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A Novel Approach to Hexagonal Fuzzy Number Ranking in Multi-Objective Fuzzy Linear Programming Problem Solving

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Abstract:

This paper presents a ranking process based on hexagonal fuzzy numbers (HFNs) applied to linear programming multi-objective problems with fuzzy measurements. By utilizing this ranking method, any fuzzy linear multi-objective programming problem can be converted into a classical value problem, facilitating the determination of a suitable exact solution. This method offers valuable insights for planners operating in uncertain organizational environments. In an society where numerous opportunities and variables such as manufacturing, inventory, commercial management, costing, and miscellaneous other parameters are involved, this ranking procedure proves to be an efficient approach. The paper includes a computational example to represent the method and provide inferences. Additionally, the well-known Gauss–Seidel iterative method for solving linear systems of equations is discussed.

Introduction: In the realm of linear programming multi-objective problems, decision-makers frequently encounter uncertainty, which can be effectively modelled using fuzzy numbers. Hexagonal fuzzy numbers (HFNs) are particularly adept at representing such uncertainties due to their flexible and detailed structure. The proposed ranking techniques for HFNs provides a systematic approach to convert fuzzy MOLPP into crisp value problems, thereby simplifying the process of finding optimal solutions.

By employing the proposed ranking method, hexagonal fuzzy numbers can be efficiently ranked and compared. This transformation enables the application of traditional optimization techniques to resolve the equivalent crisp problem. A suitable example further illustrates the practical application of this method, demonstrating its effectiveness in deriving actionable insights in complex decision-making scenarios. The paper incorporates the well-known Gauss–Seidel iterative method for solving linear systems of equations, showcasing its integration with the proposed ranking procedure for enhanced problem-solving capabilities.

Objectives: This paper presents a ranking process based on hexagonal fuzzy numbers (HFNs) applied to linear programming multi-objective problems with fuzzy measurements. By utilizing this ranking method, any fuzzy linear multi-objective programming problem can be converted into a classical value problem, facilitating the determination of a suitable exact solution.

Methods: By employing the proposed ranking method, hexagonal fuzzy numbers can be efficiently ranked and compared. This transformation enables the application of traditional optimization techniques to resolve the equivalent crisp problem. A suitable example further illustrates the practical application of this method, demonstrating its effectiveness in deriving actionable insights in complex decision-making scenarios. The paper incorporates the well-known Gauss–Seidel iterative method for solving linear systems of equations, showcasing its integration with the proposed ranking procedure for enhanced problem-solving capabilities.

Results: Based on the table above, we have achieved identical outcomes using both the current approach and the proposed hexagonal WARI ranking method. This technique minimizes solution ambiguity, offering to management an optimal basis for decisions regarding product production or alternative adjustments. It remains effective even with increased variables and parameters, proving simpler and more efficient than prior methods. Furthermore, its applicability extends seamlessly to numerous input parameters. Compared to the existing ranking method, our proposed hexagonal WARI ranking method is equally optimized.

Conclusion: This method allows for the straightforward conversion of hexagonal fuzzy numbers to crisp numbers. We suggest that this proposed method is both easy to use and an excellent additional ranking method.

Keywords: Proposed ranking method, Hexagonal fuzzy numbers, Decision making process, Gauss-Seidel method.

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1. Introduction

In the realm of linear programming multi-objective problems, decision-makers frequently encounter uncertainty, which can be effectively modelled using fuzzy numbers. Hexagonal fuzzy numbers (HFNs) are particularly adept at representing such uncertainties due to their flexible and detailed structure. The proposed ranking techniques for HFNs provides a systematic approach to convert fuzzy MOLPP into crisp value problems, thereby simplifying the process of finding optimal solutions.

By employing the proposed ranking method, hexagonal fuzzy numbers can be efficiently ranked and compared. This transformation enables the application of traditional optimization techniques to resolve the equivalent crisp problem. A suitable example further illustrates the practical application of this method, demonstrating its effectiveness in deriving actionable insights in complex decision-making scenarios. The paper incorporates the well-known Gauss–Seidel iterative method for solving linear systems of equations, showcasing its integration with the proposed ranking procedure for enhanced problem-solving capabilities.

A new approach for solving Fuzzy transportation problems employing a ranking function by [1] Amarpreet K et al. [2] introduced the technique for addressing the transportation problem. They employed hexagonal Fuzzy numbers using the Centroid positioning strategy. Chen et al. [3] have investigated that is not momentum for enrolment work concerning ordinary Fuzzy numbers. [4] utilized the area based Centroid on the distance strategy for ranking. He introduced the ides of generalized Fuzzy numbers. The positioning of ordinary Fuzzy numbers was initially presented by Jain [5]. Zadeh [6] pioneered the introduction of the fuzzy concept. Malini et al. [7] introduced a positioning method utilizing octagonal Fuzzy numbers. [8] investigated the distance strategy by employing a positioning method based on the Centroid. Rajarajeswari et al. [9] suggested a method involving generalized Fuzzy numbers based on region, mode, difference, spread, and tackled the issue using generalized hexagonal Fuzzy numbers. Chu et al. and Wang et al. Roseline et al. [10] introduced a positioning method for generalized trapezoidal Fuzzy numbers. Singh Pushpinder [11] introduced an innovative strategy for ranking Fuzzy numbers in generalized concept. [12] introduced a revised approach for positioning Fuzzy numbers, incorporating a gap in between the Centroid and a specific point.

2. Objectives

This paper presents a ranking process based on hexagonal fuzzy numbers (HFNs) applied to linear programming multi-objective problems with fuzzy measurements. By utilizing this ranking method, any fuzzy linear multi-objective programming problem can be converted into a classical value problem, facilitating the determination of a suitable exact solution.

3. Methods

3.1 Proposed Ranking of Hexagonal Fuzzy Number

The centroid of fuzzy number is conceptualized as the equilibrium point of the hexagon. This hexagon is partitioned into five distinct plane figures: a triangle denoted as ABQ, REF, QBC and RDE. Another space marked as QCDR.

The centroid of the five-plane figure is,

G1 = $\left(\frac{a1+a2+a3}{3}; \frac{w}{6}\right)$, G2 = $\left(\frac{a4+a5+a6}{3}; \frac{w}{6}\right)$, $G3 = \left(\frac{a2+2a3}{3}; \frac{w}{2}\right)$, G4 = $\left(\frac{a5+2a4}{3}; \frac{w}{2}\right)$,

G5 = $\left(\frac{a3+a4}{2}; \frac{w}{2}\right)$

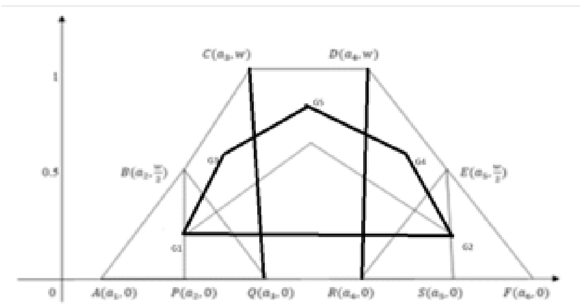

Figure 1 Hexagonal Fuzzy Number

$R(AH) = (x_0)(y_0) = \frac{(2a1+4a2+9a3+9a4+4a5+2a6)}{30} * \frac{11w}{30}$ -------------- (1)

3.2 Technique for Solving Fuzzy Linear Programming Problems with Multiple Objectives:

This paper explores a multi-objective fuzzy linear programming problem within constraints featuring fuzzy coefficients. Additionally, the objectives addressed in this study encompass a mixture of both maximization and minimization criteria.

We solve into a model whose standard form

Maximize $Z_1 = C_1 x$

Minimize $Z_2 = C_2 x$

Subject to

$A_{11} x \leq b$, $x \geq 0$,

Where $C_{ij} = (C_{i1}, C_{i2}, C_{i3} \ldots C_{in})$ is an n dimensional crisp row vector, $A_H = a_{ij}$ is an mxn fuzzy matrix.

$b = (b_1, b_2, b_3 \ldots b_m)^T$ is an m dimensional fuzzy line vector and $x = (x_1, x_2, x_3 \ldots x_n)^T$ is an n dimensional decision variable vector.

We examine a bi-objective fuzzy linear programming problem with constraints characterized by fuzzy coefficients, represented as

Maximize $Z_1 = C_{11}x_1 + C_{12}x_2 + \ldots + C_{1n}x_n$

Minimize $Z_2 = C_{21}x_1 + C_{22}x_2 + \ldots + C_{2n}x_n$

Subject to

$a_{i1}x_1 + a_{i2}x_2 + a_{i3}x_3 + \ldots + a_{in}x_n \leq b_i$

$x_1, x_2, x_3 \ldots x_n \geq 0$, $i = 1,2,3 \ldots m$

Where fuzzy number are hexagonal then,

$a_{i1} = a_{i11}, a_{i12}, a_{i13}, a_{i14}, a_{i15}, a_{i16}$

$a_{i2} = a_{i21}, a_{i22}, a_{i23}, a_{i24}, a_{i25}, a_{i26}$

…

…

$a_{in} = a_{in1}, a_{in2}, a_{in3}, a_{in4}, a_{in5}, a_{in6}$

$b_i = b_{i1}, b_{i2}, b_{i3}, b_{i4}, b_{i5}, b_{i6}$ ------------------------- (2)

The MOFLPP has been converted into a MOLPP according to the ranking algorithm.

Maximize $Z_1 = C_{11}x_1 + C_{12}x_2 + \ldots + C_{1n}x_n$

Minimize $Z_2 = C_{21}x_1 + C_{22}x_2 + \ldots + C_{2n}x_n$

Subject to

$\frac{1}{30}$ $[2(a_{11}x_1 + a_{21}x_2 + a_{31}x_3 + a_{41}x_4 + a_{51}x_5 + a_{61}x_6) +$

$4(a_{12}x_1 + a_{22}x_2 + a_{32}x_3 + a_{42}x_4 + a_{52}x_5 + a_{62}x_6) +$

$9(a_{13}x_1 + a_{23}x_2 + a_{33}x_3 + a_{43}x_4 + a_{53}x_5 + a_{63}x_6) +$

$9(a_{14}x_1 + a_{24}x_2 + a_{34}x_3 + a_{44}x_4 + a_{54}x_5 + a_{64}x_6) +$

$4(a_{15}x_1 + a_{25}x_2 + a_{35}x_3 + a_{45}x_4 + a_{55}x_5 + a_{65}x_6) +$

$2(a_{16}x_1 + a_{26}x_2 + a_{36}x_3 + a_{46}x_4 + a_{56}x_5 + a_{66}x_6)$

$\leq 2b_{i1} + 4b_{12} + 9b_{i3} + 9b_{i4} + 4b_{i5} + 2b_{i6}$] * $[\frac{11w}{30}]$

$x_1, x_2, x_3 \ldots x_n \geq 0$, i=1,2,3…m ------------------ (3)

3.3 Solve a set of simultaneous linear equations:

The Gauss–Seidel method, alternatively known as the Liebmann method or the method of successive displacement, belongs to the domain of numerical linear algebra. It serves as an iterative technique employed for solving systems of linear equations. Iterative methods like the Gauss-Seidel method provide users with control over round-off errors. Moreover, when the physics of the problem is well understood, initial guesses required in iterative methods can be selected more strategically, resulting in quicker convergence.

$a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + \ldots + a_{1n}x_n = C_1$

$a_{21}x_1 + a_{22}x_2 + a_{23}x_3 + \ldots + a_{2n}x_n = C_2$

…

$a_{n1}x_1 + a_{n2}x_2 + a_{n3}x_3 + \ldots + a_{nn}x_n = C_n$

This kind of system frequently lends itself to an iterative approach, wherein it is initially reformulated into a specific form.

$x_1 = \frac{1}{a11}(C_1 - a_{12}x_2 - a_{13}x_3 + \ldots - a_{1n}x_n)$

$x_2 = \frac{1}{a22}(C_2 - a_{21}x_1 - a_{23}x_3 + \ldots - a_{2n}x_n)$

…

$X_n = \frac{1}{ann}(C_n - a_{n1}x_1 - a_{n2}x_2 + \ldots - a_{n,n-1}x_{n-1})$

Initially let us assume that $x_2=x_3=\ldots=x_n=0$ and find x_1. Let it be x_1*. Putting x_1* for x_1 and $x_3=x_4=\ldots=x_n=0$, we get the value for x_2. In this way we can find the approximation value.

$x_1^* = \frac{1}{a11}(C_1 - a_{12}x_2 - a_{13}x_3 + \ldots - a_{1n}x_n)$

$x_2^* = \frac{1}{a22}(C_2 - a_{21}x_1^* - a_{23}x_3 + \ldots - a_{2n}x_n)$

…

$X_n = \frac{1}{ann}(C_n - a_{n1}x_1^* - a_{n2}x_2^* + \ldots - a_{n,n-1}x_{n-1}^*)$ ----- (4)

Using (2) and (3), this can be converted into single objective problem subject to the constraints with transformed crisp number coefficients and hence solved accordingly. Similarly, multi-objective problems with more than two objectives can also be solved using the above procedure, here in very first stage itself the problem is transformed into a crisp problem by using proposed hexagonal ranking number named Hexagonal WARI's ranking method and afterword's using (4) to solve a set of simultaneous linear equations by Gauss-Seidel method.

3.4 Algorithm to solve multi-objective fuzzy linear programming problem

Step 1: we choose the multi-objective fuzzy linear programming problem.

Step 2: We transform the selected multi-objective fuzzy linear programming problem into the subsequent hexagonal fuzzy linear programming problem.

Step 3: Using the order scale above equation (2), first transform the LP values, which are all hexagonally fuzzy in an objective function and constraints, to clear values.

Step 4: Using WARI's approach (1), the LP problem with crisp value can be solved by applying the suggested hexagonal ranking.

Step 5: To solve the simultaneous linear equations by using the Gauss–Seidel method (4) to identify the optimal solution.

3.5 Working Example:

A manufacturing company has recently halted the production of a particular product due to unfavourable market conditions, leading to a significant excess in production capacity. To make use of this spare capacity, the company plans to boost the production of one or two of its remaining products. The currently available capacities are:

Milling capacity: (12000,13000,14000,14000,15000,16000) hours

Lathe capacity: (11500,12000,12500,12500,13000,13500) hours

The number of machine hours required for each of the products are,

Machine type	Machine hours required	
	Product A	Product B
Milling	(180,190,200,200,210,220)	(230,240,250,250,260,270)
Lathe	(280,290,300,300,310,320)	(180,190,200,200,210,220)

The maximum profit realized from each of the two products are Rs.75, Rs.90 and the minimum profit realized from each of the two products are Rs.60, Rs.75. The production manager aims to allocate the available capacities between the two products to achieve both maximum and minimum profit.

The above-mentioned example is a production planning process.

Maximize $Z_1 = 75x_1 + 90x_2$

Minimize $Z_2=60x_1+75x_2$

Subject to,

$a_{11}\ x_1+a_{12}\ x_2 \le b_1$

$a_{21}\ x_1+a_{22}\ x_2 \le b_2$

$x_1,x_2 \ge 0$

Where,

a_{11} = (180,190,200,200,210,220)

a_{12} = (230,240,250,250,260,270)

a_{21} = (280,290,300,300,310,320)

a_{22} = (180,190,200,200,210,220)

b_1 = (12000,13000,14000,14000,15000,16000)

b_2 = (11500,12000,12500,12500,13000,13500)

Subject to,

(180,190,200,200,210,220) x_1 + (230,240,250,250,260,270) $x_2 \le$ (12000,13000,14000,14000,15000,16000)

(280,290,300,300,310,320) x_1 + (180,190,200,200,210,220) $x_2 \le$ (11500,12000,12500,12500,13000,13500)

$x_1,x_2,x_3,x_4,x_5,x_6 \ge 0$

By (3), this can be transformed into a crisp LPP as,

Maximize Z1= $75x_1+90x_2$

Minimize Z2=$60x_1+75x_2$

Subject to,

$73.33x_1+91.67x_2 \le 5133.33$

$110x_1+73.33x_2 \le 4583.33$

By using (4) Gauss Seidel method to solve the above equations

$x_1=\frac{1}{110}(4583.33-73.33\ x_2)$

$x_2=\frac{1}{91.67}(5133.33-73.33\ x_1)$

Iteration	$x_1=\frac{1}{110}(4583.33-73.33\ x_2)$	$x_2=\frac{1}{91.67}(5133.33-73.33\ x_1)$
1	X_1= 41.67	X_2=22.66
2	X_1= 26.56	X_2=34.75
3	X_1= 18.50	X_2=41.20

4	X_1= 14.20	X_2=44.64
5	X_1= 11.91	X_2=46.47
6	X_1= 10.69	X_2=47.45
7	X_1= 10.03	X_2=47.90
8	X_1= 9.68	X_2=48.26
9	X_1= 9.49	X_2=48.41
10	X_1= 9.39	X_2=48.49
11	X_1= 9.34	X_2=48.53
12	X_1= 9.31	X_2=48.55
13	X_1= 9.3	X_2=48.56
14	**X_1= 9.29**	**X_2=48.57**
15	**X_1= 9.29**	**X_2=48.57**

In the above iteration 14 & 15 are repeated and similar values.

The solutions are x_1= 9.29 & x_2= 48.57

Maximize $Z_1= 75x_1+90x_2$

Max Z_1= 5068.05

Minimize $Z_2=60x_1+75x_2$

Min Z_2= 4200.15

3.6 Table: Comparison of results obtained by using existing and proposed Hexagonal WARI's ranking method

Method	X1	X2	Max Z1	Min Z2
Existing method	9.29	48.57	5068.05	4200.15
Proposed Hexagonal WARI's ranking method	9.29	48.57	5068.05	4200.15

4 Results

Based on the table above, we have achieved identical outcomes using both the current approach and the proposed hexagonal WARI ranking method. This technique minimizes solution ambiguity, offering to management an optimal basis for decisions regarding product production or alternative adjustments. It remains effective even with increased variables and parameters, proving simpler and more efficient than prior methods. Furthermore, its applicability extends seamlessly to numerous input parameters. Compared to the existing ranking method, our proposed hexagonal WARI ranking method is equally optimized.

5 Discussion

This method allows for the straightforward conversion of hexagonal fuzzy numbers to crisp numbers. We suggest that this proposed method is both easy to use and an excellent additional ranking method.

Refrences

[1] Amarpreet, K., and Amit, K. "A New Method for Solving Fuzzy Transportation Problems Using Ranking Function," Applied Mathematical Modelling, 2011, 35(12), pp. 5652-5661.

[2] Annie, S., and Malini, D. "Solving Transportation Problems with Hexagonal Fuzzy Numbers Using BCM and Different Ranking Techniques," International journal of Engineering research and Application, 2016, 6(2), pp. 76-81.
[3] Chen, S. "Operations on Fuzzy Numbers with Function Principal," Tamkang, Journal of management sciences, 1985, 6, pp. 13-25
[4] Chu, T. , and Tsao, C. " Ranking of Fuzzy Numbers with an Area between the Centroid and Original Points," Computers and Mathematics with application, 2002, 43, pp. 111-117.
[5] Jain, R. "Decision Making in the Presence of Fuzzy Variables," IEEE Transactions on systems, Man and Cybernetics, 1976, 6, pp. 698-703.
[6] L.A. Zadeh, Fuzzy sets, Inf. Control 8 (1965) 338–353.
[8] Malini, P., and Ananthnarayan, M. "Solving Fuzzy Transportation Problem Using Ranking of Octagonal Fuzzy Numbers," International journal of pure and applied Mathematics, 2016, 110(2), pp. 275-282.
[9] Phanibhushan, R., and Ravishankar, N. "Ranking Fuzzy Numbers with Distance Method Using Circumcentre of Centroids and an Index of Modeling, "Advances in fuzzy system, 2011, 10, pp. 1155-1161.
[10] Rajarajeshwari, P., and Sudha, S. "Ordering Generalized Hexagonal Fuzzy Numbers Using Rank, Mode, Divergence and Spread," IOSR Journal Mathematics, 2014, 10(3), pp. 15-22.
[11] Roseline, S., and Amitraj, H. "GFHM to GTFTP with Ranking of GFN," International journal of applied mathematics, 2014, 3(1), pp. 5-12.
[12] Singh P. " A Novel Method for Ranking Generalized Fuzzy Numbers, Journal of Information Sciences and Engineering, 2015, 3, pp. 1373-1385
[13] Wang, Y., and Lee, S. "The Revised Method of Ranking Fuzzy Numbers with an Area between Cetroid and Original Points," Computers and Management with Application, 2008, 55 (9), pp. 2033-2042.