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A Modified Fuzzy Labeling Graph using Geometric Mean and Harmonic mean and its Application

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Abstract:
The Modified Fuzzy Labeling Graph (MFLG) has gained popularity in the field of image processing due to its ability to accurately represent complex images. However, there are limitations to the MFLG approach that can be addressed through the use of geometric mean and harmonic measures. In this paper, the authors propose a new idea on a modified fuzzy labeling graph. A Modified fuzzy (MF_G) graph σ is a function from V to [0,1] and μ is a function from E to [0,1] where $\forall\ x, y \in V$. The modified fuzzy labeling has an edge value, which is less than or equal to the maximum of two endpoints that is $\mu\ (u,v)\ \leq\ \sigma\ (u) \vee \sigma\ (v)$. To obtain the result, the edge weight is assigned as Geometric labeling mean, where W(e) is equal to the square root of two end points. The edge weight is assigned as Harmonic mean labeling, which is $\mu(u,v)\ =\ \frac{2\sigma\ (u)*\sigma\ (v)}{\sigma\ (u)+\sigma\ (v)}$. The study examines the modified fuzzy labeling for the different graphs including Alternate Pentagon snake graph, Alternate Double Pentagon snake graph, and Alternate Triple Pentagon snake graphs using Geometric mean; and the discussed TPS_n, DTPS_n, $ATPS_n$, $TTPS_n$ and Quadrilateral snake graph using Harmonic mean. Graph that permits modified fuzzy (MFL) labeling is called a MF_G. Some of the properties and results thereof have been explained. The authors also discuss an application of critical path method in Operation Research for students Higher studies problem.

Keywords: Modified fuzzy graph, Geometric mean labeling, Harmonic mean labeling, Pentagon snake graph, Triangular Prism snake graph, Quadrilateral snake graph.

1. Introduction

Researches on the labelings of a graph G = (V,E), are motivated by the numerous practical problems involved in real life scenarios. For, these need to abide by many different conditions. Over the last three decades or more, loads of literature related to several kinds of labelings of graphs addressing different graph labeling problems have become available. The Geometric Labeling of Graphs, in which the vertices are allocated with values which are subject to some conditions, poses several practical problems. Nevertheless, these are also of interest owing to their achievements.

Some of the utterly fascinating graph labeling problems have three components: (i) a group of numbers from which the labels are selected; (ii) a rule for allocating a value to every edge; (iii) a condition that should be fulfilled by these values.

Fuzzy labeling graphs are a powerful tool in graph theory that allows for the representation of uncertain or imprecise data. However, traditional fuzzy labeling graphs can suffer from limitations such as repetition and inconsistency. To address these issues, a modified fuzzy labeling graph using geometric mean and harmonic measures has been proposed. The Modified

Fuzzy Labeling Graph is an effective tool for managing and analyzing huge datasets. This technique, which incorporates both geometric mean and harmonic mean, gives a more accurate representation of the data, allowing for improved decision-making and analysis. Overall, the combination of these measures with the MFLG approach represents a significant advancement in image-processing technology and has wide-ranging applications across a variety of industries and fields.

2. Review of Literature

This study takes into account only those graphs that are simple, finite, connected, and undirected. Here is a detailed review of the literature in this niche.

Deshmukh and Shaikh discussed detailed Geometric mean graph results from various corona graphs [1]. The authors have discussed combining two graphs which admit the Geometric mean. In [2] Viji, Somasundaram, and Sandhya provided a description of the geometric labeling of some disjoined graphs along with an illustration. Geometric mean labeling using several operations as described in [3].

In 2014, Sandhya and Somasundaram [4] explored geometric mean labeling using a double quadrilateral snake. In [5], the authors investigated various additional findings and behavior of some standard graphs with regards to labeling of super geometric mean. The Geometric labeling mean behaviour of subdivision on several standard graphs are investigated in [6]. Apart from proving some properties of the modified fuzzy resolving set of the fuzzy graph and fuzzy labeling graph, this study also considered the weak interconnectivity between the fuzzy graph nodes. In [7], the authors explained the fuzzy modified-resolving set and resolving number as an extension of the concept of resolving numbers.

In [8], the authors provided a detailed analysis of the various types of graph labeling and introduced the Geometric labeling concept. Some graphs and attributes with superharmonic labeling mean were described in [10-12]. In 2012, the authors in [13] developed the idea of mean cordial labeling of graphs, and then in 2017, other researchers proposed the cordial labeling of graphs using geometric mean.

Some additional findings on Harmonic mean labeling were discussed in [14]. Certain cycle-related graphs, such as duplication, joint cycle sum, and cycle classification, which exhibit harmonic mean labeling behaviour are described in [15]. In 2010, Vaidya and Kanani established a few results on new mean graph. The concept of root square mean labeling was first presented in 2014 in [16]. The authors used a few families of graphs with specific conditions. Following that, the super root square mean graph was examined. In [17-18], the author examined new product fuzzy graph. In [19-20], the authors discuss some cycle-related graphs exhibiting harmonic mean labeling behaviour, including cycle duplication, cycle joint sum, and cycle identification. In [21-22], the authors provided additional new findings on the harmonic mean labeling of selected graph classes.

The concept of modified fuzzy labeling graph using geometric mean and harmonic mean to other existing concepts in fuzzy graph theory, a new way to calculate the overall uncertainty or fuzziness of the graph, providing a more comprehensive understanding of its characteristics.

This approach can also be seen as an extension of traditional Fuzzy graph theory. When applying the modified approach to high-dimensional data is the increased computational complexity, as processing and analyzing high-dimensional data requires more computational resources and time.

1. The use of geometric and harmonic techniques for edge weight assignment introduces a degree of fuzziness that reflects the properties of the graph.
2. The edge value constraint maintains consistency in the labeling technique by ensuring that an edge's fuzziness does not exceed that of its endpoints.
3. The main goal of this research is to investigate the performance of this modified fuzzy labeling approach on different kinds of graphs, which may be used for network analysis, decision-making, or optimisation.

3. Contribution of this study

This study examines modified fuzzy labeling for the following fuzzy graphs- Alternate Pentagon Snake Graph, Alternate Double Pentagon Snake Graph, and Alternate Triple Pentagon Snake Graph using Geometric mean and discusses their $FTPS_n$,D$FTPS_n$, $AFTPS_n$, $TFTPS_n$. It also discusses the families of Quadrilateral snake graph using Harmonic mean labeling. A graph that permits modified fuzzy (MFL) labeling is called a modified fuzzy graph. The authors used some corona graph such as F$APS_n \odot K_1$, AFP$S_n \odot K_2$, $FTPS_n \odot K_1$, AFTP$S_n \odot K_1$, that admit modified fuzzy graph and results. One of its applications on students' Higher studies problem using critical path method in Operation Research is also explained.

3.1 Definitions of Terminologies Used in the Study

Below-given are some of the definitions used in the current study.

3.1.1. Fuzzy Graph

A Fuzzy Graph (F_G), If $|V| \neq \emptyset$, σ is the function from V to [0,1] and μ is a function from Vx V to [0,1] in order that $\forall\ u, v \in V$, $\mu(u, v) \leq \sigma(u) \Lambda\ \sigma(v)$.

3.1.2. Fuzzy Graph Labeling

A Fuzzy graph labeling (F_{GL}), σ is a function from V to [0,1] and μ is a function from Vx V to [0,1] such that $\forall\ u, v \in V$ and assign the consecutive membership values for edges and vertices are $z, 2z, 3z, \ldots, pz$, such that $\mu(u, v) < \min\{\sigma(u), \sigma(v)\}$.

$F_{GL} = \begin{cases} z = 0.1 & for\ p \leq 6 \\ z = 0.01 & for\ p > 6 \end{cases}$ Where, p is the total number of vertices

3.1.3. Fuzzy Alternate Pentagon Snake Graph

A Fuzzy alternate pentagon snake graph $(FAPS_n)$ is a graph obtained from a path $u_1, u_2, \ldots, u_n$. Here every alternate edge of a path $u_i\ and\ u_{i+1}$ are replaced by a pentagon graph, where, $\mu(u_i, u_{i+1}) \leq \sigma(u_i) \Lambda\ \sigma(u_{i+1})$.

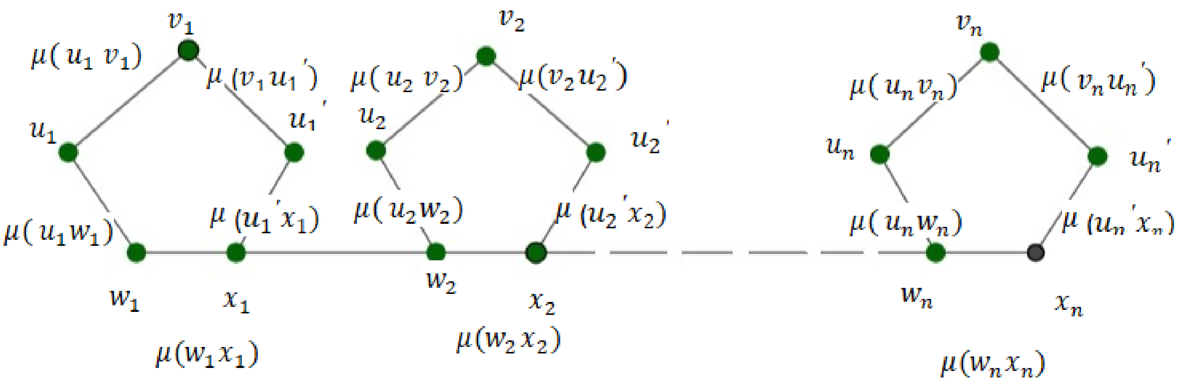

Fig 1.1. Fuzzy alternate pentagon snake graph ($FAPS_n$)

3.1.4. Fuzzy Alternate Double Pentagon Snake Graph

A Fuzzy Alternate Double pentagon snake ($FADPS_n$) is a graph obtained from a path $u_1, u_2,\ldots,u_n$ that consists of two Fuzzy Alternate Pentagon Snake graphs with a common edge- u_i *and* u_{i+1}, which is replaced by a pentagon graph.

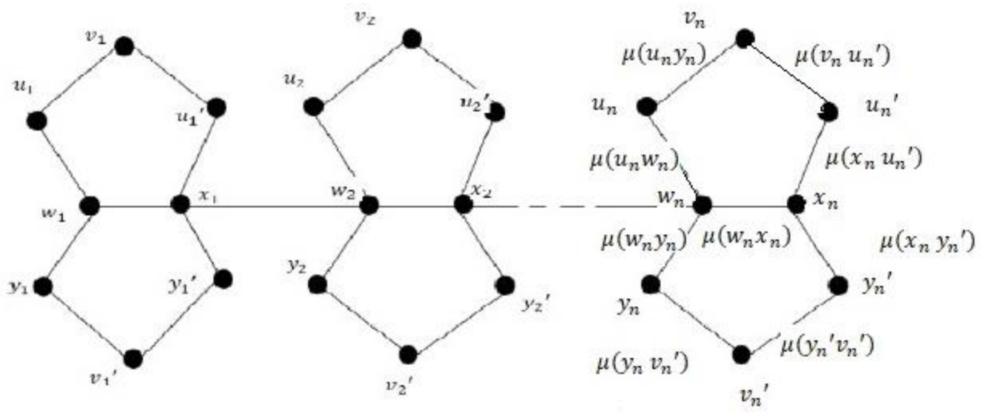

Fig 1.2. Fuzzy Alternate Double pentagon snake ($FADPS_n$)

3.1.5. The Degree of Each Vertex

The degree of every vertex $\sigma\ (x_i)$ in $F_G(x,y)$, is equal to the sum of the degrees of membership of all the edges $d(\sigma\ (x_i))$ that are incident on the vertex $\sigma\ (x_i)$.

3.1.6. Fuzzy Triangular Prism Snake graph

A Fuzzy Triangular Prism snake ($FTPS_n$) graph is a graph obtained from a path $u_1, u_2,\ldots,u_n$. Every edge of a path, u_i *and* u_{i+1}, is replaced by a fuzzy triangular prism graph. *The* $FTPS_n$ graph G(V,σ, μ), where a set of vertices together with the map σ: V → [0,1] and μ: V × V → [0,1] $\forall\ u, v \in\ V$ is such that $\mu(u,v) \le \min\{\sigma(u),\sigma(v)\}$.

3.1.7. Fuzzy Double Triangular Prism Snake graph

A Fuzzy Double Triangular Prism Snake ($FDTPS_n$) graph is a graph obtained from a path $u_1, u_2,\ldots,u_n$ that consist of two Triangular Prism graph with a common edge u_i *and* u_{i+1}, where $\mu(u_i, u_{i+1}) \le \min\{\sigma(u_i),\sigma(u_{i+1})\}$.

3.1.8. Fuzzy Alternate Triangular Prism Snake graph

A Fuzzy Alternate Triangular Prism Snake ($FATPS_n$) graph is a graph obtained from a path $u_1, u_2,\ldots,u_n,$ where every alternate edge of a path, u_i *and* u_{i+1}, is replaced by Triangular Prism graph, such that μ(u_{i+1},u_i)> 0 and μ(u_i, u_{i+1}) $= 0$ for $1 \leq i \leq n.$.

3.1.9. Fuzzy Quadrilateral Snake graph

The Fuzzy Quadrilateral snake graph (FQS_n) is a graph created by connecting each vertex to new vertices, b_i *and* c_i, and joining the vertices b_i *and* c_i for i=1,2,...,n-1. That is, a cycle FC_4 is substituted for each path edge.

3.2. Modified fuzzy labeling graph

Below given are some definitions used under the proposed modified fuzzy labeling graph.

3.2.1. Modified Fuzzy Graph

A Modified Fuzzy Graph F_mG(V, σ, μ), is a graph generated when $|V| \neq \emptyset, \sigma$ is the function from V to [0,1] and μ is a function from Vx V to [0,1] in order that $\forall\ u, v \in\ V,$ μ$(u, v) \leq \sigma(u) \vee \sigma(v)$.

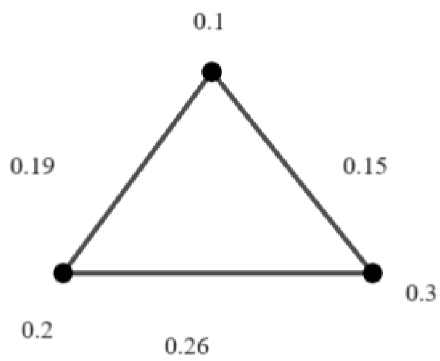

Fig1.3: Modfied fuzzy graph

3.2.2. Modified fuzzy labeling graph

A Modified fuzzy labeling graph FLG_m, is a graph generated when V is a nonempty set , σ is the function from V to [0,1] and μ is a function from Vx V to [0,1] in order that $\forall\ a, b \in$ V, $\mu(a, b) < max\{\sigma(a), \sigma(b)\}$. The process is denoted as FG_mlabeling.

Results:

For the following operations, use a modified fuzzy labeling graph.

1. Fuzzy modified mean labeling graph
2. Fuzzy modified median labeling graph
3. Fuzzy modified mode labeling graph

4. Results and Discussion

In this section, the proposed modified fuzzy labeling graph under various fuzzy graphs is defined, and then its properties are investigated and shown with an example.

4.1. Theorem 1.

The $FAPS_n$ graph admits a Modified Fuzzy Alternate Pentagon Snake labeling graph.

Proof:

The Fuzzy Alternate Pentagon Snake graph is a variation of the well-known Pentagonal Snake graph, which is a snake-like structure consisting of five vertices and four edges. The Fuzzy Alternate Pentagon Snake graph is obtained by adding edges between certain pairs of vertices in the Pentagonal Snake graph. The Modified Fuzzy Alternate Pentagon Snake labeling extends the concept of Fuzzy labeling to this new graph.Assign the label to each vertices closed range between 0 and 1. Using the geometric mean labeling of the two end points, edge values should be determined. As a result, the edge label will be distinct and less than or equal to a maximum of two end points.Consider the Fuzzy Alternate Pentagon Snake graph $FAPS_n$, by definition 3.1.3 Let p and q be the graph's total vertices and edges , where $p = 5n$ and $q = 6n$-1 edges, where n denotes the pentagonal snake graph. If a set of functions exists for a fuzzy graph $G(V, E)$, then the graph is referred to as Modified fuzzy labeling graph if there is a pair of function. Here, $\forall\, u, v \in V$, such that edge label should be less than or equal to maximum of two endpoints (i.e) $\mu\,(u, v) \leq \max\{\sigma\,(u), \sigma\,(v)\}\ for\ all\ u, v\ \epsilon\ E(G)$.

Let us define a function $f: V(G) \to [0,1]$ and edge are assigned using geometric label $E(u, v) = \sqrt{\sigma\,(u) * \sigma\,(v)}$, assume $z = 0.1$ as follows. To prove the $FAPS_n$ is a modified fuzzy Alternate Pentagon Snake labeling graph.

Assign the label to each vertices closed range between 0 and 1.The computation part is given below

$\sigma\,(u_i) = (5i - 4)z.$

$\sigma(u_i{}') = 5iz$

$\sigma\,(v_i) = (5i - 3)z$

$\sigma\,(w_i) = (5i - 2)z$

$\sigma\,(x_i) = (5i - 1)z$

Using the geometric mean labeling of the two end points, edge values should be determined

$\mu(u_i, v_i) = \sqrt{(\sigma\,(u_i) * \sigma\,(v_i))} = \sqrt{(5i - 4)z * (5i - 3)z}$

$\mu(v_i, u_i{}') = \sqrt{(\sigma\,(v_i) * \sigma(u_i{}'))} = \sqrt{(5i - 3)z * (5i)z}$

$\mu(u_i, w_i) = \sqrt{(\sigma\,(u_i) * \sigma\,(w_i))} = \sqrt{(5i - 4)z * (5i - 2)z}$

$\mu(w_i,\ x_i) = \sqrt{(\sigma\,(w_i) * \sigma\,(x_i))} = \sqrt{(5i - 2)z * (5i - 1)z}$

$\mu(x_i,\ w_{i+1}) = \sqrt{(\sigma\ (x_i) * \sigma\ (w_{i+1}))} = \sqrt{(5i-1)z * (5i+3)z}$

$\mu(u_i', x_i) = \sqrt{(\sigma(u_i') * \sigma\ (x_i))} = \sqrt{((5i)z * (5i-1)z)}$

The values of the remaining edges $\mu(v_i, u_i')$, $\mu(u_i, w_i)$, $\mu(w_i, x_i)$, $\mu(x_i, w_{i+1})$ and $\mu(u_i', x_i)$ can be thus found from the above-given formulae. It can be seen that each edge weight is less than the maximum of two end points thus ensuring that the graph is Modified Fuzzy pentagon snake which has an edge value which is less than equal to the maximum of two endpoints.

Theorem 2

The Fuzzy Alternate Pentagon Snake graph ($FAPS_n$) is not a fuzzy labeling graph.

Proof:

Let consider the fuzzy alternate Pentagon snake fuzzy graph, which is without the loss of generosity. Label the vertices in the range 0 to 1 and label the geometric mean in the edges so that the value of the edge is equal to or less than the maximum of two end vertices. which contradicts itself. Therefore,$FAPS_n$ rather than being a fuzzy labeling graph, the fuzzy Alternate Pentagon snake graph is a modified fuzzy labeling graph.

Discussion of the result:
Let us consider the Alternate Pentagon Snake Fuzzy graph APS_3. Let p and q be the graph's vertices and edges. Here, $p = 5n\ and\ q\ 6n - 1$ edges. After labeling the values for the vertices, $\sigma : V \rightarrow$ [0,1] and defining the edge label as geometric mean value, the edge value is greater than the value of one of the two nodes that is incident with it. This is shown the Figure 1.1. Thus, Alternate Pentagon Snake graph APS_3 is a modified fuzzy geometric labeling.

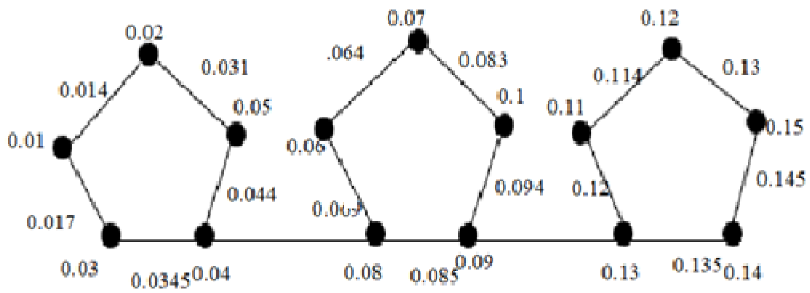

Fig 1.4. $FAPS_3$

Result 1:

- A modified fuzzy graph is obtained when a geometric mean label is applied to an edge
- The edge label using geometric mean labeling is not a fuzzy graph.

Theorem 3

The Fuzzy alternate double pentagon snake graph ($FADPS_n$) is a Modified Fuzzy labeling graph.

Proof:

Fuzzy alternate double pentagon snake graph obtained from a path $u_1, u_2,\ldots,u_n$ that consists of two Fuzzy Alternate Pentagon Snake graphs with a common edge- u_i *and* u_{i+1}, which is replaced by a pentagon graph by definition 3.1.4. Consider the p-vertices, and q-edge of fuzzy alternate double pentagon graph. The labels for the vertices range from 0 to 1. The geometric mean between the two vertices connected by the edge serves as the illustration for edge weights. The modified fuzzy graph is satisfied because the label obtained of the edges will be less than or equal to the maxima of the two end points.Below is a general illustration of the fuzzy alternate double pentagon snake graph.

Consider the Alternate Double Pentagon Snake graph. Let p and q be the graph's vertices and edges. Here $p = 8n$ and $q = 10n - 1$ edges. Let a function be defined, assuming μ(u,v)= $\begin{cases} z = 0.1, & if\ p \leq 6 \\ z = 0.01, & if\ \ p > 6 \end{cases}$ as follows:

Let the vertices be constructed as below,

$\sigma\ (u_i) = (8i - 7)z$

$\sigma\ (v_i) = (8i - 6)z$

$\sigma\ (w_i) = (8i - 5)z$

$\sigma\ (x_i) = (8i - 4)z$

$\sigma\ (y_i) = (8i - 3)z$

Similarly, make calculations for other vertices $\sigma(v_i'), \sigma(u_i'), \sigma(y_i')$.

And then, construct for edge label as follows

$\mu(u_i, v_i) = \sqrt{(\sigma\ (u_i) * \sigma\ (v_i))} = \sqrt{((8i - 7)z * 2(4i - 3)z)}$

$\mu(u_i, w_i) = \sqrt{(\sigma\ (u_i) * \sigma\ (w_i))}$

$\mu(w_i, x_i) = \sqrt{(\sigma\ (w_i) * \sigma\ (x_i))}$

Similarly, calculate the other edge values of $\mu(v_i, u_i')$, $\mu(u_i', x_i)$,$\mu(w_i y_i)$,$\mu(y_i, v_i')$, $\mu(v_i', y_i')$ and $\mu(x_i, y_i')$ using the geometric equation. Ensuring that each edge has a unique value. Additionally, the edge value must be less than or equal to the maximum of the two end points.

All edge weights are distinct, such that the edge value is less than or equal to the maximum of two end points, satisfying the Modified Fuzzy labeling graph.

Result 2:

The Fuzzy Alternate Triple Pentagon Snake Fuzzy graph is a Modified Fuzzy labeling graph.

Corollary 1:

The FG_m has $O(G) \leq S(G)$.

Proof:

By using the number of vertices and edges, the size and order of any fuzzy graph can be determined. In the Modified fuzzy graph, the edge weight is assigned as Geometric labeling mean. Here, W(e) = square root of two end points. Additionally, the size and order of a Modified fuzzy graph can be easily determined using its vertex and edge count.

In comparison to the value of one of the two nodes it is incident with, the edge label is larger. As a result, $O(G) \leq S(G)$.

Corollary 2:

The product of fuzzy graph must not be a Modified Fuzzy labeling graph.

Proof:

A product Fuzzy graph $G(x, y)$ has a vertex collection V and an edge collection E.

If $\mu(x, y) = \sigma(x) * \sigma(y)\ for\ all\ x, y \in E$, the edge value obtained is less than the two end points. This is a contradiction, which does not satisfy the Modified Fuzzy graph. This shows that the product graph is not a Modified Fuzzy graph.

Result 3:

If $FG(\sigma, \mu)$ is $FG_{m,}$ then $\sum d(\sigma(v_i)) = 2\sum \mu(u_i, v_{i+1})$.

Proof:

Consider a Fuzzy graph with n vertices $\sigma(v_1), \sigma(v_2) \ldots \ldots \ldots, \sigma(v_m)$. Every edge is incident on the vertices. Then, by the definition, the degree of every node in a F_G, is equal to the total membership degree of all edges.

Example: Consider the Modified Fuzzy graph with 5 vertices and 6 edges.

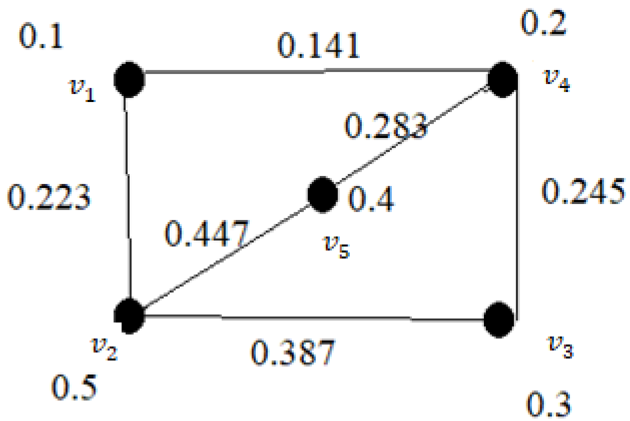

Fig 1.5 Modified fuzzy graph F_{Gm}

$$\sum_{i=1}^{5} d(\sigma(v_i)) = 3.452 ---------- (1)$$

$\sum_{i=1}^{6} \mu(u_i, v_{i+1}) = 1.726$ ------------------------ (2)

Therefore, from (1) and (2), the following equation is obtained $\sum d\big(\sigma(v_i)\big) = 2\sum \mu(u_i, v_{i+1})$.

Theorem 4:

A Fuzzy Alternate Pentagon Snake graph $FAPS_n \odot K_1$ is a modified labeling fuzzy graph

Proof:

Consider the APS_n graph, where each of the vertices is connected with the complete graph $G = APS_n \odot K_1$. It has $10n$ vertices and $11n - 1$ edges. Let a function be defined, assuming that each vertex is assigned a unique label. This function maps the vertices to the set from 0 to 1 .

The vertices are constructed as below,

$\sigma\,(x_i) = (10i - 9)z$

Similarly, for the other vertices $\sigma\,(y_i), \sigma\,(z_i), \sigma\,(s_i), \sigma\,(v_i), \sigma\,(u_i), \sigma\,(t_i), \sigma(x_i{}'), \sigma(y_i{}'), \sigma(z_i{}'),$

a function $f: E(G) \rightarrow [0,1]$ is constructed the edges as below,

$\mu(x_i, y_i) = \sqrt{(\sigma\,(x_i) * \sigma\,(y_i))} = \sqrt{(10i - 9)z * (10i - 7)z}$

Similarly, the value of all edges can be calculated using geometric mean. It can be observed that all the edge weights are distinct and the edge values are less than or equal to the maximum of two endpoints, As a result, $FAPS_n \odot K_1$ satisfying the Modified Fuzzy labeling graph.

Lemma 1.

The Fuzzy Alternate Pentagon Snake graph $FAPS_n \odot K_2$ is a Modified Labeling Fuzzy graph.

Proof: It derives from the theorem 4.

Result 4:

- The Fuzzy Alternate Double Pentagon Snake graph $FADPS_n \odot K_1$ is a modified labeling Fuzzy graph.
- The Fuzzy Alternate Double Pentagon Snake graph FADP$S_n \odot K_2$ is a Modified Labeling Fuzzy graph.

Theorem 5:

A Fuzzy Triangular Prism Snake graph $FTPS_n$ admits a Modified fuzzy labeling graph.

Proof:

Let $FG\,(\sigma, \mu)$ be a Triangular Prism Snake graph that consists of a set of nodes and edges, where each node represents a fuzzy triangular prism and each edge represents the relationship between two prisms. We need to prove that $FTPS_n$ is a modified fuzzy graph, By assigning labels to vertices from 0 to 1 and edges using the harmonic mean of the two endpoints. As the illustration given below, which has its vertices as $p = 5n + 1$ and edges as $q = 9n$. Let a function $f: V(FG) \rightarrow [0,1]$ be defined and let the values for nodes and link be assigned as

$z, 2z, 3z, \ldots, Nz.$ Here, $z = 0.1\ if\ p \leq 6\ and\ z = 0.01\ if\ p > 6$ and edge label is defined as $\mu\ (u, v)\ = \frac{2\sigma\ (u_i)*\sigma\ (v_i)}{\sigma\ (u_i)+\sigma\ (v_i)}$

$\sigma\ (u_i) = (5i - 3)z$

$\sigma\ (x_i) = (5i - 2\ )$z

Similarly, the remaining vertices can be constructed.

For calculating the edges, the following equations are employed.

$\mu(u_i, v_i) = \frac{2\sigma\ (u_i)*\sigma\ (v_i)}{\sigma\ (u_i)+\sigma\ (v_i)} = \frac{2(5i-3)z*(5i-4)z}{(5i-3)z+(5i-4)z}$

$\mu(u_i, x_i) = \frac{2\sigma\ (u_i)*\sigma\ (x_i)}{\sigma\ (u_i)+\sigma\ (x_i)} = \frac{2(5i-3z)*(5i-2)z}{(5i-3)z+(5i-2)z}$

$\mu(u_i, v_{i+1}) = \frac{2\sigma\ (u_i)*\sigma\ (v_{i+1})}{\sigma\ (u_i)+\sigma\ (v_{i+1})} = \frac{2(5i-3)*(5i+2)}{(5i-3)+(5i+2)}$

$\mu(x_i, y_i) = \frac{2\sigma\ (x_i)*\sigma\ (y_i)}{\sigma\ (x_i)+\sigma\ (y_i)} = \frac{2(5i-2)*(5i-1)}{(5i-2)+(5i-1)}$

$\mu(x_i, z_i) = \frac{2\sigma\ (x_i)*\sigma\ (z_i)}{\sigma\ (x_i)+\sigma\ (z_i)} = \frac{2(5i-2)*(5i)}{(5i-2)+(5i)}$

$\mu(y_i, z_i) = \frac{2\sigma\ (y_i)*\sigma\ (z_i)}{\sigma\ (y_i)+\sigma\ (z_i)} = \frac{2(5i-1)*(5i)}{(5i-1)+(5i)}$

$\mu(v_i, y_i) = \frac{2\sigma\ (v_i)*\sigma\ (y_i)}{\sigma\ (v_i)+\sigma\ (y_i)} = \frac{2(5i-4)*(5i-1)}{(5i-4)+(5i-1)}$

$\mu(v_{i+1}, z_i) = \frac{2\sigma\ (v_{i+1})*\sigma\ (z_i)}{\sigma\ (v_{i+1})+\sigma\ (z_i)} = \frac{2(5i+2)*(5i)}{(5i+2)+(5i)}$

It is evident from the above equations that all edge weights are distinct, such that each edge value is less than or equal to the maximum of two end points, which satisfies modified fuzzy labeling graph.

Example:

Consider the graph $FTPS_3$ with verticles $p = 5n + 1$ and edges $q = 9n.$ Let the function be $f: V \rightarrow [0,1]$ and assumption $z = 0.01$.would be represented as follows.

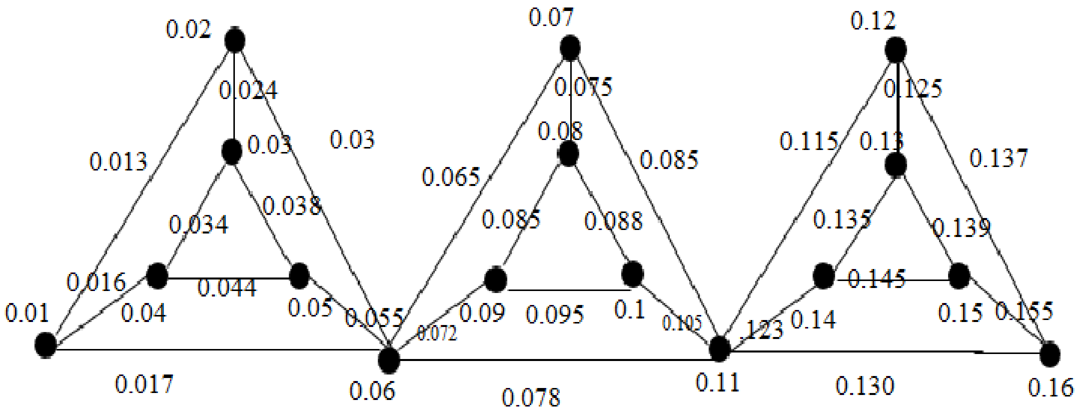

Fig 1.6 $FTPS_3$

It is observed from the above edges that μ $(u, v) \leq \sigma(u) \vee \sigma(v)$. Thus, $FTPS_3$ is a Modified Fuzzy labeling graph.

Corollary 3.

The Fuzzy Triangular Prism snake graph FTPS_n not admits fuzzy labeling graph.

Proof:

The Fuzzy Triangular Prism Snake graph by definition 3.1.6. Assign the vertices in the closed interval 0 to 1 and edges by harmonic mean with two end points, and as a result, obtain the edge label is less than or equal to the maxima of two end points and also distinct from each other.

ie, μ$(x, y) \leq \sigma(x) \wedge \sigma(y)$. Let the values for the vertices be labelled and the edge label be defined as a Harmonic mean value (i.e) μ$(u, v) = \frac{2\sigma(u)*\sigma(v)}{\sigma(u)+\sigma(v)}$ Where, μ(u, v) is not less than the minimum of two end vertices. This is a contradiction. The edge value obtained is greater than the value of one of the two nodes that it is incident with. Thus, the Triangular Prism snake graph is not fuzzy labeling graph.

Theorem 6.

A Fuzzy Double Triangular Prism Snake graph FDTPS_n admits a Modified fuzzy labeling graph.

Proof:

Let FG $(\sigma,$ μ) be a Double Triangular Prism snake graph by definition 3.1.7. We need to show that an edge label that is less than or equal to the maxima of two end points and that is also distinct from each other by assigning the vertices to the closed interval 0 to 1 and the edges by harmonic mean with two end points. Let it be considered that the vertices and edges p and q be $p = 9n + 1$ vertices and $q = 17n$ edges. Let us define a function $f: V(FG) \rightarrow [0,1]$ and assume z=0.01 as follows:

σ $(u_i) = (9i - 7)z$

σ $(v_i) = (9i - 8)z$

In this way, the rest of the vertices are constructed,

For edges, a function $f: E(FG) \rightarrow [0,1]$ is defined as follows,

μ$(u_i, v_i) = \frac{2\sigma(u_i)*\sigma(v_i)}{\sigma(u_i)+\sigma(v_i)} = \frac{2(9i-7)*(9i-8)}{(9i-7)+(9i-8)}$

μ$(u_i, x_i) = \frac{2\sigma(u_i)*\sigma(x_i)}{\sigma(u_i)+\sigma(x_i)} = \frac{2(9i-7)*(9i-6)}{(9i-7)+(9i-6)}$

Similarly, the rest of the edges are constructed.

From the above calculations, it is observed that all the edge weights are distinct and such that μ(u, v) $\sigma(u) \vee \sigma(v)$ $\forall\, u, v \in V$, which admits modified fuzzy labeling graph.

Theorem 7:

The Fuzzy Alternate Triangular Prism Snake $FATPS_n$ admits a Modified Fuzzy labeling graph.

Proof:

To prove that the Fuzzy Double Quadrilateral Snake is a modified fuzzy labeling graph, we must first assign values to its vertices and edges. To do this, we use the harmonic mean as our edge label, ensuring that the values obtained are distinct and satisfy the modified fuzzy graph definition. The resulting illustration of this process is shown here. Let F_G(σ, μ) be an Alternate Triangular prism snake. Let the vertices and edges respectively be $p= 6n$ vertices and $q = 10n - 1$. Let a functioned be defined as $f: V(F_G) \rightarrow [0,1]$ and an assumption be made as z=0.1. Then the following calculations can be made.

σ $(u_i) = (6i - 5)z$

σ $(v_i) = (6i - 4)z$

σ(v_i') = $(6i - 3)z$

σ $(x_i) = (6i - 2)z$

σ $(y_i) = (6i - 1)z$

σ $(z_i) = (6i)z$

For edges, a function $f: E(F_G) \rightarrow [0,1]$ is defined and calculations are made as follows,

$\mu(u_i, v_i)= \frac{2\sigma\,(u_i)*\sigma\,(v_i)}{\sigma\,(u_i)+\sigma\,(v_i)}$

$\mu(\,u_i, v_i')= \frac{2\sigma\,(u_i)*\sigma\,(v_i')}{\sigma\,(u_i)+\sigma\,(v_i')}$

$\mu(v_i, v_i')= \frac{2\sigma\,(v_i')*\sigma\,(u_i)}{\sigma\,(v_i')+\sigma\,(u_i)}$

$\mu(u_i, x_i)= \frac{2\sigma\,(u_i)*\sigma\,(x_i)}{\sigma\,(u_i)+\sigma\,(x_i)}$

$\mu(v_i, y_i)= \frac{2\sigma\,(u_i)*\sigma\,(y_i)}{\sigma\,(u_i)+\sigma\,(y_i)}$

$\mu(v_i', z_i)= \frac{2\sigma\,(v_i')*\sigma\,(z_i)}{\sigma\,(v_i')+\sigma\,(z_i)}$

$\mu(v_i', z_i)= \frac{2\sigma\,(v_i')*\sigma\,(z_i)}{\sigma\,(v_i')+\sigma\,(z_i)}$

$\mu(x_i, y_i)= \frac{2\sigma\,(x_i)*\sigma\,(y_i)}{\sigma\,(x_i)+\sigma\,(y_i)}$

From the above, it can be observed that all the edge weights are distinct and such that μ $(u, v) \leq \sigma\,(u) \vee \sigma\,(v)$ for all $u, v \in V$, $FATPS_n$, which admit a Modified Fuzzy labeling graph.

Theorem 8:

The Fuzzy Triple Triangular Prism Snake $FTTPS_n$ admits a Modified Fuzzy labeling graph.

Proof:

Let F_G(σ, μ) be a Triple Triangular Prism Snake graph. Let $p = 11n$ be the vertices and $q = 14n - 1$ be the edges. Let the function $f: V(F_G) \rightarrow [0,1]$ be defined and an assumption z=0.1 be made. Then calculations are performed as follows:

$\sigma\,(u_i) = (11i - 10)z$

$\sigma\,(v_i) = (11i - 9)z$

$\sigma(u_i{}') = (11i - 8)z$

$\sigma(v_i{}') = (11i - 7)z$

$\sigma(u_i{}'') = (11i - 2)z$

$\sigma(u_i{}''') = (11i - 1)z$

$\sigma\,(w_i) = (11i - 6)z$

$\sigma\,(x_i) = (11i - 5)z$

$\sigma\,(y_i) = (11i - 4)z$

$\sigma(y_i{}') = 11iz$

$\sigma(v_i{}'') = (11i - 3)z$

For calculating the edges, a function $f: E(F_G) \rightarrow [0,1]$ is defined and calculations are made as follows,

$\mu(u_i, v_i) = \frac{2\sigma\,(v_i{}') * \sigma\,(z_i)}{\sigma\,(v_i{}') + \sigma\,(z_i)}$

$\mu(\,v_i, u_i{}''') = \frac{2\sigma\,(v_i{}') * \sigma\,(z_i)}{\sigma\,(v_i{}') + \sigma\,(z_i)}$

$\mu(u_i{}', v_i{}') = \frac{2\sigma\,(v_i{}') * \sigma\,(z_i)}{\sigma\,(v_i{}') + \sigma\,(z_i)}$

Similarly, the rest of the edges are found from the above equations. It can be observed that all the edge weights are distinct and such $that\ \mu\,(u, v) \leq \sigma\,(u) \vee \sigma\,(v)$ for all $u, v \in V, FTTPS_n$, which admit a Modified fuzzy labeling.

Result 5.

- The Fuzzy Triangular Pentagon Snake graph FTP$S_n \odot K_1$ is a modified labeling fuzzy graph.
- The Fuzzy Alternate Triangular Pentagon Snake graph FAP$S_n \odot K_1$ is a modified labeling fuzzy graph

Theorem 9

The Fuzzy Quadrilateral snake graph FQS_n is a modified fuzzy labeling graph.

Proof:

Let $F_G(x_i, y_i)$ be a Fuzzy Quadrilateral snake FQS_n with $3n + 1$ nodes and $4n$ edges. Let a function $f: V(FG) \to [0,1]$ be made and an assumption $z = 0.1$ be made. The calculations are as follows:

$\sigma\ (x_i) = zi$

$\sigma\ (y_i) = (2i + 1)z$

For calculating the edges, a function $f: E(FG) \to [0,1]$ is defined and calculations are made as follows,

$\mu(x_i, x_{i+1})= \frac{2\sigma\ (x_i)*\sigma\ (x_{i+1})}{\sigma\ (x_i)+\sigma\ (x_{i+1})}$

$\mu(x_i, y_i)= \frac{2\sigma\ (x_i)*\sigma\ (y_i)}{\sigma\ (x_i)+\sigma\ (y_i)}$

Similarly $\mu(x_{i+1}, y_{i+1}), \mu(y_i, y_{i+1})$ can be found. Hence, the modified fuzzy labeling and all edges, which are distinct, are obtained.

Example:

Consider the FQS_2 vertices and and 8 edges, whose vertices are labelled as follows-

$\sigma\ (x_1) = z, \sigma\ (x_2) = 2z, \sigma\ (x_3) = 3z, \sigma\ (x_4) = 4z$

The edges are constructed as follows,

$\mu(x_1, x_2)= \frac{2\sigma\ (x_1)*\sigma\ (x_2)}{\sigma\ (x_1)+\sigma\ (x_2)}$=0.013

$\mu(x_1, y_1)= \frac{2\sigma\ (x_1)*\sigma\ (y_1)}{\sigma\ (x_1)+\sigma\ (y_1)}$=0.017

$\mu(y_1, y_2)= \frac{2\sigma\ (y_1)*\sigma\ (y_2)}{\sigma\ (y_1)+\sigma\ (y_2)}$=0.054

Similarly, all edge labels can be found. It is observed that all of them are distinct. It is found that all the edges are less than or equal to the maximum of two endpoints. Therefore, FQ$S_n$is a modified fuzzy graph.

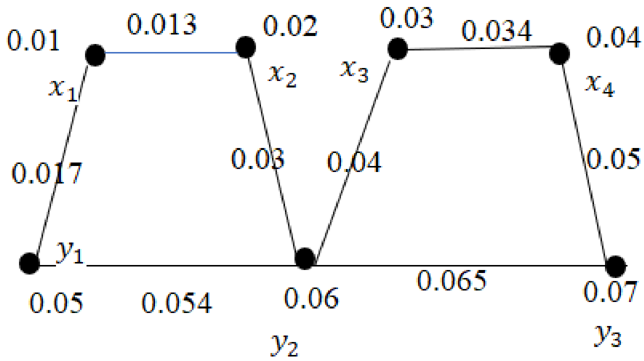

Fig 1.7 Modified Fuzzy Quadrilateral snake graph

Theorem 10

The Fuzzy Double Quadrilateral Snake FDQS_n graph is a modified fuzzy labeling graph.

Proof:

Let $F_G(x_i, y_i)$ be a double Quadrilateral Snake graph FDQS_n with $(5n+1)$ nodes and $7n$ edges. Let a function $f: V(FG) \to [0,1]$ be defined and an assumption that $z = 0.1$ be made. The calculations are carried out as follows:

$\sigma\,(x_i) = (5i-4)z$

$\sigma\,(y_i) = (5i-1)$

Similarly, it can be constructed for remaining vertices $\sigma\,(z_i), \sigma\,(w_i),$ and $\sigma\,(p_i)$.

For edges, a function $f: E(FG) \to [0,1]$ is defined and calculations are defined as follows,

$\mu(x_i, x_{i+1})= \frac{2\sigma\,(x_i)*\sigma\,(x_{i+1})}{\sigma\,(x_i)+\sigma\,(x_{i+1})}$

$\mu(x_i, y_i)= \frac{2\sigma\,(x_i)*\sigma\,(y_i)}{\sigma\,(x_i)+\sigma\,(y_i)}$

Similarly, $\mu(x_{i+1}, y_{i+1}), \mu(y_i, y_{i+1})$ are found. Hence, the modified fuzzy labeling and all the edges obtained are distinct.

Example:

Consider that FDQS_2 has 11 vertices and 14 edges. These vertices are labelled as below.

$\sigma\,(x_1) = 0.01, \sigma\,(x_2) = 0.06, \sigma\,(x_3) = 0.11, \sigma\,(y_1) = 0.02, \ldots$. and so on

to construct the edges as given below,

$\mu(x_1, x_2)= \frac{2\sigma\,(x_1)*\sigma\,(x_2)}{\sigma\,(x_1)+\sigma\,(x_2)}$=0.017

$\mu(x_1, y_1)= \frac{2\sigma\,(x_1)*\sigma\,(y_1)}{\sigma\,(x_1)+\sigma\,(y_1)}$=0.016

$\mu(y_1, z_1)= \frac{2\sigma\,(y_1)*\sigma\,(z_1)}{\sigma\,(y_1)+\sigma\,(z_1)}$=0.044

Similarly, all the edge labels are found and they are observed to be distinct. It is also noticed that all edges are less than or equal to the maximum of two endpoints. Therefore, the Double Quadrilateral snake is a Modified Fuzzy graph.

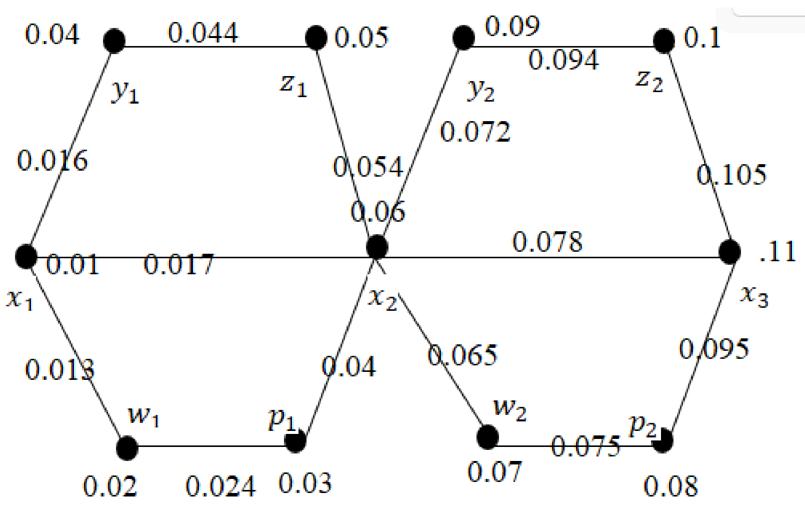

Fig 1.8 Modified fuzzy Double Quadrilateral snake graph

Lemma 2.

The Fuzzy graph $DQS_n \odot K_1$ is a Modified Labeling Fuzzy graph.

Proof:

Consider the FDQS_n graph with each of its vertices connected with the complete graph. It is defined as $FG = DQS_n \odot K_1$ and has $11n + 3$ vertices and $13n + 2$ edges. Let a function be defined and an assumption be made $\mu(u, v) = \begin{cases} z = 0.1, & if\ p \leq 6 \\ z = 0.01, & if\ p > 6 \end{cases}$ as follows:

$\sigma\,(x_i) = (11i - 10)z$

$\sigma\,(y_i) = (11i - 7)z$

Similarly, $\sigma\,(z_i), \sigma\,(w_i), \sigma\,(p_i)$, $\sigma\,(u_i)$, $\sigma\,(v_i)$, $\sigma\,(y_i')$, $\sigma\,(w_i')$, $\sigma\,(z_i')$, $\sigma\,(p_i')$ can be constructed.

For the edges, a function $f: E(G) \rightarrow [0,1]$ is defined as follows, $\sigma\,(w_1')$

$\mu(x_i, x_{i+1}) = \frac{2\sigma\,(x_i) * \sigma\,(x_{i+1})}{\sigma\,(x_i) + \sigma\,(x_{i+1})}$

$\mu(x_i, y_i) = \frac{2\sigma\,(x_i) * \sigma\,(y_i)}{\sigma\,(x_i) + \sigma\,(y_i)}$

Similarly, all the edge labels can be found. Hence, the modified fuzzy labeling in $DQS_n \odot K_1$ is obtained and all the edges are distinct.

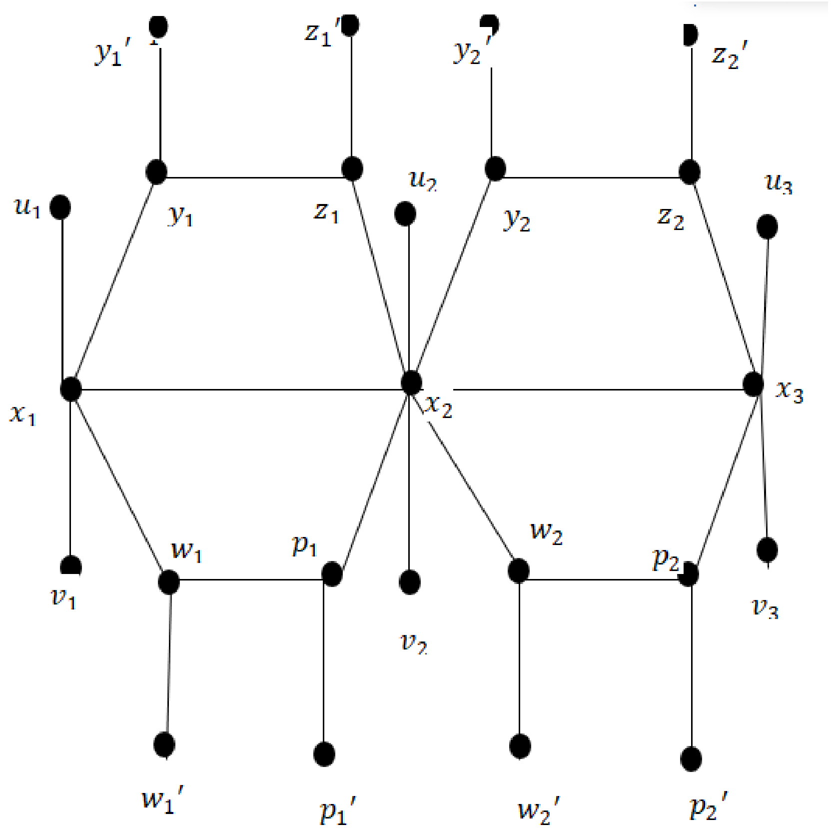

Fig 1.9 $DQS_n \odot K_1$

Lemma 3:

(i) The Fuzzy Quadrilateral Snake graph FQ$S_n \odot K_1$ is a modified labeling fuzzy graph.

(ii) A Fuzzy Alternate Quadrilateral Snake graph FAQ$S_n \odot K_1$ is a modified labeling fuzzy graph

(iii)The Fuzzy Quadrilateral snake graph FQ$S_n \odot K_2$ is a modified labeling fuzzy graph.

Application of Modified labeling in Real life

Using modified labeling, a student's future progress after the completion of his or her higher education can be predicted. The courses are represented by the vertices and their progress in higher education is indicated by the edges. The results showed that the critical path parameter had a significant impact on the overall performance of the fuzzy graph. The number of vertices also played a crucial role in determining the efficiency of the graph. Additionally, using geometric mean and harmonic mean for fuzzy graph labeling produced different results. While geometric mean produced better labeling results for smaller graphs, harmonic mean was more effective for larger graphs. Overall, this study sheds light on the importance of considering various parameters while analyzing graphs and highlights the need for further research in this area.

Consider, for instance, a class of three students who finished the first course and chose to enrol in the second. After finishing the second course, one of them goes straight to a job. The other student, also goes to a job but is not getting sufficient salary, and so, joins the third course, after the completion of which, got a job. The final student, successfully finishes the second and third courses, and then gets employed. The critical route approach was used to describe this scenario in the below-given illustration.

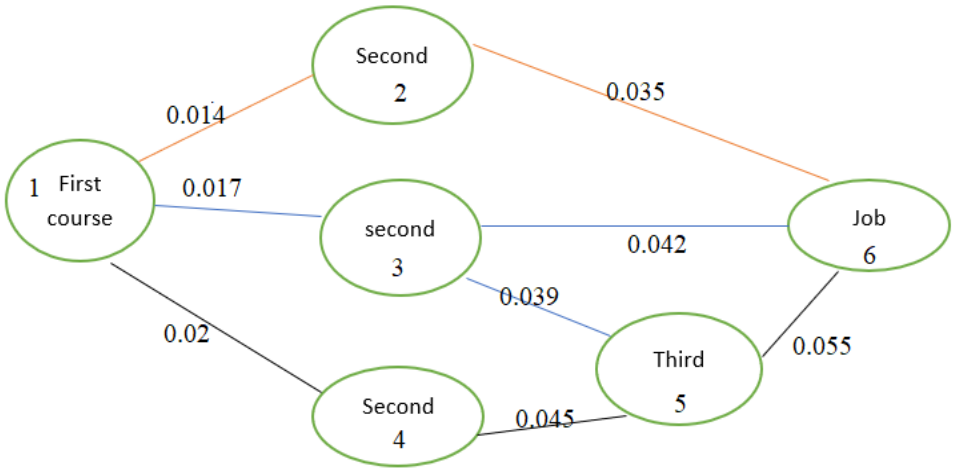

Fig 1.10. The Critical Route Approach for the Students' Progress Example

Vertices	Edges	Earliest		Latest		Total Float
		Begin	End	Begin	End	
1-2	0.014	0	0.014	0	0.085	0
1-3	0.017	0	0.017	0	0.026	0
1-4	0.02	0	0.02	0	0.02	0
3-5	0.039	0.017	0.065	0.026	0.065	0.09
4-5	0.045	0.02	0.065	0.02	0.065	0
2-6	0.035	0.014	0.12	0.085	0.12	0.071
3-6	0.042	0.017	0.12	0.026	0.12	0.09
5-6	0.055	0.065	0.12	0.065	0.12	0

Table 1.1

As shown in Table 1.1, a student who completes all the levels will progress well (get a good pay). 1-4-5-6 is the strongest level observed in the above diagram. The modified fuzzy labeling improves the efficiency of the graph-based approach by optimizing the techniques used for labeling, resulting in faster processing times and reduced computational complexity.

Conclusion

This study proposes a modified fuzzy labeling graph that is examined for some existing graphs. MFLG has been used extensively in various fields, including computer science, engineering, and biology. However, it suffers from some limitations, such as the lack of an efficient way to handle uncertainty and imprecision in the data. To address this issue, the new approach proposed by this study combines MFLG with two measures of central tendency: the geometric mean and harmonic mean. Our proposed method represents a significant improvement over existing methods for analyzing discrete data using MFLG. Here, the considered modified fuzzy labeling has an edge value, which is less than or equal to the maximum of two endpoints.

To get the result, the edge weight is assigned as Geometric labeling mean, where W(e) is equal to the square root of two endpoints. Some fuzzy graphs of TPS_n,DTPS_n, $ATPS_n$, $TTPS_n$ and families of Quadrilateral snake graph are discussed using the Harmonic mean labeling for the edges. Some fuzzy corona graph such as $APS_n \odot K_1$AP$S_n \odot K_2$, $TPS_n \odot K_1$, ATP$S_n \odot K_1$, which admit the modified fuzzy graph are used. The results thereof are obtained and an application is illustrated also illustrated. The current study can be further extended by working on the properties and operations of modified fuzzy graph. Modified magic and anti-magic labeling using modified fuzzy graph can also be investigated.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

No data was used for the research described in the article.

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