriram [17]. generated these operations on intuitionistic fuzzy matrices as well. Hamacher operators have been utilised for constructing scalar multiplication and exponentiation operators on interval-valued intuitionistic fuzzy matrices [18]. Its algebraic characteristics are also explored to some extent. By examining certain characteristics of the intutionistic fuzzy set, Wang and Liu [25] determined the operators of Einstein. Some of its characteristics were defined and explored by Selvarajan et al. [13, 14, 15]. Einstein operators are used in intuitionistic fuzzy matrices by some of its equalities. Lalitha [4] used the implication operator to develop Einstein's operation in an intuitionistic fuzzy matrix, and several aspects of the intutionistic fuzzy implication were exhibited. In this work, several properties of the Einstein operators are demonstrated in fuzzy neutrosophic soft matrices, especially those involving the scalar multiplication properties of fuzzy neutrosophic soft matrices with Einstein operators.

2. Preliminaries

In this section some basic definition of fuzzy neutrosophic soft matrix needed for the study of the paper are given.

Definition 2.1: [11] Let $\breve{U}$ be a neutrosophic set on the universal set $\check{X}$. It is defined to be as $\breve{U}= \langle Tu_{gh}, Iu_{gh}, Fu_{gh}\rangle$ such that $Tu_{gh}, Iu_{gh}, Fu_{gh}$: $\check{X} \to [0,1]$ where $0 \leq Tu_{gh} + Iu_{gh} + Fu_{gh} \leq 3$, denotes degree of truth, indeterminacy, and falsity respectively.

Definition 2.2: [8] Let $\check{X}$ be the universe of discourse and the $\check{S}$ be the set of parameters. Let $\check{P}\left(\check{X}\right)$ indicates the collections of all neutrosophic subsets of $\check{X}$. Let $\breve{D} \subseteq \check{S}$. Then the pair $\left(\check{F}_{\breve{D}}, \check{S}\right)$ is known to be fuzzy neutrosophic soft set over $\check{X}$ where the $\check{F}_{\breve{D}}$: $\check{S} \to \check{P}\left(\check{X}\right)$.

Definition 2.3: [1] Let $\check{X}$ be the universal set and let the set of parameters be $\check{S}$**.** Let $\breve{D} \subseteq \check{S}$. Then the pair $\left(\check{F}_{\breve{D}}, \check{S}\right)$ be a fuzzy neutrosophic soft set over $\check{X}$. Then the subset $\check{X} \times \check{S}$ is defined as $\check{R}_{\breve{D}} = \left\{(\check{x}, s);\ s \in \breve{D},\ \check{x} \in \check{f}_{\breve{D}}(\mathrm{s})\right\}$ which is relation form of $\left(\check{f}_{\breve{D}}, \check{S}\right)$. The truth, indeterminacy, falsity functions are written as $T\check{R}_{\breve{D}}$, $I\check{R}_{\breve{D}}$, $F\check{R}_{\breve{D}}$: $\check{X} \times \check{S} \to [0,1]$, $T\check{R}_{\breve{D}}(\check{x}, s)$, $I\check{R}_{\breve{D}}(\check{x}, s)$, $F\check{R}_{\breve{D}}(\check{x}, s) \in [0,1]$ where $\check{x} \in \check{X}$ for every $s \in \check{S}$.

Suppose $\langle T_{gh}, I_{gh}, F_{gh}\rangle = T_{gh}\left(\check{x}_g, s_h\right)$, $I_{gh}\left(\check{x}_g, s_h\right)$, $F_{gh}\left(\check{x}_g, s_h\right)$ which could be defined in matrix form as follows

$$\left[\langle T_{gh}, I_{gh}, F_{gh}\rangle\right]_{g\times h} = \begin{bmatrix} \langle T_{11}, I_{11}, F_{11}\rangle & \langle T_{12}, I_{12}, F_{12}\rangle & \dots & \langle T_{1h}, I_{1h}, F_{1h}\rangle \\ \langle T_{21}, I_{21}, F_{21}\rangle & \langle T_{22}, I_{22}, F_{22}\rangle & \dots & \langle T_{2h}, I_{2h}, F_{2h}\rangle \\ \vdots & \vdots & \dots & \vdots \\ \langle T_{g1}, I_{g1}, F_{g1}\rangle & \langle T_{g2}, I_{g2}, F_{g2}\rangle & \dots & \langle T_{gh}, I_{gh}, F_{gh}\rangle \end{bmatrix}$$

the above defined matrix is known to be a fuzzy neutrosophic soft matrix $\left(\check{f}_{\breve{D}}, \check{S}\right)$ over $\check{X}$.

Definition 2.4: [1, 22] If $\breve{U} = \langle Tu_{gh}, Iu_{gh}, Fu_{gh}\rangle$, $\breve{V}= \langle Tv_{gh}, Iv_{gh}, Fv_{gh}\rangle$ are two comparable fuzzy neutrosophic soft matrices with the same order. Then there are two cases

Case(i) If $\breve{U} \leq \breve{V}$, then $Tu_{gh} \leq Tv_{gh}$, $Iu_{gh} \leq Iv_{gh}$, $Fu_{gh} \geq Fv_{gh}$

Case(ii) If $\breve{U} \geq \breve{V}$, then $Tu_{gh} \geq Tv_{gh}$, $Iu_{gh} \geq Iv_{gh}$, $Fu_{gh} \leq Fv_{gh}$

Definition 2.5: [1, 22] The fuzzy neutrosophic soft matrix $\breve{J}= \langle 1,1,0\rangle$ whose all entries are known as universal matrix and fuzzy neutrosophic soft matrix $\breve{O}= \langle 0,0,1\rangle$ whose all entries are known as zero matrix. Then the identity matrix $\breve{I}$ is defined by $\langle T\mu_{gh}, I\mu_{gh}, F\mu_{gh}\rangle$ where

$$\langle T\mu_{gh}, I\mu_{gh}, F\mu_{gh}\rangle = \begin{cases} \langle 1,1,0\rangle, & g = h \\ \langle 0,0,1\rangle, & g \neq h \end{cases}$$

3. Einstein Operator on Fuzzy Neutrosophic Soft Matrix of Type A

In this section, some results about the defined einstine operators are discussed and proved.

Definition 3.1: Let $\breve{U}= \langle Tu_{gh}, Iu_{gh}, Fu_{gh}\rangle$ and $\breve{V}= \langle Tv_{gh}, Iv_{gh}, Fv_{gh}\rangle \in \mathbb{N}_{a\times s}$ then the component wise einstein sum and component wise einstein multiplication is defined as

(i) $$\breve{U} \oplus_{\mathscr{E}} \breve{V} = \left[\frac{Tu_{gh}+ Tv_{gh}}{1+Tu_{gh}.Tv_{gh}}, \frac{Iu_{gh}+ Iv_{gh}}{1+Iu_{gh}.Iv_{gh}}, \frac{Fu_{gh}.Fv_{gh}}{1+(1-Fu_{gh})(1-Fv_{gh})}\right] \tag{3.1}$$

(ii) $$\breve{U} \odot_{\mathscr{E}} \breve{V} = \left[\frac{Tu_{gh}.Tv_{gh}}{1+(1-Tu_{gh})(1-Tv_{gh})}, \frac{Iu_{gh}.Iv_{gh}}{1+(1-Iu_{gh})(1-Iv_{gh})}, \frac{Fu_{gh}+ Fv_{gh}}{1+Fu_{gh}.Fv_{gh}}\right] \tag{3.2}$$

Property 3.1

Let $\breve{U}$, $\breve{V}$ and $\breve{W}$ be any three comparable fuzzy neutrosophic soft matrices with the same order, then the following properties hold hood.

(i) $\breve{U} \oplus_{\mathscr{E}} \breve{V} = \breve{V} \oplus_{\mathscr{E}} \breve{U}$

(ii) $\breve{U} \odot_{\mathscr{E}} \breve{V} = \breve{V} \odot_{\mathscr{E}} \breve{U}$

(iii) $(\breve{U} \oplus_{\mathscr{E}} \breve{V}) \oplus_{\mathscr{E}} \breve{W} = \breve{U} \oplus_{\mathscr{E}} (\breve{V} \oplus_{\mathscr{E}} \breve{W})$

(iv) $(\breve{U} \odot_{\mathscr{E}} \breve{V}) \odot_{\mathscr{E}} \breve{W} = \breve{U} \odot_{\mathscr{E}} (\breve{V} \odot_{\mathscr{E}} \breve{W})$

Property 3.2

For any fuzzy neutrosophic soft matrix $\breve{U}$, then the following properties hold hood.

(i) $\breve{U} \oplus_{\mathscr{E}} \breve{O} = \breve{U} = \breve{O} \oplus_{\mathscr{E}} \breve{U}$

(ii) $\breve{U} \odot_{\mathscr{E}} \breve{J} = \breve{U} = \breve{O} \odot_{\mathscr{E}} \breve{J}$

(iii) $\breve{U} \odot_{\mathscr{E}} \breve{O} = \breve{O}$

(iv) $\breve{U} \oplus_{\mathscr{E}} \breve{J} = \breve{J}$

Therefore, $(\mathbb{N}_{mn}, \oplus_{\mathscr{E}})$ and $(\mathbb{N}_{mn}, \odot_{\mathscr{E}})$ forms a commutative monoid.

The operators do not obey the De Morgan's law over transpose.

Property 3.3

If $\breve{U}$ and $\breve{V}$ are two comparable fuzzy neutrosophic soft matrix with same order, then the following properties hold hood

(i) $\left(\breve{U} \oplus_{\mathscr{E}} \breve{V}\right)^{\breve{T}} = \breve{U}^{\breve{T}} \oplus_{\mathscr{E}} \breve{V}^{\breve{T}}$

(ii) $\left(\breve{U} \odot_{\mathscr{E}} \breve{V}\right)^{\breve{T}} = \breve{U}^{\breve{T}} \odot_{\mathscr{E}} \breve{V}^{\breve{T}}$

Where, $\breve{U}^{\breve{T}}$, $\breve{V}^{\breve{T}}$ is the transpose of $\breve{U}$ and $\breve{V}$.

Property 3.4

For any fuzzy neutrosophic soft matrix $\breve{U}$, then the following properties holds.

(i) $\breve{U} \oplus_{\mathscr{E}} \breve{U} \geq \breve{U}$

(ii) $\breve{U} \odot_{\mathscr{E}} \breve{U} \leq \breve{U}$

Proof:

(i) $\breve{U} \oplus_{\mathscr{E}} \breve{U} \geq \breve{U}$

Then gh^th^ element of $\breve{U} \oplus_{\mathscr{E}} \breve{U}$ is

$$= \left[\frac{Tu_{gh} + Tu_{gh}}{1 + \left(Tu_{gh}\right)^2}, \frac{Iu_{gh} + Iv_{gh}}{1 + Iu_{gh}.Iv_{gh}}, \frac{\left(Fu_{gh}\right)^2}{1 + (1 - Fu_{gh})^2}\right]$$

Since $2 \geq \left(1 + Tu_{gh}{}^2\right)$

$$2Tu_{gh} \geq Tu_{gh}\left(1 + Tu_{gh}{}^2\right)$$

$\frac{2Tu_{gh}}{\left(1 + Tu_{gh}{}^2\right)} \geq Tu_{gh}$ for every g and h

$$\frac{2Iu_{gh}}{\left(1 + Iu_{gh}{}^2\right)} \geq Iu_{gh}$$

Also, $\frac{\left(Fu_{gh}\right)^2}{1 + (1 - Fu_{gh})^2} \leq \left(Fu_{gh}\right)^2 \leq Fu_{gh}$ for every g and h

Hence, by Definition (2.4) Case(ii) $\breve{U} \oplus_{\mathscr{E}} \breve{U} \geq \breve{U}$.

The proof of (ii) is similar of (i).

4. Some Results of Einstein Scalar multiplication of Fuzzy Neutrosophic Soft Matrix.

In this section, we constructed Einstine scalar multiplication and investigated their algebraic properties.

Note: Using the Equations (1) and (2) the following equations are obtained for any integer

$p > 0$.

$$\breve{U} \oplus_{\mathscr{E}} \breve{U} = \left[\frac{Tu_{gh} + Tu_{gh}}{1 + Tu_{gh}.Tu_{gh}}, \frac{Iu_{gh} + Iu_{gh}}{1 + Iu_{gh}.Iu_{gh}}, \frac{Fu_{gh}{}^2}{1 + \left(1 - Fu_{gh}\right)^2}\right]$$

$2\breve{U}$ =

$$\left[\frac{\left(1+Tu_{gh}\right)^{2}-\left(1-Tu_{gh}\right)^{2}}{\left(1+Tu_{gh}\right)^{2}+\left(1-Tu_{gh}\right)^{2}},\frac{\left(1+Iu_{gh}\right)^{2}-\left(1-Iu_{gh}\right)^{2}}{\left(1+Iu_{gh}\right)^{2}+\left(1-Iu_{gh}\right)^{2}},\frac{2Fu_{gh}{}^{2}}{\left(2-Fu_{gh}\right)^{2}+Fu_{gh}{}^{2}}\right]$$

Similarly,

$3\breve{U}$ =

$$\left[\frac{\left(1+Tu_{gh}\right)^{3}-\left(1-Tu_{gh}\right)^{3}}{\left(1+Tu_{gh}\right)^{3}+\left(1-Tu_{gh}\right)^{3}},\frac{\left(1+Iu_{gh}\right)^{3}-\left(1-Iu_{gh}\right)^{3}}{\left(1+Iu_{gh}\right)^{3}+\left(1-Iu_{gh}\right)^{3}},\frac{2Fu_{gh}{}^{3}}{\left(2-Fu_{gh}\right)^{3}+Fu_{gh}{}^{3}}\right]$$

In general, the Einstein scalar multiplication is defined by

$p\breve{U}$ =

$$\left[\frac{\left(1+Tu_{gh}\right)^{p}-\left(1-Tu_{gh}\right)^{p}}{\left(1+Tu_{gh}\right)^{p}+\left(1-Tu_{gh}\right)^{p}},\frac{\left(1+Iu_{gh}\right)^{p}-\left(1-Iu_{gh}\right)^{p}}{\left(1+Iu_{gh}\right)^{p}+\left(1-Iu_{gh}\right)^{p}},\frac{2Fu_{gh}{}^{p}}{\left(2-Fu_{gh}\right)^{p}+Fu_{gh}{}^{p}}\right]$$

Property 4.1:

If $\breve{U}$ and $\breve{V}$ are two comparable fuzzy neutrosophic soft matrix of same order and p_1, p_2 be any positive integers where p > 0, then the following properties hold hood.

(i) $p\left(\breve{U} \oplus_{\mathscr{E}} \breve{V}\right) = p\breve{U} \oplus_{\mathscr{E}} p\breve{V}$

(ii) $\left(p_1\breve{U} \oplus_{\mathscr{E}} p_2\breve{U}\right) = (p_1 \oplus_{\mathscr{E}} p_2)\breve{U}$

(iii) $(p_1p_2)\breve{U} = p_1\left(p_2\breve{U}\right)$

Proof:

(i) $$\left(\breve{U} \oplus_{\mathscr{E}} \breve{V}\right) = \left[\frac{Tu_{gh}+Tv_{gh}}{1+Tu_{gh}.Tv_{gh}},\frac{Iu_{gh}+Iv_{gh}}{1+Iu_{gh}.Iv_{gh}},\frac{Fu_{gh}.Fv_{gh}}{1+(1-Fu_{gh})(1-Fv_{gh})}\right]$$

$$=\begin{bmatrix}\dfrac{\left(1+Tu_{gh}\right)\left(1+Tv_{gh}\right)-\left(1-Tu_{gh}\right)\left(1-Tv_{gh}\right)}{\left(1+Tu_{gh}\right)\left(1+Tv_{gh}\right)+\left(1-Tu_{gh}\right)\left(1-Tv_{gh}\right)}, \\ \dfrac{\left(1+Iu_{gh}\right)\left(1+Iv_{gh}\right)-\left(1-Iu_{gh}\right)\left(1-Iv_{gh}\right)}{\left(1+Iu_{gh}\right)\left(1+Iv_{gh}\right)+\left(1-Iu_{gh}\right)\left(1-Iv_{gh}\right)}, \\ \dfrac{2\,Fu_{gh}Fv_{gh}}{\left(2-Fu_{gh}\right)\left(2-Fv_{gh}\right)+Fu_{gh}\,Fv_{gh}}\end{bmatrix} \qquad (4.1)$$

From Equation (3) where,

$\left(1+Tu_{gh}\right)\left(1+Tv_{gh}\right) = T\Gamma_{gh}$, $\left(1-Tu_{gh}\right)\left(1-Tv_{gh}\right) = T\rho_{gh}$, $\left(1+Iu_{gh}\right)\left(1+Iv_{gh}\right) = I\gamma_{gh}$, $\left(1-Tu_{gh}\right)\left(1-Tv_{gh}\right) = I\lambda_{gh}$, $\left(2-Fu_{gh}\right)\left(2-Fv_{gh}\right) = F\mathfrak{r}_{gh}$, $Fu_{gh}\,Fv_{gh} = F\chi_{gh}$.

Then $\left(\breve{U} \oplus_{\mathscr{E}} \breve{V}\right) = \left[\frac{T\Gamma_{gh}-T\rho_{gh}}{T\Gamma_{gh}+T\rho_{gh}},\frac{I\gamma_{gh}-I\lambda_{gh}}{I\gamma_{gh}+I\lambda_{gh}},\frac{2F\chi_{gh}}{F\mathfrak{r}_{gh}+F\chi_{gh}}\right]$

Equation (3) follows that

$$p\left(\breve{U} \oplus_{\mathscr{E}} \breve{V}\right) = \begin{bmatrix} \dfrac{\left(1+\frac{T\Gamma_{gh}-T\rho_{gh}}{T\Gamma_{gh}+T\rho_{gh}}\right)^{p}-\left(1-\frac{T\Gamma_{gh}-T\rho_{gh}}{T\Gamma_{gh}+T\rho_{gh}}\right)^{p}}{\left(1+\frac{T\Gamma_{gh}-T\rho_{gh}}{T\Gamma_{gh}+T\rho_{gh}}\right)^{p}+\left(1-\frac{T\Gamma_{gh}-T\rho_{gh}}{T\Gamma_{gh}+T\rho_{gh}}\right)^{p}}, \\ \dfrac{\left(1+\frac{I\gamma_{gh}-I\lambda_{gh}}{I\gamma_{gh}+I\lambda_{gh}}\right)^{p}-\left(1-\frac{I\gamma_{gh}-I\lambda_{gh}}{I\gamma_{gh}+I\lambda_{gh}}\right)^{p}}{\left(1+\frac{I\gamma_{gh}-I\lambda_{gh}}{I\gamma_{gh}+I\lambda_{gh}}\right)^{p}+\left(1-\frac{I\gamma_{gh}-I\lambda_{gh}}{I\gamma_{gh}+I\lambda_{gh}}\right)^{p}}, \\ \dfrac{2-\left(\frac{2F\chi_{gh}}{F\mathfrak{r}_{gh}+F\chi_{gh}}\right)^{p}}{\left(2-\frac{2F\chi_{gh}}{F\mathfrak{r}_{gh}+F\chi_{gh}}\right)^{p}+\left(\frac{2F\chi_{gh}}{F\mathfrak{r}_{gh}+F\chi_{gh}}\right)^{p}} \end{bmatrix}$$

$$= \left[\frac{T\Gamma_{gh}{}^{p}-T\rho_{gh}{}^{p}}{T\Gamma_{gh}{}^{p}+T\rho_{gh}{}^{p}}, \frac{I\gamma_{gh}{}^{p}-I\lambda_{gh}{}^{p}}{I\gamma_{gh}{}^{p}+I\lambda_{gh}{}^{p}}, \frac{2F\chi_{gh}{}^{p}}{F\mathfrak{r}_{gh}{}^{p}+F\chi_{gh}{}^{p}}\right]$$

$$= \begin{bmatrix} \dfrac{\left(1+Tu_{gh}\right)^{p}\left(1+Tv_{gh}\right)^{p}-\left(1-Tu_{gh}\right)^{p}\left(1-Tv_{gh}\right)^{p}}{\left(1+Tu_{gh}\right)^{p}\left(1+Tv_{gh}\right)^{p}+\left(1-Tu_{gh}\right)^{p}\left(1-Tv_{gh}\right)^{p}}, \\ \dfrac{\left(1+Iu_{gh}\right)^{p}\left(1+Iv_{gh}\right)^{p}-\left(1-Iu_{gh}\right)^{p}\left(1-Iv_{gh}\right)^{p}}{\left(1+Iu_{gh}\right)^{p}\left(1+Iv_{gh}\right)^{p}+\left(1-Iu_{gh}\right)^{p}\left(1-Iv_{gh}\right)^{p}}, \\ \dfrac{2\,Fu_{gh}{}^{p}\,Fv_{gh}{}^{p}}{\left(2-Fu_{gh}\right)^{p}\left(2-Fv_{gh}\right)^{p}+Fu_{gh}{}^{p}\,Fv_{gh}{}^{p}} \end{bmatrix} \tag{4.2}$$

As,

$$p\breve{U} = \left[\frac{\left(1+Tu_{gh}\right)^{p}-\left(1-Tu_{gh}\right)^{p}}{\left(1+Tu_{gh}\right)^{p}+\left(1-Tu_{gh}\right)^{p}}, \frac{\left(1+Iu_{gh}\right)^{p}-\left(1-Iu_{gh}\right)^{p}}{\left(1+Iu_{gh}\right)^{p}+\left(1-Iu_{gh}\right)^{p}}, \frac{2Fu_{gh}{}^{p}}{\left(2-Fu_{gh}\right)^{p}+Fu_{gh}{}^{p}}\right] \text{ and}$$

$$p\breve{V} = \left[\frac{\left(1+Tv_{gh}\right)^{p}-\left(1-Tv_{gh}\right)^{p}}{\left(1+Tv_{gh}\right)^{p}+\left(1-Tv_{gh}\right)^{p}}, \frac{\left(1+Iv_{gh}\right)^{p}-\left(1-Iv_{gh}\right)^{p}}{\left(1+Iv_{gh}\right)^{p}+\left(1-Iv_{gh}\right)^{p}}, \frac{2Fv_{gh}{}^{p}}{\left(2-Fv_{gh}\right)^{p}+Fv_{gh}{}^{p}}\right]$$

where,

$s_{gh} = \left(1+Tu_{gh}\right)^{p}$, $t_{gh} = \left(1-Tu_{gh}\right)^{p}$, $q_{gh} = \left(1+Iu_{\mathrm{gh}}\right)^{\mathrm{p}}$, $l_{gh} = \left(1-Iu_{\mathrm{gh}}\right)^{\mathrm{p}}$, $d_{gh} = \left(2-Fu_{gh}\right)^{p}$, $e_{gh} = Fu_{gh}{}^{p}$.
$c_{gh} = \left(1+Tv_{gh}\right)^{p}$, $a_{gh} = \left(1-Tv_{gh}\right)^{p}$, $f_{gh} = \left(1+Iv_{gh}\right)^{\mathrm{p}}$, $k_{gh} = \left(1-Iv_{\mathrm{gh}}\right)^{\mathrm{p}}$, $m_{gh} = \left(2-Fv_{gh}\right)^{p}$, $n_{gh} = Fv_{gh}{}^{p}$.

By the Definition (3.1) (i) Einstein sum

$$P\breve{U} \oplus_{\mathscr{E}} p\breve{V} = \left[\frac{s_{gh}{}^{p}-t_{gh}{}^{p}}{s_{gh}{}^{p}+t_{gh}{}^{p}}, \frac{q_{gh}{}^{p}-l_{gh}{}^{p}}{q_{gh}{}^{p}+l_{gh}{}^{p}}, \frac{2e_{gh}{}^{p}}{d_{gh}{}^{p}+e_{gh}{}^{p}}\right]$$

$$\oplus_{\mathscr{E}} \left[\frac{c_{gh}{}^{p}-a_{gh}{}^{p}}{c_{gh}{}^{p}+a_{gh}{}^{p}}, \frac{f_{gh}{}^{p}-k_{gh}{}^{p}}{f_{gh}{}^{p}+k_{gh}{}^{p}}, \frac{2n_{gh}{}^{p}}{m_{gh}{}^{p}+n_{gh}{}^{p}}\right]$$

$$= \left[\begin{array}{c} \dfrac{\left(\dfrac{{s_{gh}}^{p}-{t_{gh}}^{p}}{{s_{gh}}^{p}+{t_{gh}}^{p}}\right)+\left(\dfrac{{c_{gh}}^{p}-{a_{gh}}^{p}}{{c_{gh}}^{p}+{a_{gh}}^{p}}\right)}{1+\left(\dfrac{{s_{gh}}^{p}-{t_{gh}}^{p}}{{s_{gh}}^{p}+{t_{gh}}^{p}}\right)\left(\dfrac{{c_{gh}}^{p}-{a_{gh}}^{p}}{{c_{gh}}^{p}+{a_{gh}}^{p}}\right)}, \\ \dfrac{\left(\dfrac{{q_{gh}}^{p}-{l_{gh}}^{p}}{{q_{gh}}^{p}+{l_{gh}}^{p}}\right)+\left(\dfrac{{f_{gh}}^{p}-{k_{gh}}^{p}}{{f_{gh}}^{p}+{k_{gh}}^{p}}\right)}{1+\left(\dfrac{{q_{gh}}^{p}-{l_{gh}}^{p}}{{q_{gh}}^{p}+{l_{gh}}^{p}}\right)\left(\dfrac{{f_{gh}}^{p}-{k_{gh}}^{p}}{{f_{gh}}^{p}+{k_{gh}}^{p}}\right)}, \\ \dfrac{\left(\dfrac{2{e_{gh}}^{p}}{{d_{gh}}^{p}+{e_{gh}}^{p}}\right)\left(\dfrac{2{n_{gh}}^{p}}{{m_{gh}}^{p}+{n_{gh}}^{p}}\right)}{1+\left(1-\dfrac{2{e_{gh}}^{p}}{{d_{gh}}^{p}+{e_{gh}}^{p}}\right)\left(1-\dfrac{2{n_{gh}}^{p}}{{m_{gh}}^{p}+{n_{gh}}^{p}}\right)} \end{array}\right]$$

$$= \left[\frac{{s_{gh}}^{p}\,{c_{gh}}^{p}-\,{t_{gh}}^{p}\,{a_{gh}}^{p}}{{s_{gh}}^{p}\,{c_{gh}}^{p}+\,{t_{gh}}^{p}\,{a_{gh}}^{p}}, \frac{{q_{gh}}^{p}\,{f_{gh}}^{p}-{l_{gh}}^{p}{k_{gh}}^{p}}{{q_{gh}}^{p}\,{f_{gh}}^{p}+{l_{gh}}^{p}{k_{gh}}^{p}}, \frac{2\,{e_{gh}}^{p}{n_{gh}}^{p}}{{d_{gh}}^{p}{m_{gh}}^{p}+{e_{gh}}^{p}{n_{gh}}^{p}}\right]$$

$$= \left[\begin{array}{c} \dfrac{(1+Tu_{gh})^{p}\,(1+Tv_{gh})^{p}-(1-Tu_{gh})^{p}\,(1-Tv_{gh})^{p}}{(1+Tu_{gh})^{p}\,(1+Tv_{gh})^{p}+(1-Tu_{gh})^{p}\,(1-Tv_{gh})^{p}}, \\ \dfrac{(1+Iu_{gh})^{p}(1+Iv_{gh})^{p}-(1-Iu_{gh})^{p}(1-Iv_{gh})^{p}}{(1+Iu_{gh})^{p}(1+Iv_{gh})^{p}+(1-Iu_{gh})^{p}(1-Iv_{gh})^{p}}, \\ \dfrac{2\,{Fu_{gh}}^{p}\,{Fv_{gh}}^{p}}{(2-Fu_{gh})^{p}\,(2-Fv_{gh})^{p}+{Fu_{gh}}^{p}\,{Fv_{gh}}^{p}} \end{array}\right] \tag{4.3}$$

Therefore, from Equation (4.2) and (4.3) we get

$p(\breve{U} \oplus_{\mathscr{E}} \breve{V}) = p\breve{U} \oplus_{\mathscr{E}} p\breve{V}$

(ii) As,

$$p_1\breve{U} = \left[\frac{(1+Tu_{gh})^{p_1}-(1-Tu_{gh})^{p_1}}{(1+Tu_{gh})^{p_1}+(1-Tu_{gh})^{p_1}}, \frac{(1+Iu_{gh})^{p_1}-(1-Iu_{gh})^{p_1}}{(1+Iu_{gh})^{p_1}+(1-Iu_{gh})^{p_1}}, \frac{2{Fu_{gh}}^{p_1}}{(2-Fu_{gh})^{p_1}+{Fu_{gh}}^{p_1}}\right]$$

$$p_2\breve{U} = \left[\frac{(1+Tu_{gh})^{p_2}-(1-Tu_{gh})^{p_2}}{(1+Tu_{gh})^{p_2}+(1-Tu_{gh})^{p_2}}, \frac{(1+Iu_{gh})^{p_2}-(1-Iu_{gh})^{p_2}}{(1+Iu_{gh})^{p_2}+(1-Iu_{gh})^{p_2}}, \frac{2{Fu_{gh}}^{p_2}}{(2-Fu_{gh})^{p_2}+{Fu_{gh}}^{p_2}}\right]$$

Where,

$a_{gh}=(1+\,Tu_{gh})^{p_1}$, $b_{gh}=(1-\,Tu_{gh})^{p_1}$, $t_{gh}=(1+\,Iu_{gh})^{p_1}$, $\delta_{gh}=(1-\,Iu_{gh})^{p_1}$, $\gamma_{gh}=(2-\,Fu_{gh})^{p_1}$, $\beta_{gh}={Fu_{gh}}^{p_1}$, $m_{gh}=(1+\,Tu_{gh})^{p_2}$, $n_{gh}=(1-\,Tu_{gh})^{p_2}$, $r_{gh}=(1+\,Iu_{gh})^{p_2}$, $j_{gh}=(1-\,Iu_{gh})^{p_2}$, $w_{gh}=(2-\,Fu_{gh})^{p_2}$, $y_{gh}={Fu_{gh}}^{p_2}$.

Then, $p_1\breve{U} = \left[\dfrac{a_{gh}-b_{gh}}{a_{gh}+b_{gh}}, \dfrac{t_{gh}-\delta_{gh}}{t_{gh}+\delta_{gh}}, \dfrac{2\beta_{gh}}{\gamma_{gh}+\beta_{gh}}\right]$ and $p_2\breve{U} = \left[\dfrac{m_{gh}-n_{gh}}{m_{gh}+n_{gh}}, \dfrac{r_{gh}-j_{gh}}{r_{gh}+j_{gh}}, \dfrac{2y_{gh}}{w_{gh}+y_{gh}}\right]$

$$(p_1 \oplus_{\mathscr{E}} p_2)\breve{U} = \left[\begin{array}{c} \dfrac{(1+Tu_{gh})^{p_1+p_2}-(1-Tu_{gh})^{p_1+p_2}}{(1+Tu_{gh})^{p_1+p_2}+(1-Tu_{gh})^{p_1+p_2}}, \\ \dfrac{(1+Iu_{gh})^{p_1+p_2}-(1-Iu_{gh})^{p_1+p_2}}{(1+Iu_{gh})^{p_1+p_2}+(1-Iu_{gh})^{p_1+p_2}}, \\ \dfrac{2{Fu_{gh}}^{p_1+p_2}}{(2-Fu_{gh})^{p_1+p_2}+{Fu_{gh}}^{p_1+p_2}} \end{array}\right] \tag{4.4}$$

By the Definition (3.1) (i) Einstein sum it follows that

$$\left(p_1\breve{U} \oplus_{\mathscr{E}} p_2\breve{U}\right) = \left[\begin{array}{c} \dfrac{\left(\dfrac{a_{gh}-b_{gh}}{a_{gh}+b_{gh}}\right)+\left(\dfrac{m_{gh}-n_{gh}}{m_{gh}+n_{gh}}\right)}{1+\left(\dfrac{a_{gh}-b_{gh}}{a_{gh}+b_{gh}}\right)\left(\dfrac{m_{gh}-n_{gh}}{m_{gh}+n_{gh}}\right)}, \\ \dfrac{\left(\dfrac{t_{gh}-\delta_{gh}}{t_{gh}+\delta_{gh}}\right)+\left(\dfrac{r_{gh}-j_{gh}}{r_{gh}+j_{gh}}\right)}{1+\left(\dfrac{t_{gh}-\delta_{gh}}{t_{gh}+\delta_{gh}}\right)\left(\dfrac{r_{gh}-j_{gh}}{r_{gh}+j_{gh}}\right)}, \\ \dfrac{\left(\dfrac{2\beta_{gh}}{\gamma_{gh}+\beta_{gh}}\right)\left(\dfrac{2y_{gh}}{w_{gh}+y_{gh}}\right)}{1+\left(1-\dfrac{2\beta_{gh}}{\gamma_{gh}+\beta_{gh}}\right)\left(1-\dfrac{2y_{gh}}{w_{gh}+y_{gh}}\right)} \end{array}\right]$$

$$= \left[\frac{a_{gh}\,m_{gh}-b_{gh}n_{gh}}{a_{gh}\,m_{gh}+b_{gh}n_{gh}}, \frac{t_{gh}\,r_{gh}-\delta_{gh}\,j_{gh}}{t_{gh}\,r_{gh}+\delta_{gh}\,j_{gh}}, \frac{2\beta_{gh}\;y_{gh}}{\gamma_{gh}w_{gh}+\beta_{gh}y_{gh}}\right]$$

$$= \left[\begin{array}{c} \dfrac{\left(1+Tu_{gh}\right)^{p_1+p_2}-\left(1-Tu_{gh}\right)^{p_1+p_2}}{\left(1+Tu_{gh}\right)^{p_1+p_2}+\left(1-Tu_{gh}\right)^{p_1+p_2}}, \\ \dfrac{\left(1+Iu_{gh}\right)^{p_1+p_2}-\left(1-Iu_{gh}\right)^{p_1+p_2}}{\left(1+Iu_{gh}\right)^{p_1+p_2}+\left(1-Iu_{gh}\right)^{p_1+p_2}}, \\ \dfrac{2Fu_{gh}{}^{p_1+p_2}}{\left(2-Fu_{gh}\right)^{p_1+p_2}+Fu_{gh}{}^{p_1+p_2}} \end{array}\right] \tag{4.5}$$

Therefore, from Equation (4.4) and (4.5) we get

$\left(p_1\breve{U} \oplus_{\mathscr{E}} p_2\breve{U}\right) = (p_1 \oplus_{\mathscr{E}} p_2)\breve{U}$.

(iii) To prove $(p_1p_2)\breve{U} = p_1\left(p_2\breve{U}\right)$

As, $p_2\breve{U} = \left[\dfrac{\left(1+Tu_{gh}\right)^{p_2}-\left(1-Tu_{gh}\right)^{p_2}}{\left(1+Tu_{gh}\right)^{p_2}+\left(1-Tu_{gh}\right)^{p_2}}, \dfrac{\left(1+Iu_{gh}\right)^{p_2}-\left(1-Iu_{gh}\right)^{p_2}}{\left(1+Iu_{gh}\right)^{p_2}+\left(1-Iu_{gh}\right)^{p_2}}, \dfrac{2Fu_{gh}{}^{p_2}}{\left(2-Fu_{gh}\right)^{p_2}+Fu_{gh}{}^{p_2}}\right]$

Let $\mathrm{p}_{gh} = \left(1+Tu_{gh}\right)^{p_2}$, $x_{gh} = \left(1-Tu_{gh}\right)^{p_2}$, $z_{gh} = \left(1+Iu_{gh}\right)^{p_2}$, $g_{gh} = \left(1-Iu_{gh}\right)^{p_2}$, $\eta_{gh} = \left(2-Fu_{gh}\right)^{p_2}$, $\zeta_{gh} = Fu_{gh}{}^{p_2}$

Then $p_2\breve{U} = \left[\dfrac{\mathrm{p}_{gh}-x_{gh}}{\mathrm{p}_{gh}+x_{gh}}, \dfrac{z_{gh}-g_{gh}}{z_{gh}+g_{gh}}, \dfrac{2\zeta_{gh}}{\eta_{gh}+\zeta_{gh}}\right]$

$$p_1\left(p_2\breve{U}\right) = \left[\frac{\left(1+\frac{\mathrm{p}_{gh}-x_{gh}}{\mathrm{p}_{gh}+x_{gh}}\right)^{p_1}-\left(1-\frac{\mathrm{p}_{gh}-x_{gh}}{\mathrm{p}_{gh}+x_{gh}}\right)^{p_1}}{\left(1+\frac{\mathrm{p}_{gh}-x_{gh}}{\mathrm{p}_{gh}+x_{gh}}\right)^{p_1}+\left(1-\frac{\mathrm{p}_{gh}-x_{gh}}{\mathrm{p}_{gh}+x_{gh}}\right)^{p_1}}, \frac{\left(1+\frac{z_{gh}-g_{gh}}{z_{gh}+g_{gh}}\right)^{p_1}-\left(1-\frac{z_{gh}-g_{gh}}{z_{gh}+g_{gh}}\right)^{p_1}}{\left(1+\frac{z_{gh}-g_{gh}}{z_{gh}+g_{gh}}\right)^{p_1}+\left(1-\frac{z_{gh}-g_{gh}}{z_{gh}+g_{gh}}\right)^{p_1}}, \frac{2\left(\frac{2\zeta_{gh}}{\eta_{gh}+\zeta_{gh}}\right)^{p_1}}{\left(2-\frac{2\zeta_{gh}}{\eta_{gh}+\zeta_{gh}}\right)^{p_1}+\left(\frac{2\zeta_{gh}}{\eta_{gh}+\zeta_{gh}}\right)^{p_1}}\right]$$

$$= \left[\frac{\mathrm{p}_{gh}{}^{p_1}-x_{gh}{}^{p_1}}{\mathrm{p}_{gh}{}^{p_1}+x_{gh}{}^{p_1}}, \frac{z_{gh}{}^{p_1}-g_{gh}{}^{p_1}}{z_{gh}{}^{p_1}+g_{gh}{}^{p_1}}, \frac{2\zeta_{gh}{}^{p_1}}{\eta_{gh}{}^{p_1}+\zeta_{gh}{}^{p_1}}\right]$$

$$= \left[\frac{\left(1+Tu_{gh}\right)^{p_1p_2}-\left(1-Tu_{gh}\right)^{p_1p_2}}{\left(1+Tu_{gh}\right)^{p_1p_2}+\left(1-Tu_{gh}\right)^{p_1p_2}}, \frac{\left(1+Iu_{gh}\right)^{p_1p_2}-\left(1-Iu_{gh}\right)^{p_1p_2}}{\left(1+Iu_{gh}\right)^{p_1p_2}+\left(1-Iu_{gh}\right)^{p_1p_2}}, \frac{2Fu_{gh}{}^{p_1p_2}}{\left(2-Fu_{gh}\right)^{p_1p_2}+Fu_{gh}{}^{p_1p_2}}\right]$$

$$= (p_1p_2)\breve{U}$$

Property 4.2:

For any fuzzy neutrosophic soft matrix $\breve{U}$ and for any positive integer $p>0$, $q>0$ if $p>q$, then $p\breve{U}> q\breve{U}$.

Proof:

Let $\breve{U}=\langle Tu_{gh}, Iu_{gh}, Fu_{gh}\rangle$ be a fuzzy neutrosophic soft matrix.

Let $p\breve{U}=\langle Ta_{gh}, Ia_{gh}, Fa_{gh}\rangle$ and $q\breve{U}=\langle Tb_{gh}, Ib_{gh}, Fb_{gh}\rangle$.

Let $\check{f}(\breve{x}) = \frac{(1+u)^{\breve{x}}-(1-u)^{\breve{x}}}{(1+u)^{\breve{x}}+(1-u)^{\breve{x}}}$, $\breve{g}(\breve{x}) = \frac{2v^{\breve{y}}}{(2-v)^{\breve{y}}+(v)^{\breve{y}}}$

Then, $\check{f}'(\breve{x}) = \frac{2\,(1+u)^{\breve{x}}\ (1-u)^{\breve{x}}\ \left(ln\,(1+u)^{\breve{x}}-(1-u)^{\breve{x}}\right)}{\left[(1+u)^{\breve{x}}+(1-u)^{\breve{x}}\right]^{2}}$, which is >0, u$\in[0,1]$, $\breve{x}>0$

$\breve{g}'(\breve{x})= \frac{2\,v^{\breve{y}}\ (2-v)^{\breve{y}}\ ln\frac{\breve{y}}{2-\breve{y}}}{\left[(2-v)^{\breve{y}}+v^{\breve{y}}\right]^{2}} < 0$, $v\in[0,1]$, $\breve{y}>0$

Which shows $\check{f}(\breve{x})$ and $\breve{g}(\breve{x})$ are the increasing and decreasing functions respectively.

So, if p > q then $\check{f}(p) > \check{f}(q)$ and $\breve{g}(p) > \breve{g}(q)$

That is $Ta_{gh}>Tb_{gh}$, $Ia_{gh} > Ia_{gh}$, $Fa_{gh} < Fa_{gh}$

Therefore, $p\breve{U}> q\breve{U}$.

Property 4.3:

Let $\breve{U}=\langle Ta_{gh}, Ia_{gh}, Fa_{gh}\rangle$, $\breve{V}=\langle Tb_{gh}, Ib_{gh}, Fb_{gh}\rangle$, $\breve{W}=\langle Tc_{gh}, Ic_{gh}, Fc_{gh}\rangle$, and

$\breve{G}=\langle Td_{gh}, Id_{gh}, Fd_{gh}\rangle$ be a comparable fuzzy neutrosophic soft matrices of same order such that $\breve{U}\geq\breve{W}$ and $\breve{V}\geq\breve{G}$, then the following properties hold.

(i) $\breve{U}\oplus_{\mathscr{E}}\breve{W} \geq \breve{V}\oplus_{\mathscr{E}}\breve{G}$

(ii) $\breve{U}\odot_{\mathscr{E}}\breve{W} \geq \breve{V}\odot_{\mathscr{E}}\breve{G}$

Proof:

Let $\breve{U}\geq\breve{W}$ and $\breve{V}\geq\breve{G}$

That is $Ta_{gh}\geq Tc_{gh}$, $Ia_{gh}\geq Ic_{gh}$, $Fa_{gh}\leq Fc_{gh}$ and $Tb_{gh}\geq Td_{gh}$, $Ib_{gh}\geq Id_{gh}$, $Fb_{gh}\leq Fd_{gh}$.

Then, $\left(Ta_{gh}-Tc_{gh}\right)\left(1-Tb_{gh}Td_{gh}\right)+\left(Tb_{gh}-Td_{gh}\right)\left(1-Ta_{gh}Tc_{gh}\right)\geq 0$

(i.e.) $\frac{Ta_{gh}+Tb_{gh}}{1+Tb_{gh}Td_{gh}} \geq \frac{Tc_{gh}+Td_{gh}}{1+Tc_{gh}Td_{gh}}$,

$\left(Ia_{gh}-Ic_{gh}\right)\left(1-Ib_{gh}Id_{gh}\right)+\left(Ib_{gh}-Id_{gh}\right)\left(1-Ia_{gh}Ic_{gh}\right)\geq 0$

(i.e.) $\frac{Ia_{gh}+Ib_{gh}}{1+Ib_{gh}Id_{gh}} \geq \frac{Ic_{gh}+Id_{gh}}{1+Ic_{gh}Id_{gh}}$, and

$$Fa_{gh}\,Fb_{gh}\left(1-Fc_{gh}\right)\leq Fc_{gh}\,Fd_{gh}\left(1-Fa_{gh}\right),\;Fa_{gh}\,Fb_{gh}\left(1-Fd_{gh}\right)\leq Fc_{gh}\,Fd_{gh}\left(1-Fb_{gh}\right)$$

$$\Leftrightarrow Fa_{gh}\,Fb_{gh}\left(1-Fc_{gh}\right)+Fa_{gh}\,Fb_{gh}\left(1-Fd_{gh}\right)\leq Fc_{gh}\,Fd_{gh}\left(1-Fa_{gh}\right)+Fc_{gh}\,Fd_{gh}\left(1-Fb_{gh}\right)$$

$$\Leftrightarrow Fa_{gh}\,Fb_{gh}\,(1-Fc_{gh})+Fa_{gh}\,Fb_{gh}\,(1-Fd_{gh})+Fa_{gh}Fb_{gh}Fc_{gh}Fd_{gh}\leq Fc_{gh}\,Fd_{gh}\,(1-Fa_{gh})+Fc_{gh}\,Fd_{gh}\,(1-Fb_{gh})+Fa_{gh}Fb_{gh}Fc_{gh}Fd_{gh}$$

$$\Leftrightarrow Fa_{gh}\,Fb_{gh}\left(1+(1-Fc_{gh})\,(1-Fd_{gh})\right)\leq Fc_{gh}\,Fd_{gh}\left(1+(1-Fa_{gh})\,(1-Fb_{gh})\right)$$

$$\Leftrightarrow \frac{Fa_{gh}\,Fb_{gh}}{\left(1+(1-Fa_{gh})\,(1-Fb_{gh})\right)}\leq\frac{Fc_{gh}\,Fd_{gh}}{\left(1+(1-Fc_{gh})\,(1-Fd_{gh})\right)}$$

Therefore, $\frac{Ta_{gh}+Tb_{gh}}{1+Tb_{gh}Td_{gh}}\geq\frac{Tc_{gh}+Td_{gh}}{1+Tc_{gh}Td_{gh}},\ \frac{Ia_{gh}+Ib_{gh}}{1+Ib_{gh}Id_{gh}}\geq\frac{Ic_{gh}+Id_{gh}}{1+Ic_{gh}Id_{gh}},$

$$\frac{Fa_{gh}\,Fb_{gh}}{\left(1+(1-Fa_{gh})\,(1-Fb_{gh})\right)}\leq\frac{Fc_{gh}\,Fd_{gh}}{\left(1+(1-Fc_{gh})\,(1-Fd_{gh})\right)}$$

Hence $\breve{U}\oplus_{\mathscr{E}}\breve{W}\geq\check{V}\oplus_{\mathscr{E}}\breve{G}$.

The proof of (ii) is similar of (i).

5. Conclusion

In this article, einstein sum and einstein multiplication of fuzzy neutrosophic soft matrices are defined and some of the properties are proved. The set of all fuzzy neutrosophic soft matrices forms a commutative monoid using these operators. Scalar multiplication of these operators is also proved.

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