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Seperation Axioms in Micro Semi Pre-Topological Spaces

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Abstract: The primary objetive of this paper is to introduce a new type of separation axioms namely, micro semi pre-T_0, micro semi pre-T_1, micro semi pre-T_2 and to obtain their properties. Also, the relationship between these spaces with related concepts are investigated.

Introduction: In 2019, S. Chandrasekar [3] introduction of micro topology as a straightforward expansion from nano topology, emphasizing micro pre-open and micro semi-open sets. H.Z. Ibrahim [6] introduced micro β-open sets in micro topological spaces. In 2020, Hariwan Z. Ibrahim [7] introduced the separation axioms on micro topology with particular focus on micro-open sets. In this paper some new types of spaces are define and study in micro topological spaces namely, micro semi pre T_0, micro semi pre T_1, micro semi pre T_2, using the concept of micro semi pre open sets. Also, the properties of these spaces and the relationship between these spaces with related concepts are found.

Conclusions: In this paper introduces and investigates new types of micro semi pre topological spaces, namely micro semi pre T_0, micro semi pre T_1, and micro semi pre T_2, extending concepts from previous works on micro topology. The properties and relationships of these spaces are explored, shedding light on their separation axioms and connections with related concepts like micro-T_0, micro-T_1, and micro-T_2 spaces. The study presented here not only expands the theoretical framework of micro topology but also provides insights into the interplay between different types of micro topological spaces. Overall, this paper contributes to the ongoing development of micro topology by introducing and analyzing micro semi pre topological spaces, leading the way for further research in this area and potentially finding applications in various mathematical and scientific disciplines.

In Future, connecting separation axioms in topology to the behavior of an RLC circuit within micro topology provides a rigorous mathematical framework to distinguish and analyze different states and transitions. This approach not only enhances our understanding of the circuit's dynamics but also offers a robust method to explore micro particle interactions and configurations within a topological space.

Keywords: micro α-T_k space (where k = 0, 1, 2) and micro β-T_k space (where k = 0, 1, 2).

1. Introduction

Generalized closed sets, initialized by Levine [7] in 1970, laid the groundwork for various advancements in topological studies. Lellis Thivagar [2] expanded upon this by introducing Nano topology, which utilizing approximations and boundary regions to define closed sets, interior and closure using nano open sets. In 1975, Maheshwari and Prasad [11] Ashish Kar and Bhattacharyya [1] gave the idea about the weak separation axioms. In 2019, A notion of Separation Axioms on Nano Topological Spaces introduced by Sathishmohan et al. [8] established the concepts of separation axioms for nano semi pre-open sets and acquired interesting findings.

In 2019, S. Chandrasekar [3] introduction of micro topology as a straightforward expansion from nano topology, emphasizing micro pre-open and micro semi-open sets. H.Z. Ibrahim [6] introduced micro β-open sets in micro topological spaces. In 2020, Hariwan Z. Ibrahim [7] introduced the separation axioms on micro topology with particular focus on micro-open sets. In this paper some new types of spaces are define and study in micro topological spaces namely, micro semi pre T_0, micro semi pre T_1, micro semi pre T_2, using the concept of micro semi pre open sets. Also, the properties of these spaces and the relationship between these spaces with related concepts are found.

2. Preliminaries

The following outlines essential concepts and prerequisites required for the progression of this work.

Definition 2.1.[2] Let U be a non-empty finite set of objects called the universe R be an equivalence relation on U named as the indiscerniblity relation. Elements belonging to the same equivalence class are said to be indiscernible with one another. The pair (U, R) is said to be the approximation space. Let $X \subseteq U$.

(i) The Lower approximation of X with respect to R is the set of all objects, which can be for certain classified as X with respect to R and it is denoted by $L_R(X)$. That is, $L_R(X) = \{\bigcup_{x\in U} \{R(x): R(x) \subseteq X\}\}$, where $R(x)$ denotes the equivalence class determined by x.

(ii) The Upper approximation of X with respect to R is the set of all objects, which can be for certain classified as X with respect to R and it is denoted by $U_R(X)$. That is, $U_R(X) = \{\bigcup_{x\in U} \{R(x): R(x) \cap X \neq \phi\}\}$

(iii) The Boundary region of X with respect to R is the set of all objects which can be classified as neither as X nor as not X with respect to R and it is denoted by $B_R(X)$. That is, $B_R(X) = U_R(X) - L_R(X)$

Definition 2.2. [2] Let U be the universe; R be an equivalence relation on U and $\tau_R(X) = \{U, \phi, L_R(X), U_R(X), B_R(X)\}$ where $X \subseteq U$. $\tau_R(X)$ satisfies the following axioms:

1. U and $\phi \in \tau_R(X)$

2. The union of elements of any sub collection of $\tau_R(X)$ is in $\tau_R(X)$.

3. The intersection of the elements of any finite sub collection of $\tau_R(X)$ is in $\tau_R(X)$

That is, $\tau_R(X)$ forms a topology on U is called the nano topology on U with respect to X. We call $\{U, \tau_R(X)\}$ is called the nano topological space.

Definition 2.3. [3] $(U, \tau_R(X))$ is a Nano topological space here $\mu_R(X) = \{N \cup (N' \cap \mu): N, N' \in \tau_R(X)\}$ and called it micro topology of $\tau_R(X)$ by μ where $\mu \notin \tau_R(X)$

Definition 2.4. [3] The Micro topology $\mu_R(X)$ satisfies the following axioms.

1. U and $\phi \in \mu_R(X)$

2. The union of elements of any sub collection of $\tau_R(X)$ is in $\tau_R(X)$.

3. The intersection of the elements of any finite sub collection of $\mu_R(X)$ is in $\mu_R(X)$

Then $\mu_R(X)$ is called Micro topology on U with respect to X. The triplet $(U, \tau_R(X), \mu_R(X))$ is called Micro topological spaces and the elements of $\mu_R(X)$ are called Micro open sets and the complement of a Micro open set is called a Micro closed set.

Definition 2.5. [5] Let $(U, \tau_R(X), \mu_R(X))$ be a micro topological space and $A \subseteq U$. Then A is called micro β-open set if A ⊆ Mic-cl(Mic-int(Mic-cl(A))). The complement of micro β-open set is called micro β-closed.

Definition 2.6. [9] If $(U, \tau_R(X), \mu_R(X))$ be a micro topological space if A ⊆ U, then

(i) The union of all micro β-open sets contained in A is called the micro β-interior of A and is denoted by Mic-βint(A).

(ii) The intersection of all micro β-closed sets containing A is called the micro β-closure of A and denoted by Mic-βcl(A).

Definition 2.7.[6] Let $(U, \tau_R(X), \mu_R(X))$ be a micro topological space. Then U is said to be:

(1) Micro T_0 if for each pair of distinct points x, y in U, there exist a micro-open set L such that either $x \in$ L and $y \notin$ L or $x \notin$ L and $y \in$ L
(2) Micro T_1 if for each pair of distinct points x, y in U, there exist two micro-open sets L and K such that $x \in$ L but $y \notin$ L and $y \in$ K but $x \notin$ K
(3) Micro T_2 if for each distinct points x, y in U, there exist two disjoint micro-open sets L and K containing x and y respectively.

Definition 2.8. [10] Let $(U, \tau_R(X), \mu_R(X))$ be a micro topological space. Then

(1) A space U is said to be micro semi-T_0 (resp. micro pre- T_0) if for each couple of distinct points $x, y \in$ U, ∃ a micro semi-open (resp. micro pre-open) set H such that either $x \in$ H and $y \notin$ H or $x \notin$ H and $y \in$ H and micro semi semi-T_0 is denoted by Mic-ST_0 and micro pre-T_0 is denoted by Mic-PT_0.
(2) A space U is said to be micro semi-T_1 (resp. micro pre-T_1) space, for each copule of distinct points x, y in U, ∃ micro semi-open (resp. micro pre-open) sets H and I such that $x \in$ H, $y \notin$ H and

$y \in$ I, $x \notin$ I, where micro semi -T_1 is denoted by Mic-ST_1 and micro pre-T_1 as Mic-PT_1.

(3) A space U is said to be micro semi-T_2 (resp. micro pre-T_2) space, for each copule of distinct points x, y in U, ∃ a disjoint micro semi-open (resp. micro pre-open) sets sets H and I where micro semi- T_2 is denoted by Mic-ST_2 and micro pre-T_2 as Mic-PT_2.

3. Micro Semi pre-T_0 Space

This section discusses a few characteristics of micro α-T_0 space and micro semi pre-T_0 space. Also studied the comparison between micro semi pre-T_0 space with micro-T_0, micro semi-T_0, micro pre-T_0 spaces.

Definition 3.1. Let ((U,τ_R(X), μ_R(X)) be a micro topological space. Then U is said to be:

(i) Micro α-T_0 (or Mic- αT_0) if for each pair of distinct points a, b in U, ∃ a micro α-open set such that either $a \in Q$ and $b \notin Q$ or $a \notin Q$ and $b \in Q$.

(ii) Micro β-T_0 (or Mic- βT_0) if for each pair of distinct points a, b in U, ∃ a micro β-open set such that either $a \in Q$ and $b \notin Q$ or $a \notin Q$ and $b \in Q$.

Example 3.2. Let $U = \{\gamma, \beta, \eta, \zeta\}$, $U/R = \{\{\gamma\}, \{\zeta\}, \{\delta, \eta\}\}$, $X = \{\gamma, \delta\}$, $\tau_R(X) = \{\phi, U, \{\gamma\}, \{\delta, \eta\}, \{\gamma, \delta, \eta\}$ and $\mu_R(X) = \{\phi, U, \{\gamma\}, \{\delta\}, \{\gamma, \delta\}, \{\delta, \eta\}, \{\gamma, \delta, \eta\}\}$ be a micro topology on U. We have

$MSO(U,X) = \phi, U, \{\gamma\}, \{\delta\}, \{\gamma, \delta\}, \{\gamma, \zeta\}, \{\delta, \eta\}, \{b, d\}, \{\gamma, \delta, \eta\}, \{\gamma, \delta, \zeta\}, \{\delta, \eta, \zeta\}$

$MPO(U,X) = \{\phi, U, \{\gamma\}, \{\delta\}, \{\gamma, \delta\}, \{\delta, \eta\}, \{\gamma, \delta, \eta\}, \{\gamma, \delta, \zeta\}$.

$M\alpha O(U,X) = \{\phi, U, \{\gamma\}, \{\delta\}, \{\gamma, \delta\}, \{\delta, \eta\}, \{\gamma, \delta, \eta\}, \{\gamma, \delta, \zeta\}$.

$M\beta O(U,X) = \{\phi, U, \{\gamma\}, \{\delta\}, \{\gamma, \delta\}, \{\gamma, \zeta\}, \{\delta, \eta\}, \{b, d\}, \{\gamma, \delta, \eta\}, \{\gamma, \delta, \zeta\}, \{\delta, \eta, \zeta\}$

(1) Mic-αT_0 space: Let $a = \{\gamma\}$ and $b = \{\eta\}$, a, b are distinct points in U, ∃ a micro α-open set $G = \{\gamma, \delta, \zeta\} \ni a \in Q, b \notin Q$

(2) Mic-βT_0 space: Let $a = \{\gamma\}$ and $b = \{\eta\}$, a, b are distinct points in U, ∃ a micro δ-open set $Q = \{\gamma, \delta, \zeta\} \ni a \in Q, b \notin Q$

Theorem 3.3. ((U,τ_R(X), μ_R(X)) be a micro topological space then every micro T_0 (resp. Semi T_0, Pre-T_0, Mic-αT_0) spaces is Mic-βT_0 space

Proof: Let U be a Mic-T_0 space and a, b be two different points of U, as U is Mic-T_0 space ∃ a micro-open set Q ∋ $a \in Q$ and $b \notin Q$. But every micro-open (resp. semi open, pre open, α-open) set is micro β-open set. Hence Q is micro semi pre-open set such that $a \in Q$, $b \notin Q$. Therefore, U be a Mic-βT_0 space.

The converse of the above theorem is not generally true, as demonstrated in the following example.

Example 3.4. Let $U = \{\gamma, \delta, \eta, \zeta\}$, $U/R = \{\{\gamma\}, \{\eta\}, \{\delta, \zeta\}\}$, $X = \{\eta\}$, $\tau_R(X) = \{\phi, U, \{\eta\}$ and μ_R(X) $= \{\phi, U, \{\eta\}, \{\gamma, \zeta\}, \{\gamma, \eta, \zeta\}$ be a micro topology on U. We have

$MSO(U,X) = \{\phi, U, \{\eta\}, \{\gamma, \zeta\}, \{\delta, \eta\}, \{\gamma, \delta, \zeta\}, \{\gamma, \eta, \zeta\}$

$MPO(U,X) = \{\phi, U, \{\gamma\}, \{\eta\}, \{\zeta\}, \{\gamma, \eta\}, \{\gamma, \zeta\}, \{\eta, \zeta\}, \{\gamma, \delta, \eta\}, \{\gamma, \eta, \zeta\}, \{\delta, \eta, \zeta\}\}$.

$M\alpha O(U,X) = \{\phi, U, \{\eta\}, \{\gamma,\zeta\}, \{\gamma,\eta,\zeta\}$

$M\beta O(U,X) = \{\phi, U, \{\gamma\}, \{\eta\}, \{\zeta\}, \{\gamma,\delta\}, \{\gamma,\eta\}, \{\gamma,\zeta\}, \{\delta,\eta\}, \{\delta,\zeta\}, \{\eta,\zeta\}, \{\gamma,\delta,\eta\}, \{\gamma,\delta,\zeta\},$

$\{\gamma,\eta,\zeta\}, \{\delta,\eta,\zeta\}\}$.

Mic-βT_0 space: Let $a = \{\gamma\}$ and $b = \{\zeta\}$, a, b are distinct elements in U, ∃ a micro β-open set $G = \{\gamma,\delta,\eta\} \ni a \in G, b \notin G$. So U is Mic-β$T_0$ space but U is not Mic-T_0 and Mic-ST_0 space.

Example 3.5. Consider the micro topology given in the example, 3.2. Let $a = \{\gamma\}$ and $b = \{\zeta\}$, a, b are distinct elements in U, ∃ a micro β-open set $Q = \{\gamma,\delta,\eta\} \ni a \in Q, b \notin Q$. So U is Mic-β$T_0$ space but U is not Mic-T_0 and Mic-ST_0 space.

Theorem 3.6. A micro topological space U is Mic-βT_0 space iff the micro semi pre closure of any two distinct singletons are distinct.

Proof: Let a, b ∈ U and $a \neq b$ with U as Mic-βT_0 space. We have to show that Mic-βcl{x} ≠ Mic-βcl{b}. Consider the set $A = U - \{a\}$, it is clear that Mic-cl(A) is either A or U. If Mic-cl(A) = A then A is Mic-closed and hence Mic β-closed. Therefore $U - A = \{a\}$ is a Mic β-open set which contains a but not b. So, a not in Mic-βcl{b}. But a ∈ Mic-βcl({a}) and hence Mic-βcl({a}) ≠ Mic-βcl({b}). If Mic-cl(A) = U, then A is Mic β-open so $U - A = \{a\}$ is Mic β-closed. Therefore Mic-βcl({a}) = {a}. Since $y \notin \{a\}$ and b ∈ Mic-βcl({b}), it follows that Mic-βcl({a}), Mic-βcl({b}) are distinct.

Conversely, assume that a and b are distinct singletons in U in which their micro semi pre closure are distinct. Therefore ∃ c ∈ U for which c ∈ Mic-βcl({a}) but c ∉ Mic-βcl({b}). We claim that a not in Mic-βcl({b}). For if a ∈ Mic-βcl({b}) then Mic-βcl({a}) is a subset of Mic-βcl({b}) and this implies c ∈ Mic-βcl({b}). This contradicts the fact that c ∉ Mic-βcl({b}). Therefore, our assumption is wrong. (i.e.,) a not in Mic-βcl({b}) implies a ∈ U − Mic-βcl({b}) and Mic-βcl({b}) is a Mic β-open set containing a but not b. Hence U is Mic-βT_0 space.

Theorem 3.7. Let $(U, \tau_R(X), \mu_R(X))$be a micro topological space. If U is Mic-βT_0 space and V is a subspace of U then V is also Mic-βT_0 space.

Proof: Assume U be Mic-βT_0 space. To show that V is Mic-βT_0 space. Choose distinct points a, b from V and $a \neq b$. Since V ⊆ U, we have $a, b \in U$. But U is Mic-βT_0 space, so ∃ a micro semi pre-open set H ∋ $a \in H$ and $b \notin H$. Then V ∩ L is a micro semi pre-open set in V ∋ $a \in V \cap L$ and $b \notin V \cap L$. Hence V is also a Mic-βT_0 space.

4. Micro Semi Pre - T_1 Space

This section discusses a few characteristics of micro α-T_1 space and micro semi pre-T_1 space. Also to study the comparison between micro semi pre-T_1 space with micro-T_1, micro semi-T_1, micro pre-T_1 space and obtain their basic results.

Definition 4.1. Let (U, τ_R(X), μ_R(X)) be a micro topological space. Then, U is said to be:

(i) Micro α-T_1 (or Mic-αT_1) if for for each pair of distinct points x, y in U, there exist two micro α-open sets Q and R such that $x \in$ Q but $y \notin$ Q and $y \in$ R but $x \notin$ R.

(ii) Micro β-T_1 (or Mic-βT_1) if for for each pair of distinct points x, y in U, there exist two micro β-open sets Q and R such that $x \in$ Q but $y \notin$ Q and $y \in$ R but $x \notin$ R.

Example 4.2. Let $U = \{\gamma, \delta, \eta, \zeta\}$, $U/R = \{\{\delta\}, \{\gamma, \eta, \zeta\}\}$, $X = \{\delta\}$, $\tau_R(X) = \{\phi, U, \{\delta\}\}$ and $\mu_R(X) = \{\phi, U, \{\delta\}, \{\gamma, \eta\}, \{\gamma, \delta, \eta\}\}$ be a micro topology on U. We have

$MSO(U, X) = \{\phi, U, \{\delta\}, \{\gamma, \eta\}, \{\delta, \zeta\}, \{\gamma, \delta, \eta\}, \{\gamma, \eta, \zeta\}$

$MPO(U, X) = \{\phi, U, \{\gamma\}, \{\delta\}, \{\eta\}, \{\gamma, \delta\}, \{\gamma, \eta\}, \{\delta, \eta\}, \{\gamma, \delta, \eta\}, \{\gamma, \delta, \zeta\}, \{\delta, \eta, \zeta\}$.

$M\alpha O(U, X = \{\phi, U, \{\delta\}, \{\gamma, \eta\}, \{\gamma, \delta, \eta\}$

$M\beta O(U, X) = \{\phi, U, \{\gamma\}, \{\delta\}, \{\eta\}, \{\gamma, \delta\}, \{\gamma, \eta\}, \{\gamma, \zeta\}, \{\delta, \eta\}, \{\delta, \zeta\}, \{\eta, \zeta\}, \{\gamma, \delta, \eta\}, \{\gamma, \delta, \zeta\}$,

$\{\gamma, \eta, \zeta\}, \{\delta, \eta, \zeta\}$.

(1) Mic-γT_1 space: Let $a = \{\gamma\}$ and $b = \{\delta\}$ are distinct points in U, ∃ distinct micro α-open sets $Q = \{\gamma, \eta\}$ and $R = \{\delta\} \ni a \in R$ but $b \notin Q$ and $b \in R$ but $a \notin Q$.

(2) Mic-δT_1 space: Let $a = \{\gamma\}$ and $b = \{\eta\}$ are distinct points in U, ∃ distinct micro β-open sets $Q = \{\gamma, \delta\}$ and $R = \{\eta, \zeta\} \ni a \in Q$ but $b \notin R$ and $b \in Q$ but $a \notin R$.

Theorem 4.3. Let (U, τ_R(X), μ_R(X)) be a micro topological space then every micro T_1 (resp. semi-T_1, pre-T_1, αT_1) spaces is Mic-βT_1 space.

Proof: Let U be a Mic-T_1 space and let a, b be two distinct points in U. Then by hypothesis, ∃ distinct micro-open sets Q and R $\ni a \in Q$ and $b \in R$. As Every micro-open set is micro β-open (resp. semi-open, pre-open, α-open) set and hence Q and R are distinct micro β-open sets ∋ $a \in Q$ and $b \in R$. Therefore, U be a Mic-βT_1 space.

Converse of the above theorem is need not be true in general is shown in the following example.

Example 4.4. Consider the micro topology given in the example 4.2.

(1) Let $a = \{\gamma\}$ and $b = \{\eta\}$, are distinct points in U, ∃ a distinct micro β-open sets $Q = \{\gamma, \delta\}$ and $R = \{\delta, \eta, \zeta\} \ni$ a ∈ Q, $b \notin R$ and b ∈ R, $a \notin Q$. So U is Mic-βT_1 space but not Mic-T_1 (Mic-semi T1, Mic-αT1) space.

(2) Let $a = \{\delta\}$ and $b = \{\zeta\}$ are distinct points in U, ∃ a distinct micro β-open sets $Q = \{\gamma, \delta, \eta\}$ and $Q = \{\delta, \eta, \zeta\} \ni a \in Q$, $b \notin R$ and $b \in R$, $a \notin R$. So U is Mic-βT_1 space but not Mic-pre T_1 space.

Theorem 4.5. Let $(U, \tau_R(X), \mu_R(X))$ be a micro topological space. Then U is Mic-βT_1 space if and only if the singletons are micro closed sets.

Proof: Let U be Mic-βT_1 space and x is any point of U. Suppose $y \in U - \{x\}$, then $x \neq y$ and so there exist a micro β-open set L such that $y \in L$ but $x \notin L$. Consequently $y \in L \subseteq U - \{x\}$, that is $U - \{x\} = \cup \{L: y \in U - \{x\}\}$ which is micro β-open.
Conversely, suppose {x} is micro closed for every x ∈ U. Let x, y ∈ U with $x \neq y$ implies $y \in U - \{x\}$. Hence $U - \{x\}$ is a micro β-open set contains y but not x. Similarly, $U - \{y\}$ is a micro β-open set contains x but not y. Accordingly U is Mic-βT_1 space

Theorem 4.6. A a micro topological space $(U, \tau_R(X), \mu_R(X))$ is Mic-βT_1 space iff each one-point set is Mic-β-closed.

Proof: Assume U is Mic-βT_1 space. Let a ∈ U and {a} is a singleton subset of U. Therefore, for every $b \in U - \{a\}$, ∃ a Mic β-open set V ∋ $b \in V$ and $a \notin V$, the sets {a} and V are disjoint and $V \subseteq U - \{a\}$. Thus $b \in V \subseteq U - \{a\}$. So, $U - \{a\}$, which is a Mic-β-open set which implies {a} is Mic-β-closed set. Conversely, assume that each one-point set is Mic-β-closed.

Conversely, let a, b are distinct points in U. By hypothesis, we get two Mic-β-open sets $U - \{a\}$ and $U - \{b\}$ containing a and b respectively. so, $a \in U - \{b\}$ and $a \notin U - \{a\}$; $b \in U - \{a\}$ and $b \notin U - \{b\}$. Therefore $(U, \tau_R(X), \mu_R(X))$ is Mic-βT_1 space.

Theorem 4.7. Let $(U, \tau_R(X), \mu_R(X))$be a micro topological space. Every subspace of Mic-βT1 space is Mic-βT_1 space.

Proof: Let U be Mic-βT_1 space. To show that V is Mic-βT_1 space.

Let V be a subspace of U. Let a, b are distinct points in V. Since V ⊆ U, we have a, b are points of U. But U is Mic-βT1 space, so ∃ a micro β- open sets Q and R ∋ $x \in Q$ but $y \notin Q$ and $y \in R$ but $a \notin R$. Let $I = V \cap Q$ and $J = V \cap R$. Then I and J are micro β- open sets a ∈ I but $b \notin I$ and $b \in R$ but $a \notin R$. Hence V is Mic-βT_1 space.

5. Micro Semi Pre - T_2 Space

This section is to define and study a few characteristics of micro α-T_2 space and micro semi pre-T_2 space. Also, the comparison between micro semi pre-T_2 space with micro-T_2, micro semi-T_2, micro pre-T_2 space and obtain their basic results.

Definition 5.1. Let $(U, \tau_R(X), \mu_R(X))$ be a micro topological space. Then U is said to be:

(i) Micro α-T_2 (or Mic-αT_2) if for each distinct points a, b in U, there exists two disjoint micro α-open sets Q and R containing a and b respectively.

(ii) Micro β-T_2 (or Mic-βT_2) if for each distinct points a, b in U, there exists two disjoint micro β-open sets Q and R containing a and b respectively.

Example 5.2. Let $U = \{\gamma, \delta, \eta, \zeta\}$, $U/R = \{\{\delta\}, \{\gamma, \eta, \zeta\}\}$, $X = \{\delta\}$, $\tau_R(X) = \{\phi, U, \{\delta\}\}$ and $\mu R(X) = \{\phi, U, \{\delta\}, \{\gamma, \eta\}, \{\gamma, \delta, \eta\}\}$ be a micro topology on U. Then

$MSO(U, X) = \{\phi, U, \{\delta\}, \{\gamma, \eta\}, \{\delta, \zeta\}, \{\gamma, \delta, \eta\}, \{\gamma, \eta, \zeta\}$

$MPO(U, X) = \{\phi, U, \{\gamma\}, \{\delta\}, \{\eta\}, \{\gamma, \delta\}, \{\gamma, \eta\}, \{\delta, \eta\}, \{\gamma, \delta, \eta\}, \{\gamma, \delta, \zeta\}, \{\delta, \eta, \zeta\}$.

$M\alpha O(U, X) = \{\phi, U, \{\delta\}, \{\gamma, \eta\}, \{\gamma, \delta, \eta\}$

$M\beta O(U, X) = \{\phi, U, \{\gamma\}, \{\delta\}, \{\eta\}, \{\gamma, \delta\}, \{\gamma, \eta\}, \{\gamma, \zeta\}, \{\delta, \eta\}, \{\delta, \zeta\}, \{\eta, \zeta\}, \{\gamma, \delta, \eta\}, \{\gamma, \delta, \zeta\},$

$\{\gamma, \eta, \zeta\}, \{\delta, \eta, \zeta\}$.

(1)Mic-αT_2 space: Let $a = \{\gamma\}$ and $b = \{\delta\}$, are distinct points in U, ∃ disjoint micro α-open sets $Q = \{\gamma, \eta\}$ and $R = \{\delta\}$ ∋ $a \in Q$ and $b \in R$.

(2)Mic-βT$_2$ space: Let $a = \{\gamma\}$ and $b = \{\eta\}$ are distinct points in U, ∃ disjoint micro β-open sets $Q = \{\gamma, \delta\}$ and $R = \{\eta, \zeta\} \ni a \in Q$ and $b \in R$.

Theorem 5.3. Let $(U, \tau_R(X), \mu_R(X))$ be a micro topological space then every micro-T$_2$ (resp. micro semi-T$_2$, micro pre-T$_2$, micro α-T$_2$) space is Mic-βT$_2$ space

Proof: Let U be a Mic-T$_2$ space and let a and b are distinct elements in U. Then ∃ disjoint micro- open sets Q and R ∋ $a \in Q$ and $b \in R$. As Every micro-open set is micro β-open (semi-open, pre-open, α-open) set and hence Q and R are disjoint micro β-open sets ∋ $a \in Q$ and b ∈ R. Therefore, U be a Mic-βT$_2$ space.

The following example illustrates that the converse of the above theorem does not hold true in general.

Example 5.4. Consider the micro topology given in the example 5.2

(1) Let $a = \{\gamma\}$ and $b = \{\eta\}$, are distinct points in U, ∃ a disjoint micro β-open sets $Q = \{\gamma\}$ and $R = \{\delta, \eta, \zeta\} \ni a \in Q, b \notin R$ and $b \in R, a \notin Q$. So U is Mic-βT$_2$ space but not Mic-T$_2$ (Mic-semi T$_2$ and Mic-αT$_2$) space.

(2) Let $a = \{\beta\}$ and $b = \{\zeta\}$, are distinct points in U, ∃ a disjoint micro β-open sets $Q = \{\beta\}$ and $R = \{\gamma, \eta, \zeta\} \ni a \in Q, b \notin R$ So U is Mic-βT$_2$ space but not Mic-pre T$_2$ space.

Theorem 5.5. Let $(U, \tau_R(X), \mu_R(X))$ be a micro topological space. Every subspace of Mic-βT$_2$ space is Mic-βT$_2$ space.

Proof: Let U be Mic-βT$_2$ space. To show that V is Mic-βT$_2$ space.

Let V be a subspace of U. Let a, b are distinct elements in V. Since V ⊆ U, we have a, b ∈ U.

But U is Mic-βT$_2$ space, so ∃ disjoint Mic β-open sets Q and R ∋ $x \in Q$ and $y \in R$. Thus, we have Q ∩ V, R ∩ V are Mic β-open sets in V ∋ $(Q \cap V) \cap (R \cap V) = (Q \cap R) \cap V = \emptyset \cap V = \emptyset$. Also, $x \in Q \cap V$ and $y \in R \cap V$. Hence V is Mic-βT$_2$ space.

Theorem 5.6. $(U, \tau_R(X), \mu_R(X))$ be a micro topological space. ThenEvery Mic-βT$_2$ (resp. Mic-αT$_2$) space is Mic-βT$_1$ (resp. Mic-αT$_1$) space.

(1) Every Mic-βT$_1$ (resp. Mic-αT$_1$) space is Mic-βT$_0$ (resp. Mic-αT$_0$) space.

(2) Every Mic-semi T$_1$ space (resp. Mic-pre T$_1$) is Mic-βT$_0$ space.

(3) Every Mic-semi T$_2$ space (resp. Mic-pre T$_2$) is Mic-βT$_1$ space.

Proof: (1) and (2) The proof is straightforward from the definitions.

(3) Let U is Mic-semi T$_1$ (resp. Mic-pre T$_1$) space. Let a, b are distinct pints in U.

Thus, there is distinct micro semi-open (resp. micro pre-open) sets Q and R ∋ a ∈ Q but $b \notin Q$ and b ∈ R but a ∉ R. Every micro semi-open (resp. Mic pre-open) set is micro β-open set. So, Q and R are micro β-open sets ∋ either a ∈ Q but b ∉ Q or $b \notin R$ but a ∉ R. So U is Mic-βT$_0$ space.

(4) Let U is Mic-semi T$_2$ (resp. Mic-pre T$_2$) space. Let a, b are distinct points in U. Thus, there is

disjoint micro semi-open (resp. micro pre-open) sets Q and R ∋ $a \in Q$ but $b \notin Q$ and $b \in R$ but $a \notin R$. Every micro semi-open (resp. Mic pre-open) set is micro β-open set. So, Q and R are distinct micro β-open sets ∋ either a ∈ Q but $b \notin R$ or $b \notin R$ but $a \notin Q$. So U is Mic-βT_1 space.

Remark: The following figure illustrates the relationship between separation axioms and other micro semi pre separation axioms derived from the above results.

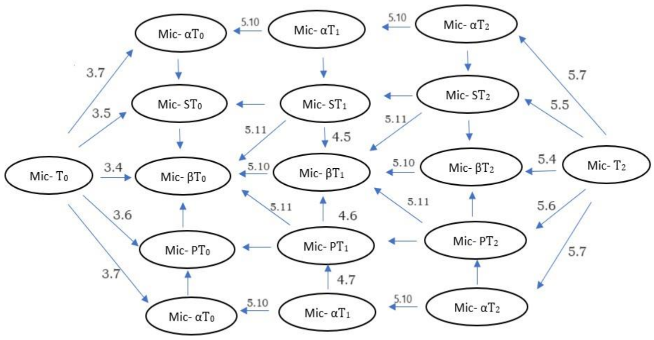

FIGURE 1. Relationship Between micro-Separation axioms

Refrences

[1] Ashish Kar and P. Bhattacharyya, Some Weak Separation Axioms, Bulletin of the Calcutta Mathematical Society, 82, 1990, 415 - 422. https://zbmath.org/0736.54013

[2] Lellis Thivagar. M and Carmel Richard, On Nano Forms of Weakly Open Sets, International Journal of Mathematics and Statistics Invention, 1(1), 2013, 31-37.

[3] Chandrasekar.S, On Micro–Topological Spaces, Journal of new theory 26, 2019, 23 - 31.

[4] Chandrasekar.S and Swathi.S, Micro α-open Sets in Micro–Topological Spaces, International Journal of Research in Advent Technology, 6(10), 2018, 2633-2637.

[5] Hariwan Z. Ibrahim, Micro β-open sets in Micro-Topology, General Letters in Mathematics, 8(1), 2020, 8-15.

[6] Hariwan Z. Ibrahim, Micro Seperation Axioms, International Journal of Analysis and Applications 18(4), 2020, 572 - 585.

[7] Levine. N, Generalized Closed sets in Topology, Rendiconti del Circolo Matematico di Palermo, 19(2), 1970, 89-96.

[8] Sathishmohan. P, Rajendran. V, Vignesh Kumar. C, Dhanasekaran. P.K, A Few Separation Axioms on Nano Topological Spaces, nternational Journal of Recent Technology and Engineering, 7(5), 2019, 461 - 463.

[9] Sathishmohan. P, Poongothai. G, Rajalakshmi. K and Thiruchelvi. M, On Micro-Semi Pre-Operators in Micro Topological Spaces, Indian Journal of Natural Sciences, 15(84), 2024, 74900-74907.

[10] Sathishmohan. P, Stanley Roshan. S, Rajalakshmi.K, Poongothai. G, Mythili. S, Micro Pre-Separation Axioms in Circuit of Light System on Four Stroke Engines (Communicated)

[11] Maheshwari S. N and Prasad. R, On s-normal spaces, Bulletin Mathematique de la Societe des Sciences Mathematiques de la Republique Socialiste de Roumanie, 22(70), 1978, 27 - 29.