Advances in Nonlinear Variational Inequalities
ISSN: 1092-910X
Vol 28 No. 2 (2025)

Rough Ideals in Near Algebra

K. Rajani[1a], P. Narasimha Swamy[2b*], B. Harika[3c]
[1, 2] Department of Mathematics, GITAM Deemed to be University, Hyderabad, India
[3] Department of Humanities and Sciences, Malla Reddy College of Engineering and Technology, Hyderabad, India
[a] rajanireddy2u@gmail.com, [*b] swamy.pasham@gmail.com, [c] harika.bhurgula84@gmail.com
*ORCID iD: 0000-0002-6210-4767 [*]Corresponding author

Article History:

Received: *05-08-2024*

Revised: *23-09-2024*

Accepted: *04-10-2024*

Abstract:

This article proposes and explores the concepts of rough sub near algebras, rough ideals and various other rough ideals in near algebra (NA) and discusses some of their distinctive features. Also our discussion delves into the relationships between upper and lower rough ideals and the upper and lower approximations (ULAs) of their homomorphism images.

Keywords: Rough sub near algebra**;** rough ideal

Mathematics Subject Classification: 16Y30, 03E72, 03G25.

---

1. Introduction

There are several ways to tackle the issue of what to think and do when there is not adequate information. The most effective way of dealing with insufficient information is definitely L. A. Zadeh's fuzzy set theory [23]. Rough set theory (RST) emerges as a new mathematical approach for handling uncertain information, complementing established frameworks like probability theory, fuzzy set theory and evidence theory, and providing a robust instrument for data analysis and processing in the face of ambiguity and uncertainty and it was introduced by Z. Pawlak [12]. Numerous fields, including finance, voice recognition, image processing and medical data analysis, have benefited from the use of RST. A rough set is a subset of the universe that is described by the pair of sets called ULAs. One of the approaches in generalizing the notion of rough sets is integrating it with abstract algebra. Many authors have extended RST by substituting different algebraic structures for the universal set, examining the roughness properties and behaviors of these structures, and thereby expanding the scope of the theory.

Rough ideal (RI), an extended concept of an ideal in a semi group (SG) was proposed by N. Kuroki [7], and additionally he provided some characteristics associated with these ideals. V. Selven [18] introduced the notion of RIs in semi-rings, building on the Bourne relation associated with an ideal of a semi-ring. Rough prime (primary) ideals (RP(Pr)Is) and rough fuzzy prime (primary) ideals (RFP(Pr)Is) in a SG have been defined by Qi – Mei Xiao [15], who also provided certain features of these ideals. In addition to presenting several features of the ULAs in a ring, B. Davvaz [3] put forth the idea of rough sub ring with respect to an ideal of a ring, which is an extended notion of a subring in a ring. Osman Kazanci [11] suggested RP(Pr)Is and RFP(Pr)Is in a ring and he added extra features for such ideals. Neelima [10] introduced the concept of rough semi prime ideal (RSPIs), rough bi ideal and gave some properties of anti-homomorphism on these RIs. V. S. Subha [21, 22] has made significant contributions to the field of near rings and gamma near rings, exploring various RIs in these algebraic structures.

Brown [2] pioneered the idea of NA. With the possible exception of one distributive property, an algebraic system with two binary operations that admits field as a right operator domain and satisfies all of the ring's axioms is called a NA. A few variants of NAs and their structures were examined by Brown [2]. The study of NAs has potential applications in physics; quantum mechanical models have been established where the operators are just a NA. Besides to being an axiomatic question, an investigation of NAs is essential for physical reasons.

In this investigation, we replace universe set by a near algebra and propose novel notions of rough sub near algebra (RSNA), rough ideal, and related rough ideals within the near algebra and highlight their distinctive properties. We examine the interconnections between upper (lower) rough prime, semi-prime, and primary ideals and the upper (lower) approximations of their homomorphism images.

2. Preliminaries

In this section, we provide an over view of the fundamental definitions related to NA, drawing from various sources in the literature.

Definition 2.1. [17] A linear space (LS) X over a field F is called an *algebra* over the field F if multiplication is defined such that

(i) X forms a SG with respect to multiplication,

(ii) $(\eta_{11} + \eta_{12})\,\eta_{13} = (\eta_{11}\,\eta_{13}) + (\eta_{12}\,\eta_{13})$,

$\eta_{11}(\eta_{12} + \eta_{13}) = (\eta_{11}\eta_{12}) + (\eta_{11}\,\eta_{13})$ for all $\eta_{11}, \eta_{12}, \eta_{13} \in X$ and

(iii) $\kappa(\eta_{11}\eta_{12}) = (\kappa\eta_{11})\eta_{12} = \eta_{11}(\kappa\eta_{12})$ for all $\kappa \in F$ and $\eta_{11}, \eta_{12} \in X$.

Definition 2.2. [2] A LS N over a field F is called a *(right) near algebra* over the field F if multiplication is defined such that

(i) N forms a SG with respect to multiplication,

(ii) $(\eta_{11} + \eta_{12})\,\eta_{13} = (\eta_{11}\,\eta_{13}) + (\eta_{12}\,\eta_{13})$ for all $\eta_{11}, \eta_{12}, \eta_{13} \in N$ and

(iii) $\kappa\,(\eta_{11}\eta_{12}) = (\kappa\eta_{11})\eta_{12}$ for all $\kappa \in F$ and $\eta_{11}, \eta_{12} \in N$.

Throughout this paper, we focus exclusively on right near algebras, which we will simply refer to as near algebras for brevity.

Definition 2.3. [18] A non-empty subset S of a NA N over a field F is said to be a *sub near-algebra* (SNA) of N if it forms a NA over F, inheriting the operations of N.

Remark 2.4. [18] A non- empty subset S of a NA N over a field F is a *SNA* of N if and only if $\eta_{11} - \eta_{12} \in S, \kappa\eta_{11} \in S, \eta_{11}\eta_{12} \in S$ for all $\eta_{11}, \eta_{12} \in S$ and $\kappa \in F$.

Definition 2.5. [18] A non - empty subset I of a NA N is referred to as a *near algebra ideal* (NAI) of N if

(i) I is a linear subspace of the LS N,
(ii) $\xi_{11}\eta_{11} \in I$ for every $\eta_{11} \in N$, $\xi_{11} \in I$ and
(iii) $\eta_{12}(\eta_{11} + \xi_{11}) - \eta_{12}\eta_{11} \in I$ for every $\eta_{11}, \eta_{12} \in N$ and $\xi_{11} \in I$.

I is called a *right ideal* of N if it satisfies (i) and (ii).

I is called a *left ideal* of N if it satisfies (i) and (iii).

Definition 2.6. [18] Let N and N' be two NAs over a field F. A mapping $\varsigma: N \to N'$ is called a *near algebra homomorphism* if

(i) $\varsigma(\eta_{11} + \eta_{12}) = \varsigma(\eta_{11}) + \varsigma(\eta_{12})$
(ii) $\varsigma(\kappa\eta_{11}) = \kappa\varsigma(\eta_{11})$
(iii) $\varsigma(\eta_{11}\eta_{12}) = \varsigma(\eta_{11})\varsigma(\eta_{12})$ for every $\eta_{11}, \eta_{12} \in N$ and $\kappa \in F$.

A homomorphism which is one – one is called a *monomorphism*. A homomorphism which is onto is called an *epimorphsim*. A monomorphism which is onto is called an *isomorphism*.

N is a NA over a field F throughout this work and preliminary definitions on rough sets can be found in [12], [13], [3], [6].

3. Rough Ideals in Near Algebra

In this section, we define rough ideal in a near algebra and established some important results to this notion.

Definition 3.1. An *equivalence relation (ER)* θ on NA N is a reflexive, symmetric and transitive binary relation on N. If θ is an ER on N then the *equivalence class* of $\eta_{11} \in N$ is the set $\{\eta_{12} \in N | (\eta_{11}, \eta_{12}) \in \theta\}$. We write it as $[\eta_{11}]_\theta$.

Definition 3.2. Let θ be an ER on N, then θ is called a *full congruence relation (FCR)* if $(\eta_{11}, \eta_{12}) \in \theta$ implies $(\eta_{11} + \tau_{11}, \eta_{12} + \tau_{11}), (\kappa\eta_{11}, \kappa\eta_{12}), (\eta_{11}\tau_{11}, \eta_{12}\tau_{11})$ and $(\tau_{11}\eta_{11}, \tau_{11}\eta_{12}) \in \theta$ for all $\kappa \in F$ and $\tau_{11} \in N$.

Proposition 3.3. Let θ be a FCR on a NA N, then $(\eta_{11}, \eta_{12}) \in \theta$ and $(\tau_{11}, \tau_{12}) \in \theta$ implies

$(\eta_{11} + \tau_{11}, \eta_{12} + \tau_{12}) \in \theta$ and $(\eta_{11}\tau_{11}, \eta_{12}\tau_{12}) \in \theta$ for all $\eta_{11}, \eta_{12}, \tau_{11}, \tau_{12} \in N$.

Proof. It is straight forward.

Theorem 3.4. Let θ be a FCR on a NA N. If $\eta_{11}, \eta_{12} \in N$ and $\kappa \in F$ then,

(i) $[\eta_{11}]_\theta + [\eta_{12}]_\theta = [\eta_{11} + \eta_{12}]_\theta$,
(ii) $[\kappa\eta_{11}]_\theta = \kappa[\eta_{11}]_\theta$,
(iii) $\{\tau_{11}\tau_{12} | \tau_{11} \in [\eta_{11}]_\theta, \tau_{12} \in [\eta_{12}]_\theta \subseteq [\eta_{11}\eta_{12}]_\theta$.

Proof. Proofs of (i) and (ii) are straight forward.

(iii) Let $\tau = \tau_{11}\tau_{12}$ be such that $\tau_{11} \in [\eta_{11}]_\theta$, $\tau_{12} \in [\eta_{12}]_\theta$. Then $(\tau_{11}, \eta_{11}) \in \theta$ and $(\tau_{12}, \eta_{12}) \in \theta$. Since θ is a FCR, $(\tau_{11}\tau_{12}, \eta_{11}\eta_{12}) \in \theta$. This implies $\tau = \tau_{11}\tau_{12} \in [\eta_{11}\eta_{12}]_\theta$ and hence $\{\tau_{11}\tau_{12} | \tau_{11} \in [\eta_{11}]_\theta, \tau_{12} \in [\eta_{12}]_\theta\} \subseteq [\eta_{11}\eta_{12}]_\theta$ proving (iii).

Remark 3.5. Let P and Q be non empty subsets of N and PQ represent the set of all finite sums $\{p_{11}q_{11} + p_{12}q_{12} + \cdots .. + p_{1n}q_{1n} | n \in N, p_{1j} \in P, q_{1j} \in Q\}$.

Definition 3.6. A FCR θ on N is called *complete* if $[\eta_{11}\eta_{12}]_\theta = \{\tau_{11}\tau_{12} | \tau_{11} \in [\eta_{11}]_\theta, \tau_{12} \in [\eta_{12}]_\theta\}$ for all $\eta_{11}, \eta_{12} \in N$.

Definition 3.7. Let θ be a FCR on N and S, a subset of N. Then the sets

$LApr_\theta(S) = \{\eta_{11} \in N | [\eta_{11}]_\theta \subseteq S\}$ and $UApr_\theta(S) = \{\eta_{11} \in N | [\eta_{11}]_\theta \cap S \neq \emptyset\}$

are called respectively the θ *−lower and* θ *−upper apprroximations* of the set S.

Remark 3.8. For any non – empty S of N, we have $LApr_\theta(S) \subseteq S \subseteq UApr_\theta(S)$.

Definition 3.9. For any non – empty S of N , $Apr_\theta(S) = (LApr_\theta(S), UApr_\theta(S))$ is called a *rough set (RS)* with respect to θ if $LApr_\theta(S) \neq UApr_\theta(S)$.

Definition 3.10. A non-empty subset S of the NA N is called an *upper rough sub near algebra (URSNA)* of N if $UApr_\theta(S)$ is a SNA of N.

Definition 3.11. A non-empty subset S of the NA N is called a *lower rough sub near algebra (LRSNA)* of N if $LApr_\theta(S)$ is a SNA of N.

Theorem 3.12. Let θ be a FCR on N. If S is a SNA of N then $UApr_\theta(S)$ is a SNA of N.

Proof. Assume that $\xi_{11}, \xi_{12} \in UApr_\theta(S)$ and for all $\kappa \in F$. Then $[\xi_{11}]_\theta \cap S \neq \emptyset$ and $[\xi_{12}]_\theta \cap S \neq \emptyset$. Hence there exists $\tau_{11} \in [\xi_{11}]_\theta \cap S$ and $\tau_{12} \in [\xi_{12}]_\theta \cap S$. So $\tau_{11} \in [\xi_{11}]_\theta$, $\tau_{11} \in S$ and $\tau_{12} \in [\xi_{12}]_\theta, \tau_{12} \in S$. Since S is a SNA of N, $\tau_{11} - \tau_{12} \in S$, $\kappa\tau_{11} \in S$, $\tau_{11}\tau_{12} \in S$. Now, $\tau_{11} - \tau_{12} \in [\xi_{11}]_\theta - [\xi_{12}]_\theta = [\xi_{11} - \xi_{12}]_\theta$. Hence $\tau_{11} - \tau_{12} \in [\xi_{11} - \xi_{12}]_\theta \cap S$, which implies $[\xi_{11} - \xi_{12}]_\theta \cap S \neq \emptyset$ or $\xi_{11} - \xi_{12} \in UApr_\theta(S)$.

Also $\kappa\tau_{11} \in \kappa[\xi_{11}]_\theta = [\kappa\xi_{11}]_\theta$. Therefore $\kappa\tau_{11} \in [\kappa\xi_{11}]_\theta \cap S$ and hence $[\kappa\xi_{11}]_\theta \cap S \neq \emptyset$ or $\kappa\xi_{11} \in UApr_\theta(S)$.

Since θ is a FCR, $\{\tau_{11}\tau_{12} | \tau_{11} \in [\xi_{11}]_\theta, \tau_{12} \in [\xi_{12}]_\theta\} \subseteq [\xi_{11}\xi_{12}]_\theta$ and hence $\tau_{11}\tau_{12} \in [\xi_{11}\xi_{12}]_\theta$. Thus $\tau_{11}\tau_{12} \in [\xi_{11}\xi_{12}]_\theta \cap S$. Therefore $[\xi_{11}\xi_{12}]_\theta \cap S \neq \emptyset$ or $\xi_{11}\xi_{12} \in UApr_\theta(S)$. Hence $UApr_\theta(S)$ is a SNA of N.

Theorem 3.13. Let θ be a complete congruence relation (CCR) on N. If $LApr_\theta(S)$ is non-empty and S is a SNA of N, then $LApr_\theta(S)$ is a SNA of N.

Proof. Let $\xi_{11}, \xi_{12} \in LApr_\theta(S)$ and $\kappa \in F$. Then $[\xi_{11}]_\theta \subseteq S$ and $[\xi_{12}]_\theta \subseteq S$. Now,

$[\xi_{11} + \xi_{12}]_\theta = [\xi_{11}]_\theta + [\xi_{12}]_\theta \subseteq S + S = S$, this implies $\xi_{11} + \xi_{12} \in LApr_\theta(S)$.

Also, $[\kappa\xi_{11}]_\theta = \kappa[\xi_{11}]_\theta \subseteq \kappa S \subseteq S$ and hence $\kappa\xi_{11} \in LApr_\theta(S)$. Since θ is a CCR, $[\xi_{11}\xi_{12}]_\theta = \{\tau_{11}\tau_{12} | \tau_{11} \in [\xi_{11}]_\theta, \tau_{12} \in [\xi_{12}]_\theta \subseteq S$, thus $\xi_{11}\xi_{12} \in LApr_\theta(S)$. Therefore $LApr_\theta(S)$ is a SNA of N.

Definition 3.14. A non-empty subset I of a NA N is called an *upper rough ideal (URI)* if $UApr_\theta(I)$ is an ideal of N and *lower rough ideal (LRI)* if $LApr_\theta(I)$ is an ideal of N.

Example 3.15. Consider the linear space $Z_2 X Z_2$ over the field Z_2. Define multiplication on $Z_2 X Z_2$ as $(\xi_{11}, \xi_{12})(\tau_{11}, \tau_{12}) = (\xi_{11}, \xi_{12})$ for all $(\xi_{11}, \xi_{12}), (\tau_{11}, \tau_{12}) \in Z_2 X Z_2$. Then $Z_2 X Z_2$ is a NA over the field Z_2. Define θ on Z_2XZ_2 as $(\xi_{11},\xi_{12})\ \theta\ (\tau_{11},\tau_{12})$ if and only if $\xi_{11}+_2\xi_{12}=\tau_{11}+_2\tau_{12}$. Then θ is a full congruence relation on Z_2XZ_2 with the equivalence classes $C_1=\{(0,0),(1,1)\}$ and $C_2=\{(0,1),(1,0)\}$ Let $I=\{(0,0),(0,1),(1,0)\}$. Then $UApr_\theta(I)=Z_2XZ_2$ is an ideal of Z_2XZ_2 and $LApr_\theta(I)=\{(0,1),(1,0)\}$ is not an ideal of Z_2XZ_2. Hence I is an URI of Z_2XZ_2 and not a LRI of Z_2XZ_2 .

Theorem 3.16. Let θ be a FCR on N. If I is an ideal of N then $UApr_\theta(I)$ is an ideal of N.

Proof. Suppose that $\xi_{11},\xi_{12}\in UApr_\theta(I)$, $\kappa\in F$ and $\eta_{11},\eta_{12}\in N$, then $[\xi_{11}]_\theta\cap I\neq\emptyset$ and $[\xi_{12}]_\theta\cap I\neq\emptyset$. Hence there exists $\tau_{11}\in[\xi_{11}]_\theta\cap I$ and $\tau_{12}\in[\xi_{12}]_\theta\cap I$. So $\tau_{11}\in[\xi_{11}]_\theta$, $\tau_{11}\in I$ and $\tau_{12}\in[\xi_{12}]_\theta$, $\tau_{12}\in I$. Since I is an ideal of N, $\tau_{11}-\tau_{12}\in I$, $\kappa\in I$, $\tau_{11}\eta_{11}\in I$ and $\eta_{12}(\eta_{11}+\tau_{11})-\eta_{12}\eta_{11}\in I$.

Now, $\tau_{11}-\tau_{12}\in[\xi_{11}]_\theta-[\xi_{12}]_\theta=[\xi_{11}-\xi_{12}]_\theta$. Hence $\tau_{11}-\tau_{12}\in[\xi_{11}-\xi_{12}]_\theta\cap I$, which implies $[\xi_{11}-\xi_{12}]_\theta\cap I\neq\emptyset$ or $\xi_{11}-\xi_{12}\in UApr_\theta(I)$.

Also, $\kappa\tau_{11}\in\ \kappa[\xi_{11}]_\theta=[\kappa\xi_{11}]_\theta$. Therefore $\kappa\tau_{11}\in[\kappa\xi_{11}]_\theta\cap I$ and hence $[\kappa\xi_{11}]_\theta\cap I\neq\emptyset$ or

$\kappa\xi_{11}\in UApr_\theta(I)$.

Since $(\tau_{11},\xi_{11})\in\theta$, then $(\tau_{11}\eta_{11},\xi_{11}\eta_{11})\in\theta$ or $\tau_{11}\eta_{11}\in[\xi_{11}\eta_{11}]_\theta$. Hence $\tau_{11}\eta_{11}\in[\xi_{11}\eta_{11}]_\theta\cap I$ and therefore $[\xi_{11}\eta_{11}]_\theta\cap I\neq\emptyset$ or $\xi_{11}\eta_{11}\in UApr_\theta(I)$.

Now $(\tau_{11},\xi_{11})\in\theta$ implies$(\tau_{11}+\eta_{11},\xi_{11}+\eta_{11})\in\theta$ which inturn implies

$(\eta_{12}(\tau_{11}+\eta_{11}),\eta_{12}(\ \xi_{11}+\eta_{11}))\in\theta$ and hence $(\eta_{12}(\tau_{11}+\eta_{11})-\eta_{12}\eta_{11},\eta_{12}(\ \xi_{11}+\eta_{11})-\eta_{12}\eta_{11})\in\theta$. Thus $\eta_{12}(\tau_{11}+\eta_{11})-\eta_{12}\eta_{11}\in[\eta_{12}(\xi_{11}+\eta_{11})-\eta_{12}\eta_{11}]_\theta$ and hence

$[\eta_{12}(\xi_{11}+\eta_{11})-\eta_{12}\eta_{11}]_\theta\cap I\neq\emptyset$.

Therefore $\eta_{12}(\eta_{11}+\xi_{11})-\eta_{12}\eta_{11}\in UApr_\theta(I)$ and thus $UApr_\theta(I)$ is an ideal of N.

Remark 3.17. The previously stated theorem do not typically hold in the reverse direction. In example 3.15., $I=\{(0,0),(0,1),(1,0)\}$ is not an ideal of Z_2XZ_2 but $UApr_\theta(I)=Z_2XZ_2$ is an ideal of Z_2XZ_2.

Theorem 3.18. Let θ be a FCR on N and I be an ideal of N. If $LApr_\theta(I)$ is a non - empty set, then it is equal to I.

Proof. Since $LApr_\theta(I)\subseteq I$ and we now show that, $I\subseteq LApr_\theta(I)$. Let $\xi_{11}\in LApr_\theta(I)$ and $\tau_{11}\in I$. Then $[0]_\theta=[\xi_{11}-\xi_{11}]_\theta=[\xi_{11}]_\theta+[-\xi_{11}]_\theta\subseteq I+I=I$ and hence $a+[0]_\theta\subseteq\ a+I=I$. Since $\xi_{11}\in\tau_{11}+[0]_\theta$ iff $\xi_{11}-\tau_{11}\in[0]_\theta$ iff $(\xi_{11}-\tau_{11},0)\in\theta$ iff $(\xi_{11},\tau_{11})\in\theta$ iff $\xi_{11}\in[\tau_{11}]_\theta$ and hence $[\tau_{11}]_\theta\subseteq I$, which implies $\tau_{11}\in LApr_\theta(I)$. Therefore $I\subseteq LApr_\theta(I)$ and thus $LApr_\theta(I)=I$.

Definition 3.19. Let I be a subset of N and $(LApr_\theta(I),UApr_\theta(I))$, a rough set. If $LApr_\theta(I)$ and $UApr_\theta(I)$ are ideals of N, then we call $(LApr_\theta(I),UApr_\theta(I))$ a *rough ideal*.

Theorem 3.20. Let I be an ideal of N and $LApr_\theta(I)$ is a non - empty set. Then $(LApr_\theta(I),UApr_\theta(I))$ is a RI of N.

Proof. It is straight forward.

Theorem 3.21. Let I and J be ideals of N and $LApr_{\theta}(I \cap J)$is a non-empty set. Then $(LApr_{\theta}(I \cap J), UApr_{\theta}(I \cap J))$ is a RI of N.

Proof. It is straight forward

Theorem 3.22. Let N and N' be NAs over a field F and $\varsigma: N \to N'$ be an epimorphism. Let ρ be a FCR on N' and S be a subset of N. Then

(i) $\theta = \{(\eta_{11}, \eta_{12}) \in NXN' | (\varsigma(\eta_{11}), \varsigma(\eta_{12})) \in \rho\}$ is a FCR on N.
(ii) If ς is one - one and ρ is complete, then θ is complete.
(iii) $\varsigma(UApr_{\theta}(S)) = UApr_{\rho}(\varsigma(S))$.
(iv) $\varsigma(LApr_{\theta}(S)) \subseteq LApr_{\rho}(\varsigma(S))$. If ς is one - one, then $\varsigma(LApr_{\theta}(S)) = LApr_{\rho}(\varsigma(S))$.

Proof. (i) Let $(\eta_{11}, \eta_{12}) \in \theta$, $\eta \in N$ and $\kappa \in F$. Then $(\varsigma(\eta_{11}), \varsigma(\eta_{12})) \in \rho$ and $\varsigma(\eta) \in N'$. Since ρ is a FCR on N', we get $(\varsigma(\eta_{11}) + \varsigma(\eta), \varsigma(\eta_{12}) + \varsigma(\eta)) \in \rho$, $(\kappa\varsigma(\eta_{11}), \kappa\varsigma(\eta_{12})) \in \rho$, $(\varsigma(\eta_{11})\varsigma(\eta), \varsigma(\eta_{12})\varsigma(\eta)) \in \rho$ and $(\varsigma(\eta)\varsigma(\eta_{11}), \varsigma(\eta)\varsigma(\eta_{12})) \in \rho$.

Then $(\varsigma(\eta_{11} + \eta), \varsigma(\eta_{12} + \eta)) \in \rho$, $(\varsigma(\kappa\eta_{11}), \varsigma(\kappa\eta_{12})) \in \rho$, $(\varsigma(\eta_{11}\eta), \varsigma(\eta_{12}\eta)) \in \rho$ and $(\varsigma(\eta\eta_{11}), \varsigma(\eta\eta_{12})) \in \rho$. Hence $(\eta_{11} + \eta, \eta_{12} + \eta) \in \theta$, $(\kappa\eta_{11}, \kappa\eta_{12}) \in \theta$, $(\eta_{11}\eta, \eta_{12}\eta) \in \theta$ and $(\eta\eta_{11}, \eta\eta_{12}) \in \theta$.This implies θ is a FCR on N.

(ii). We have $\{\eta_{11}\eta_{12} | \eta_{11} \in [\tau_{11}]_{\theta}, \eta_{12} \in [\tau_{12}]_{\theta}\} \subseteq [\tau_{11}\tau_{12}]_{\theta}$ and we show that $[\tau_{11}\tau_{12}]_{\theta} \subseteq \{\eta_{11}\eta_{12} | \eta_{11} \in [\tau_{11}]_{\theta}, \eta_{12} \in [\tau_{12}]_{\theta}\}$ to prove θ is complete. Let $\tau_{13} \in [\tau_{11}\tau_{12}]_{\theta}$. Then $(\tau_{13}, \tau_{11}\tau_{12}) \in \theta$.

By the definition, $(\varsigma(\tau_{13}), \varsigma(\tau_{11}\tau_{12})) \in \rho$. Thus $\varsigma(\tau_{13}) \in [\varsigma(\tau_{11}\tau_{12})]_{\rho} = [\varsigma(\tau_{11})\varsigma(\tau_{12})]_{\rho}$

$= \{\varsigma(\eta_{11})\varsigma(\eta_{12}) | \varsigma(\eta_{11}) \in [\varsigma(\tau_{11})]_{\rho}, \varsigma(\eta_{12}) \in [\varsigma(\tau_{12})]_{\rho}\}$

Hence there exists $\eta_{11}, \eta_{12} \in N$ such that $\varsigma(\tau_{13}) = \varsigma(\eta_{11})\varsigma(\eta_{12}) = \varsigma(\eta_{11}\eta_{12})$ and $\varsigma(\eta_{11}) \in [\varsigma(\tau_{11})]_{\rho}$, $\varsigma(\eta_{12}) \in [\varsigma(\tau_{12})]_{\rho}$. Since ς is one one, we have $\tau_{13} = \eta_{11}\eta_{12}$ and $\eta_{11} \in [\tau_{11}]_{\theta}, \eta_{11} \in [\tau_{12}]_{\theta}$. Thus $\tau_{13} \in \{\eta_{11}\eta_{12} | \eta_{11} \in [\tau_{11}]_{\theta}, \eta_{11} \in [\tau_{12}]_{\theta}\}$ and therefore $[\tau_{11}\tau_{12}]_{\theta} \subseteq \{\eta_{11}\eta_{12} | \eta_{11} \in [\tau_{11}]_{\theta}, \eta_{12} \in [\tau_{12}]_{\theta}\}$. Hence θ is complete.

(iii). Let $\tau_{12} \in \varsigma(UApr_{\theta}(S))$. Then there exists $\tau_{11} \in UApr_{\theta}(S)$ such that $\varsigma(\tau_{11}) = \tau_{12}$.

Now $\tau_{11} \in UApr_{\theta}(S)$, implies $[\tau_{11}]_{\theta} \cap S \neq \emptyset$. Let $\eta_{11} \in [\tau_{11}]_{\theta} \cap S$. Then $(\eta_{11}, \tau_{11}) \in \theta$ and $\eta_{11} \in S$. By the definition, $(\varsigma(\eta_{11}), \varsigma(\tau_{11})) \in \rho$ and $\varsigma(\eta_{11}) \in \varsigma(S)$. This implies, $\varsigma(\eta_{11}) \in [\varsigma(\tau_{11})]_{\rho}$ and hence $\varsigma(\eta_{11}) \in [\varsigma(\tau_{11})]_{\rho} \cap \varsigma(S)$. Thus $[\varsigma(\tau_{11})]_{\rho} \cap \varsigma(S) \neq \emptyset$. Hence $\tau_{12} = \varsigma(\tau_{11}) \in UApr_{\rho}(\varsigma(S))$ and therefore $\varsigma(UApr_{\theta}(S)) \subseteq UApr_{\rho}(\varsigma(S))$.

Conversely, let $\tau_{12} \in UApr_{\rho}(\varsigma(S))$. Then $[\tau_{12}]_{\rho} \cap \varsigma(S) \neq \emptyset$. Also there exists $\tau_{11} \in N$ such that $\varsigma(\tau_{11}) = \tau_{12}$. Therefore $[\varsigma(\tau_{11})]_{\rho} \cap \varsigma(S) \neq \emptyset$. Let $\varsigma(\eta_{11}) \in [\varsigma(\tau_{11})]_{\rho} \cap \varsigma(S)$ for some $\eta_{11} \in S$. Then $(\varsigma(\eta_{11}), \varsigma(\tau_{11})) \in \rho$. This implies $(\eta_{11}, \tau_{11}) \in \theta$ and hence $\eta_{11} \in [\tau_{11}]_{\theta}$.

Thus $[\tau_{11}]_{\theta} \cap S \neq \emptyset$. Therefore $\tau_{11} \in UApr_{\theta}(S)$ and so $\tau_{12} = \varsigma(\tau_{11}) \in \varsigma(UApr_{\theta}(S))$. Thus $UApr_{\rho}(\varsigma(S)) \subseteq \varsigma(UApr_{\theta}(S))$ and $\varsigma(UApr_{\theta}(S)) = UApr_{\rho}(\varsigma(S))$.

(iv). Let $\tau_{12} \in \varsigma\big(LApr_\theta(S)\big)$. Then there exists $\tau_{11} \in LApr_\theta(S)$ such that $\varsigma(\tau_{11}) = \tau_{12}$. Now $\tau_{11} \in LApr_\theta(S)$, implies $[\tau_{11}]_\theta \subseteq S$. Let $\eta_{12} \in [\tau_{12}]_\theta$. Then there exists $\eta_{11} \in N$ such that $\varsigma(\eta_{11}) = \eta_{12}$ and $\varsigma(\eta_{11}) \in [\varsigma(\tau_{11})]_\rho$. Thus $\eta_{11} \in [\tau_{11}]_\theta \subseteq S$ and therefore $\eta_{12} = \varsigma(\eta_{11}) \in \varsigma(S)$. Thus $[\tau_{12}]_\theta \subseteq \varsigma(S)$ or $\tau_{12} \in LApr_\rho\big(\varsigma(S)\big)$. So $\varsigma\big(LApr_\theta(S)\big) \subseteq LApr_\rho\big(\varsigma(S)\big)$. Suppose ς is one to one. Let $\tau_{12} \in LApr_\rho\big(\varsigma(S)\big)$. Then there exists $\tau_{11} \in N$ such that $\varsigma(\tau_{11}) = \tau_{12}$ and $[\varsigma(\tau_{11})]_\rho \subseteq \varsigma(S)$. Let $\eta_{11} \in [\tau_{11}]_\theta$. Then $\varsigma(\eta_{11}) \in [\varsigma(\tau_{11})]_\rho \subseteq \varsigma(S)$ and hence $\eta_{11} \in S$. Thus $[\tau_{11}]_\theta \subseteq S$ implies, $\tau_{11} \in LApr_\theta(S)$. Then $\tau_{12}= \varsigma(\tau_{11}) \in \varsigma\big(LApr_\theta(S)\big)$ and therefore $LApr_\rho\big(\varsigma(S)\big) \subseteq \varsigma\big(LApr_\theta(S)\big)$. Thus $\varsigma\big(LApr_\theta(S)\big) = LApr_\rho\big(\varsigma(S)\big)$.

Theorem 3.23. Let N and N' be NAs over a field F and $\varsigma: N \to N'$ be an epimorphism. Let ρ be a FCR on N' and I be a subset of N. Let $\theta = \{(\eta_{11}, \eta_{12}) \in NXN' | (\varsigma(\eta_{11}), \varsigma(\eta_{12})) \in \rho\}$. Then $UApr_\theta(I)$ is an ideal of N if and only if $UApr_\rho\big(\varsigma(I)\big)$ is an ideal of N'.

Proof. Let $UApr_\theta(I)$ be an ideal of N. Let $\tau'_{11}, \tau'_{12} \in UApr_\rho\big(\varsigma(I)\big), \kappa \in F$ and $\xi'_{11}, \xi'_{12} \in N'$. Then $\tau_{11}', \tau_{12}' \in \varsigma(UApr_\theta(I))$. So there exist $\tau_{11}, \tau_{12} \in UApr_\theta(I)$ such that $\tau'_{11} = \varsigma(\tau_{11})$, $\tau'_{12} = \varsigma(\tau_{12})$. Since ς is onto, there exist $\xi_{11}, \xi_{12} \in N$ such that $\xi'_{11} = \varsigma(\xi_{11})$, $\xi'_{12} = \varsigma(\xi_{12})$. Since $UApr_\theta(I)$ is an ideal of N,

$\tau_{11} - \tau_{12},\ \kappa\tau_{11},\ \tau_{11}\xi_{11},\ \xi_{12}(\xi_{11} + \tau_{11}) - \xi_{12}\xi_{11} \in UApr_\theta(I)$.

Now, $\tau'_{11} - \tau'_{12} = \varsigma(\tau_{11}) - \varsigma(\tau_{12}) = \varsigma(\tau_{11} - \tau_{12}) \in \varsigma(UApr_\theta(I) = UApr_\rho\big(\varsigma(I)\big)$,

$\kappa\, \tau'_{11} = \kappa\, \varsigma(\tau_{11}) = \varsigma(\kappa\tau_{11}) \in \varsigma(UApr_\theta(I) = UApr_\rho\big(\varsigma(I)\big)$,

$\tau'_{11}\xi'_{11} = \varsigma(\tau_{11})\, \varsigma(\xi_{11}) = \varsigma(\tau_{11}\xi_{11}) \in \varsigma(UApr_\theta(I) = UApr_\rho\big(\varsigma(I)\big)$ and

$$\begin{aligned}
\xi_{12}'(\xi_{11}' + \tau_{11}') - \xi_{12}'\xi_{11}' &= \varsigma(\xi_{12})\big(\varsigma(\xi_{11}) + \varsigma(\tau_{11})\big) - \varsigma(\xi_{12})\, \varsigma(\xi_{11}) \\
&= \varsigma(\xi_{12})\big(\varsigma(\xi_{11} + \tau_{11})\big) - \varsigma(\xi_{12}\xi_{11}) \\
&= \varsigma\big(\xi_{12}(\xi_{11} + \tau_{11})\big) - \varsigma(\xi_{12}\xi_{11}) \\
&= \varsigma(\xi_{12}(\xi_{11} + \tau_{11})) - (\xi_{12}\xi_{11})) \\
&\in \varsigma(UApr_\theta(I) = UApr_\rho\big(\varsigma(I)\big).
\end{aligned}$$

Hence $UApr_\rho\big(\varsigma(I)\big)$ is an ideal of N'.

Conversely, assume that $UApr_\rho\big(\varsigma(I)\big)$ is an ideal of N'. Let $\tau_{11}, \tau_{12} \in UApr_\theta(I)$, $\kappa \in F$ and $\xi_{11}, \xi_{12} \in N$. Then $\tau'_{11} = \varsigma(\tau_{11})$, $\tau'_{12} = \varsigma(\tau_{12}) \in \varsigma\big(UApr_\theta(I)\big) = UApr_\rho\big(\varsigma(I)\big)$ and $\xi'_{11} = \varsigma(\xi_{11}), \xi'_{12} = \varsigma(\xi_{12}) \in N'$. Since $UApr_\rho\big(\varsigma(I)\big)$ is an ideal of $N', \tau'_{11} - \tau'_{12}$, $\kappa\tau'_{11}$, $\tau'_{11}\xi'_{11}$ and $\xi'_{12}\big(\xi'_{11} + \tau'_{11}\big) - \xi'_{12}\xi'_{11} \in UApr_\rho\big(\varsigma(I)\big)$.

Now, $\varsigma(\tau_{11} - \tau_{12}) = \varsigma(\tau_{11}) - \varsigma(\tau_{12}) = \tau'_{11} - \tau'_{12} \in UApr_\rho\big(\varsigma(I)\big) = \varsigma(UApr_\theta(I))$.

$\varsigma(\kappa\tau_{11}) = \kappa\, \varsigma(\tau_{11}) = \kappa\tau'_{11} \in UApr_\rho\big(\varsigma(I)\big) = \varsigma(UApr_\theta(I)$.

$\varsigma(\tau_{11}\xi_{11}) = \varsigma(\tau_{11})\varsigma(\xi_{11}) = \tau'_{11}\xi'_{11} \in UApr_\rho\big(\varsigma(I)\big) = \varsigma(UApr_\theta(I)$ and

$\varsigma(\xi_{12}(\xi_{11}+\tau_{11})-\xi_{12}\xi_{11}) = \xi'_{12}(\xi'_{11}+\tau'_{11})-\xi'_{12}\xi'_{11} \in UApr_{\rho}(\varsigma(I)) = \varsigma(UApr_{\theta}(I)$. Thus there exist $\vartheta_1, \vartheta_2, \vartheta_3, \vartheta_4 \in UApr_{\theta}(I)$ such that $\tau_{11}-\tau_{12}=\vartheta_1, \kappa\tau_{11}=\vartheta_2, \tau_{11}\xi_{11}=\vartheta_3$ and $\xi_{12}(\xi_{11}+\tau_{11})-\xi_{12}\xi_{11}=\vartheta_4$. Thus we have $[\vartheta_1]_{\theta}\cap I\neq\emptyset, [\vartheta_2]_{\theta}\cap I\neq\emptyset$, $[\vartheta_3]_{\theta}\cap I\neq\emptyset$ and $[\vartheta_4]_{\theta}\cap I\neq\emptyset$. Also we have $\tau_{11}-\tau_{12}\in[\vartheta_1]_{\theta}, \kappa\tau_{11}\in[\vartheta_2]_{\theta}, \tau_{11}\xi_{11}\in[\vartheta_3]_{\theta}$ and $\xi_{12}(\xi_{11}+\tau_{11})-\xi_{12}\xi_{11}\in[\vartheta_4]_{\theta}$. Hence $[\tau_{11}-\tau_{12}]_{\theta}\cap I\neq\emptyset$, $[\kappa\tau_{11}]_{\theta}\cap I\neq\emptyset$, $[\tau_{11}\xi_{11}]_{\theta}\cap I\neq\emptyset$ and $[\xi_{12}(\xi_{11}+\tau_{11})-\xi_{12}\xi_{11}]_{\theta}\cap I\neq\emptyset$ and therefore $\tau_{11}-\tau_{12}$, $\kappa\tau_{11}, \tau_{11}\xi_{11}$

and $\xi_{12}(\xi_{11}+\tau_{11})-\xi_{12}\xi_{11}\in UApr_{\theta}(I)$. Thus $UApr_{\theta}(I)$ is an ideal of N.

Theorem 3.24. Let N and N' be NAs over a field F and $\varsigma: N\to N'$ be an isomorphism. Let ρ be a CCR on N' and I be a subset of N. Let $\theta=\{(\eta_{11},\eta_{12})\in NXN'|(\varsigma(\eta_{11}),\varsigma(\eta_{12}))\in\rho\}$. Then $LApr_{\theta}(I)$ is an ideal of N if and only if $LApr_{\rho}(\varsigma(I))$ is an ideal of N'.

Proof. Similar proof to the above theorem.

4. Rough Prime Ideals in Near Algebra

In this section, we define rough prime ideal in a near algebra and obtained some important properties to this notion.

Definition 4.1. An ideal P of N is called a *prime ideal (PI)* of N if $\xi_{11}\xi_{12}\in P$ for $\xi_{11},\xi_{12}\in N$ then $\xi_{11}\in P$ or $\xi_{12}\in P$. Let θ be a FCR on N. Then a subset P of N is called an *upper rough prime ideal (URPI)* of N if $UApr_{\theta}(P)$ is a PI of N and *a lower rough prime ideal (LRPI)* of N if $LApr_{\theta}(P)$ is a PI of N.

Theorem 4.2. Let P be a PI of N such that $UApr_{\theta}(P)\neq N$ and θ be a CCR on N. Then P is an URPI of N.

Proof. Since P is an ideal of N, we have $UApr_{\theta}(P)$ is an ideal of N. Let $\xi_{11}\xi_{12}\in UApr_{\theta}(P)$ for some $\xi_{11},\xi_{12}\in N$. Then $[\xi_{11}\xi_{12}]_{\theta}\cap P\neq\emptyset$. Since θ is complete, $\{\eta_{11}\eta_{12}|\eta_{11}\in[\xi_{11}]_{\theta}, \eta_{12}\in[\xi_{12}]_{\theta}\}\cap P\neq\emptyset$, therefore there exist $\eta_{11}\in[\xi_{11}]_{\theta}$ and $\eta_{12}\in[\xi_{12}]_{\theta}$ such that $\eta_{11}\eta_{12}\in P$. Since P is a PI, we have $\eta_{11}\in P$ or $\eta_{12}\in P$. Hence $[\xi_{11}]_{\theta}\cap P\neq\emptyset$ or $[\xi_{12}]_{\theta}\cap P\neq\emptyset$. Thus $\xi_{11}\in UApr_{\theta}(I)$ or $\xi_{12}\in UApr_{\theta}(I)$. So $UApr_{\theta}(I)$ is a PI of N or P is an URPI of N.

Theorem 4.3. Let P be a PI of N such that $LApr_{\theta}(P)\neq\emptyset$ and θ be a FCR on N. Then P is a LRPI of N.

Proof. By theorem 3.17, $LApr_{\theta}(P)=P$ is a PI and hence P is a LRPI of N.

Definition 4.4. Prime ideal P is called a *rough prime ideal (RPI)* of N if it is both a LRPI and URPI of N.

Remark 4.5. According to the theorems 4.2, 4.3., a PI of a NA is a RPI with regard to a CCR.

Theorem 4.6. Let N and N' be NAs over a field F and $\varsigma: N\to N'$ be an onto homomorphism. Let ρ be a CCR on N' and P be a subset of N. Let $\theta=\{(\eta_{11},\eta_{12})\in NXN'|(\varsigma(\eta_{11}),\varsigma(\eta_{12}))\in\rho\}$. Then $UApr_{\theta}(P)$ is a PI of N if and only if $UApr_{\rho}(\varsigma(P))$ is a PI of N'.

Proof. Suppose that $UApr_\theta(P)$ is a PI of N. Let $\xi_{11}, \xi_{12} \in N'$ be such that $\xi_{11}\xi_{12} \in UApr_\rho(\varsigma(P))$. Then there exists τ_{11}, τ_{12} in N such that $\varsigma(\tau_{11}) = \xi_{11}, \varsigma(\tau_{12}) = \xi_{12}$. So $[\varsigma(\tau_{11})\varsigma(\tau_{12})]_\rho \cap \varsigma(P) \neq \emptyset$. Since ρ is complete, there exists $\varsigma(\omega_{11}) \in [\varsigma(\tau_{11})]_\rho$ and $\varsigma(\omega_{12}) \in [\varsigma(\tau_{12})]_\rho$ such that $\varsigma(\omega_{11})\varsigma(\omega_{12}) = \varsigma(\omega_{11}\omega_{12}) \in \varsigma(P)$. Then we have $\omega_{11} \in [\tau_{11}]_\theta$ and $\omega_{12} \in [\tau_{12}]_\theta$. Also there exists $\omega \in P$ such that $\varsigma(\omega) = \varsigma(\omega_{11}\omega_{12})$. Thus $\omega_{11}\omega_{12} \in [\tau_{11}\tau_{12}]_\theta$ and $\omega \in [\omega_{11}\omega_{12}]_\theta$. So $\omega \in [\tau_{11}\tau_{12}]_\theta$ and therefore $[\tau_{11}\tau_{12}]_\theta \cap P \neq \emptyset$ which yields $\tau_{11}\tau_{12} \in UApr_\theta(P)$. Since $UApr_\theta(P)$ is a PI of N, we have $\tau_{11} \in UApr_\theta(P)$ or $\tau_{12} \in UApr_\theta(P)$. Thus $\xi_{11} = \varsigma(\tau_{11}) \in \varsigma(UApr_\theta(P)) = UApr_\rho(\varsigma(P))$ or $\xi_{12} = \varsigma(\tau_{12}) \in \varsigma(UApr_\theta(P)) = UApr_\rho(\varsigma(P))$. So $UApr_\rho(\varsigma(P))$ is a PI of N'. Conversely, suppose that $UApr_\rho(\varsigma(P))$ is a PI of N'. Let $\xi_{11}, \xi_{12} \in N$ be such that $\xi_{11}\xi_{12} \in UApr_\theta(P)$. Then $\varsigma(\xi_{11}\xi_{12}) = \varsigma(\xi_{11})\,\varsigma(\xi_{12}) \in \varsigma(UApr_\theta(P)) = UApr_\rho(\varsigma(P))$. Thus $\varsigma(\xi_{11}) \in UApr_\rho(\varsigma(P))$ or $\varsigma(\xi_{12}) \in UApr_\rho(\varsigma(P))$. If $\varsigma(\xi_{11}) \in UApr_\rho(\varsigma(P))$ then there exists $\tau_{11} \in UApr_\theta(P)$ such that $\varsigma(\xi_{11}) = \varsigma(\tau_{11})$. So $[\tau_{11}]_\theta \cap P \neq \emptyset$ and $\xi_{11} \in [\tau_{11}]_\theta$. Thus $[\xi_{11}]_\theta \cap P \neq \emptyset$ which yields $\xi_{11} \in UApr_\theta(P)$. Similarly, if $\varsigma(\xi_{12}) \in UApr_\rho(\varsigma(P))$ then $\xi_{12} \in UApr_\theta(P)$.

Theorem 4.7. Let N and N' be NAs over a field F and $\varsigma: N \rightarrow N'$ be an isomorphism. Let ρ be a CCR on N' and P be a subset of N. Let $\theta = \{(\eta_{11}, \eta_{12}) \in NXN' | (\varsigma(\eta_{11}), \varsigma(\eta_{12})) \in \rho\}$. Then $LApr_\theta(P)$ is a PI of N if and only if $LApr_\rho(\varsigma(P))$ is a PI of N'.

Proof. Similar proof to the above theorem.

5. Rough Semi Prime Ideals in Near Algebra

In this section, we define rough semi prime ideal in a near algebra and established some important results on rough semi prime ideal in a near algebra.

Definition 5.1. An ideal $P \neq N$ is said to be *semi prime ideal (SPI)* if for any ideal I in N, $I^n \subseteq P$ implies $I \subseteq P$, for some positive integer n. Then a subset P of N is called an *upper rough semi prime ideal (URSPI)* of N if $UApr_\theta(P)$ is a SPI of N and a *lower rough semi prime ideal (LRSPI)* of N if $LApr_\theta(P)$ is a SPI of N.

Theorem 5.2. Let θ be a CCR on N and P, a SPI of N such that $UApr_\theta(P) \neq N$ then P is an URSPI of N.

Proof. Since P is an ideal of N, we have $UApr_\theta(P)$ is an ideal of N. Let I be an ideal of N such that $I^n \subseteq UApr_\theta(P)$. Let $\xi_{11} \in I$. Then ${\xi_{11}}^n \in I^n$ and hence ${\xi_{11}}^n \in UApr_\theta(P)$. Thus $[{\xi_{11}}^n]_\theta \cap P \neq \emptyset$ or ${[\xi_{11}]_\theta}^n \cap P \neq \emptyset$. Thus there exists $\xi_{12} \in P$ which is of the form $\xi_{12} = {\tau_{11}}^n, \tau_{11} \in [\xi_{11}]_\theta$. Since P is SPI of N, $\tau_{11} \in P$. Hence $\tau_{11} \in [\xi_{11}]_\theta \cap P$ or $[\xi_{11}]_\theta \cap P \neq \emptyset$. Thus $\xi_{11} \in UApr_\theta(P)$. Therefore $I \subseteq UApr_\theta(P)$. So $UApr_\theta(P)$ is a SPI of N or P is an URSPI of N.

Theorem 5.3. Let θ be a FCR on N and P be a SPI of N such that $LApr_\theta(P) \neq \emptyset$ then P is a LRSPI of N.

Proof. By theorem 3.17, $LApr_\theta(P) = P$ is a SPI and hence P is a LRSPI of N.

Definition 5.4. A non- empty subset P of N is called a *rough semi prime ideal (RSPI)* of N if it is both a LRSPI and URSPI of N.

Remark 5.5. According to the theorems 5.2, 5.3., a SPI of a NA is a RSPI with regard to a CCR.

Theorem 5.6. Let N and N' be NAs over a field F and $\varsigma: N \to N'$ be an isomorphism. Let θ be a CCR on N and P be a subset of N. Then $UApr_\theta(P)$ is a SPI of N if and only if $\varsigma(UApr_\theta(P))$ is a SPI of N'.

Proof. Let I be an ideal of N' such that $I^n \subseteq \varsigma(UApr_\theta(P))$. Then $\varsigma^{-1}(I^n) \subseteq UApr_\theta(P)$. This implies $(\varsigma^{-1}(I))^n \subseteq UApr_\theta(P)$. Since $UApr_\theta(P)$ is a SPI, $\varsigma^{-1}(I) \subseteq UApr_\theta(P)$ and so $I \subseteq \varsigma(UApr_\theta(P))$. Thus $\varsigma(UApr_\theta(P))$ is a SPI of N'.

Conversely, Let I be an ideal of N such that $I^n \subseteq UApr_\theta(P)$. Then $\varsigma(I^n) \subseteq \varsigma(UApr_\theta(P))$ or $\varsigma(I)^n \subseteq \varsigma(UApr_\theta(P))$. Since $\varsigma(UApr_\theta(P))$ is a SPI of N', $\varsigma(I) \subseteq \varsigma(UApr_\theta(P))$ or $I \subseteq UApr_\theta(P)$. Therefore $UApr_\theta(P)$ is a SPI of N.

Theorem 5.7. Let N and N' be NAs over a field F and $\varsigma: N \to N'$ be an isomorphsim. Let θ be a CCR on N and P be a subset of N. Then $LApr_\theta(P)$ is a SPI of N if and only if $\varsigma(LApr_\theta(P))$ is a SPI of N'.

Proof. Similar proof to the above theorem.

6. Rough Primary Ideals in Near Algebra

In this section, we define rough primary ideal in a near algebra and obtained some fundamental results to this notion.

Definition 6.1. An ideal $P_R \neq N$ in a NA N is *primary ideal (PrI)* if for any $\xi_{11}, \xi_{12} \in N, \xi_{11}\xi_{12} \in P_R$ and $\xi_{11} \notin P_R$ implies ${\xi_{12}}^n \in P_R$ for some $n > 0$. Let θ be a FCR on N. Then a subset P_R of N is called an *upper rough primary ideal (URPrI)* of N if $UApr_\theta(P_R)$ is a PrI of N and *a lower rough primary ideal (LRPrI)* of N if $LApr_\theta(P_R)$ is a PrI of N.

Theorem 6.2. Let θ be a CCR on N and P_R, a PrI of N such that $UApr_\theta(P_R) \neq N$. Then P_R is an *URPrI* on N.

Proof. Let $\xi_{11}\xi_{12} \in UApr_\theta(P_R)$ and $\xi_{11} \notin P_R$. Then $[\xi_{11}\xi_{12}]_\theta \cap P_R \neq \emptyset$ and $[\xi_{11}]_\theta \cap P_R = \emptyset$. Since θ is complete, there exists $\tau_{11} \in [\xi_{11}]_\theta$ and $\tau_{12} \in [\xi_{12}]_\theta$ such that $\tau_{11}\tau_{12} \in P_R$ and $\tau_{11} \notin P_R$. Since P_R is a PrI, we have ${\tau_{12}}^n \in P_R$ for some positive integer n. Since $\tau_{12} \in [\xi_{12}]_\theta$, we have ${\tau_{12}}^n \in [{\xi_{12}}^n]_\theta$. Thus $[{\xi_{12}}^n]_\theta \cap P_R \neq \emptyset$, which yields that ${\xi_{12}}^n \in UApr_\theta(P_R)$ and hence $UApr_\theta(P_R)$ is a PrI of N or P_R is an *URPrI* on N.

Theorem 6.3. Let θ be a FCR on N and P_R, a PrI of N such that $LApr_\theta(P_R) \neq \emptyset$, then P_R is a L*RPrI*.

Proof. By theorem 3.17, $LApr_\theta(P_R) = P_R$ is a PrI and hence P_R is a LRPrI of N.

Definition 6.4. Primary ideal P_R is called a *rough primary ideal (RPrI)* of N if it is both a lower and upper rough primary ideal of N.

Remark 6.5. From theorems 6.2, 6.3., we know that a PrI is a RPrI with respect to a CCR on a NA.

Theorem 6.6. Let N and N' be NAs over a field F and $\varsigma: N \to N'$ be an onto homomorphism. Let ρ be a CCR on N' and P_R be a subset of N. Let $\theta = \{(\eta_{11}, \eta_{12}) \in NXN' | (\varsigma(\eta_{11}), \varsigma(\eta_{12})) \in \rho\}$. Then $UApr_\theta(P_R)$ is a PrI of N if and only if $UApr_\rho(\varsigma(P_R))$ is a PrI of N'.

Proof. Suppose that $UApr_\theta(P_R)$ is a PrI of N. Let $\xi_{11}, \xi_{12} \in N'$ be such that $\xi_{11}\xi_{12} \in UApr_\rho(\varsigma(P_R))$ and $\xi_{11} \notin UApr_\rho(\varsigma(P_R))$. Then there exists τ_{11}, τ_{12} in N such that $\varsigma(\tau_{11}) = \xi_{11}, \varsigma(\tau_{12}) = \xi_{12}$. So $[\varsigma(\tau_{11})\varsigma(\tau_{12})]_\rho \cap \varsigma(P_R) \neq \emptyset$ and $[\varsigma(\tau_{11})]_\rho \cap \varsigma(P_R) = \emptyset$. Proceeding as in theorem 4.6 we have $\tau_{11}\tau_{12} \in UApr_\theta(P_R)$ and $\tau_{11} \notin UApr_\theta(P_R)$. Since $UApr_\theta(P_R)$ is a PrI of N, we have ${\tau_{12}}^n \in UApr_\theta(P_R)$. Thus $\varsigma({\tau_{12}}^n) = (\varsigma(\tau_{12}))^n \in \varsigma(UApr_\theta(P_R)) = UApr_\rho(\varsigma(P_R))$ or ${\xi_{12}}^n \in UApr_\rho(\varsigma(P_R))$ and hence $UApr_\rho(\varsigma(P_R))$ is a PrI of N'.

Conversely, suppose that $UApr_\rho(\varsigma(P_R))$ is a PrI of N'. Let $\xi_{11}, \xi_{12} \in N$ be such that $\xi_{11}\xi_{12} \in UApr_\theta(P_R)$ and $\xi_{11} \notin UApr_\theta(P_R)$. Then $\varsigma(\xi_{11}\xi_{12}) = \varsigma(\xi_{11})\,\varsigma(\xi_{12}) \in \varsigma(UApr_\theta(P_R)) = UApr_\rho(\varsigma(P_R))$ and $\varsigma(\xi_{11}) \notin \varsigma(UApr_\theta(P_R)) = UApr_\rho(\varsigma(P_R))$. Since $UApr_\rho(\varsigma(P_R))$ is a PrI of N', $(\varsigma(\xi_{12}))^n = \varsigma({\xi_{12}}^n) \in UApr_\rho(\varsigma(P_R))$. Thus there exists ${\tau_{12}}^n \in UApr_\theta(P_R)$ such that $\varsigma({\xi_{12}}^n) = \varsigma({\tau_{12}}^n)$. So $[{\tau_{12}}^n]_\theta \cap P_R \neq \emptyset$ or ${\xi_{12}}^n \in [{\tau_{12}}^n]_\theta$. Hence $[{\xi_{12}}^n]_\theta \cap P_R \neq \emptyset$ which implies ${\xi_{12}}^n \in UApr_\theta(P_R)$. Thus $UApr_\theta(P_R)$ is a PrI of N.

Theorem 6.7. Let N and N' be NAs over a field F and $\varsigma: N \to N'$ be an isomorphism. Let ρ be a CCR on N' and P_R be a subset of N. Let $\theta = \{(\eta_{11}, \eta_{12}) \in NXN' | (\varsigma(\eta_{11}), \varsigma(\eta_{12})) \in \rho\}$. Then $LApr_\theta(P_R)$ is a PrI of N if and only if $LApr_\rho(\varsigma(P_R))$ is a PrI of N'.

Proof. Similar proof to the above theorem.

7. Conclusion

We introduced and explored novel concepts of rough sub near algebras and rough ideals in near algebra. This work investigated the distinctive properties of rough ideals, illustrating our findings with pertinent examples. We also explored the connection between different ideals and their corresponding homomorphism images, revealing insights into the algebraic structure and behaviour.

8. Funding

The authors expressed their gratitude to GITAM Deemed to be University, India for funding this research under GITAM SEED Grant F. No. 2021/0104.

References

[1] R. Biswas, S. Nanda, Rough groups and rough subgroups, *Bull. Pol. Ac. Math.*, 42 (1994), 251 - 254.
[2] H. Brown, Near algebra, *Illinois J. Math.*, 12 (1968), 215-227.
[3] B. Davvaz, Roughness in rings, *Inform. Sci.*, 164 (2004), 147–163.
[4] B. Davvaz, M. Mahdavipour, Roughness in modules, *Inform. Sci.*, 176 (2006), 3658- 3674.
[5] B. Davvaz , Roughness based on fuzzy ideals, *Inform. Sci.*, 176 (2006), 2417- 2437.
[6] B. Harika, K. Rajani, P. Narasimha Swamy, T. Nagaiah, I. Bhaskar, Neutrosophic near 9 algebra over neutrosophic field, *International Journal of Neutrosophic Science*, 22 (2023), 29-34.
[7] N. Kuroki, Rough ideals in semigroups, *Inform. Sci.* 100 (1997), 139–163.
[8] J. Marynirmala, D. Sivakumar, Rough ideals in rough near - rings, *Adv. Math., Sci. J*. 9 (2020), 2345-2352.

[9] Neelima C. A., Paul Isaac, Rough anti homomorphsim on a rough group, *Global Journal of Mathematical Sciences: Theory and Practical*, 6 (2014), 79-87.

[10] Neelima C. A, Paul Isaac, Rough semi prime ideals and rough bi ideals in rings, *Int Jr. of Mathematical Sciences & Applications*, Vol.4, No.1,

[11] Osman Kazanci, B. Davvaz, On the structure of rough prime (primary) ideals and rough fuzzy prime (primary) ideals in commutative rings, *Inform. Sci*., 178(2008), 1343-1354.

[12] Paul Isaac, Rough ideals and their properties, *Jl. Global research in math archives*, 1(6):90-98, June, 2014.

[13] Z. Pawlak, Rough sets, *Int. J. Comput. Inf. Sci*. 11 (1982), 341–356.

[14] Z. Pawlak, Theoretical aspects of reasoning about data, Kluwer academic publishers, Netherlands, 1991.

[15] Qi – Mei Xiao, Zhen – Liang Zhang, Rough prime ideals and rough fuzzy prime ideals in semigroups, *Inform. Sci*., 176(2006), 725-733.

[16] Qun - Feng Zhang, Al - Min Fu, Shin - Xin Zhao, Rough modules and their some properties, Proceedings of the 5th International conference on Machine Learning and Cybernetics, Dalian, 13-16 Aug, 2006.

[17] K. Rajani, P. Narasimha Swamy, Ravikumar Bandaru, Akbar Rezaei , Amal S. Alali, On Rough Near Algebras, *Palestine Journal of Mathematics*, Accepted.

[18] V. Selven, Rough ideals in semi rings, *Int Jr. of Mathematics & Applications*, Vol.2, No. 2, 2012.

[19] G. F. Simmons, Introduction to topology and modern analysis, Robert E. Krieger Publishing Company, Florida 1963.

[20] T. Srinivas, P. Narasimha Swamy, Near Algebras and gamma near algebras, Near rings, near fields and related topics, World Scientific Pub Co Pte Lt, (2017), 256-263.

[21] V. S. Subha, On rough ideals in Γ- Near Rings, *IJRAR*, 2017, Vol 4, Issue 2.

[22] V. S. Subha, N. Thillaigovindan, Rough set concepts applied to ideals in near rings, *Proceedings of Dynamic Systems*, 2012.

[23] L. A. Zadeh, Fuzzy sets, *Inform. Control*. 8 (1965), 338–353.