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Fuzzy Coloring and Total Fuzzy Coloring to Strong Intuitionistic Fuzzy Graphs – Euler, Hamiltonian and Petersen Graphs

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Abstract:

A pioneering contemplate of fuzzy coloring and total fuzzy coloring is incited to a strong intuitionistic fuzzy Euler graph, strong intuitionistic fuzzy Hamiltonian graph and to a strong intuitionistic fuzzy Petersen graph. The fuzzy chromatic number along with its fuzzy chromatic index of both vertices and edges and the total fuzzy chromatic number along with its total fuzzy chromatic index are examined by coloring to the vertices and edges. The concept of fuzzy coloring makes it easier to identify a color's name and its solidity at the vertices and edges. This coloring gives a path of a graph.

Keywords: fuzzy coloring, total fuzzy coloring, chromatic number, fuzzy chromatic number, fuzzy chromatic index.

1. Introduction

Francis Guthrie, since 1852, raised with the coloring conjecture in graph theory. Coloring has been moving with several fields of areas like, telecommunications, networking etc,. Graph coloring travels with the coloring of its vertices and its edges. The ideas of fuzzy coloring and total fuzzy coloring [5] are instigated to the vertices and edges of a strong intuitionistic fuzzy Euler graph, strong intuitionistic fuzzy Hamiltonian graph and to a strong intuitionistic fuzzy Petersen graph and then the results of chromatic number, fuzzy chromatic number along with its fuzzy chromatic index and the total chromatic number, total fuzzy chromatic number along with its total fuzzy chromatic index are arrived. The fuzzy coloring concept assists in the finding the fuzzy color solidity apllied at the vertices and edges validly.

1.1 Preliminaries

Definition 1.1.

$\hat{G} = (V, E)$is an intuitionistic fuzzy graph (IFG), where

(1) $V = \{v_1, v_2, \dots, v_n\} \ni \mu_1 : V \to [0,1]$ and $v_1 : V \to [0,1]$ notate levels of membership and non-membership of an element $\mu_i \in V$ and $v_i \in V$ and $0 \le \mu_i + v_i \le 1, \forall v_i \in V, (i = 1,2, \dots, n)$.

(2) $E \subseteq V \times V$ where $\mu_{ij} : V \times V \to [0,1]$ and $v_{ij}: V \times V \to [0,1] \ni \mu_{ij} \leq \min\{\mu_i, \mu_j\}, v_{ij} \leq \max\{v_i, v_j\}$ and $0 \leq \mu_{ij}(v_i, v_j) + v_{ij}(v_i, v_j) \leq 1 \ \forall (v_i, v_j) \in E$.

Here triplets (v_i, μ_i, v_i) and $(e_{ij}, \mu_{ij}, v_{ij})$ notate levels of the membership and non-membership of vertex v_i and the relation between the edges $e_{ij} = (v_i, v_j)$ on $V \times V$.

Definition 1.2.

The family $\Gamma = \{\gamma_1, \dots, \gamma_k\}$ of fuzzy sets on V is called a k - fuzzy coloring of

$\hat{G} = (V, \sigma, \mu)$ if

(1) $\vee \Gamma = 0$

(2) $\gamma_i \wedge \gamma_j = 0$

(3) $\forall\, xy$, the strong edge of $\hat{G}$, $\wedge \{\gamma_i(x), \gamma_j(y)\} = 0, (1 \leq i \leq k)$

Definition 1.3.

The family $\Gamma = \{\gamma_1, \dots, \gamma_k\}$ of intuitionistic fuzzy sets on V [9] is called a k - vertex coloring of $\hat{G} = (V, E)$ if

(1) $\vee \gamma_i(x) = V \ \forall\, x \in V$

(2) $\gamma_i \wedge \gamma_j = 0$

(3) $\forall\, xy$, strong edge of $\hat{G}$, $\min\{\gamma_i(\mu_1(x)), \ \gamma_i(\mu_1(y))\} = 0$; $\max\{\gamma_i(v_i(x)), \ \gamma_i(v_i(y))\} = 1, (1 \leq i \leq k)$.

The minimal utility of k of $\hat{G}$ having k - vertex coloring notated as $\chi(\hat{G})$, implies vertex chromatic number of an IFG $\hat{G}$.

Definition 1.4.

The family $\Gamma = \{\gamma_1, \dots, \gamma_k\}$ are of intuitionistic fuzzy sets on E, the k - edge coloring of $\hat{G} = (V, E)$ if

(1) $\vee \gamma_i(xy) = E \ \forall\, xy \in E$

(2) $\gamma_i \wedge \gamma_j = 0$

(3) $\forall\, xy$, incident edges of E, $\min\left\{\gamma_i\left(\mu_{ij}(xy)\right)\right\} = 0$; $\max\left\{\gamma_i\left(v_{ij}(xy)\right)\right\} = 1, (1 \leq i \leq k)$.

The minimal utility of k of $\hat{G}$ having k- edge coloring notated as $\chi'(\hat{G})$, implies edge chromatic number of an IFG $\hat{G}$.

Definition 1.5.

The family $\Gamma = \{\gamma_1, \dots, \gamma_k\}$ are of intuitionistic fuzzy sets on V and E, k - total coloring of $\hat{G} = (V, E)$ when

(1) $\vee \gamma_i(x) \vee \gamma_i(xy) = V \vee E \ \forall\, x \in V, xy \in E$

(2) $\gamma_i \wedge \gamma_i = 0$

(3) $\forall\, xy$, strong and incident vertices and edges of $\hat{G}$, $\min\left\{\gamma_i(\mu_i(x)), \gamma_i(\mu_i(y)), \gamma_i\left(\mu_{ij}(xy)\right)\right\} = 0$; $\max\{\gamma_i(v_i(x), \gamma_i(v_i(y), \gamma_i(v_{ij}(xy))\} = 1, (1 \leq i \leq k)$.

In $\hat{G}$, the minimal utility of k having a k- total coloring notated as $\chi^T(\hat{G})$, implies total chromatic number of an IFG $\hat{G}$.

Definition 1.6.

A path in a graph which covers each and every edge of the graph exactly once [1], and but may repeat the covering of the vertices is called as an Euler Graph.

A path in a graph which covers each and every vertices of the graph exactly once, and but may repeat the covering of the edges is called as a Hamiltonian Graph.

A graph with 10 vertices and 15 edges is called as a Petersen Graph which is most commonly a pentagon with five crossbars inside.

Definition 1.7.

A fuzzy Euler graph, Hamiltonian graph and Petersen graph, $E_G = (\sigma, \mu), H_G = (\sigma, \mu)\ and\ P_G = (\sigma, \mu)$ are a set of functions $\sigma: V_s \to [0,1]$ and $\mu: V_s \times V_s \to [0,1], S = 1,2, \dots, n$ where $\forall u, v \in V_s$ and $\mu(u, v) \leq \sigma(u) \wedge \sigma(v)$ for $(u, v) \in E_T, T = 1,2, \dots, n$.

Definition 1.8.

$E_{\hat{G}} = (V_s, E_T), H_{\hat{G}} = (V_s, E_T)\ and\ P_{\hat{G}} = (V_s, E_T)$ notate a strong Intuitionistic Fuzzy Euler Graph, Hamiltonian Graph and Petersen Graph.

(1) $V_S = \{v_1, v_2, \dots . v_n\}$ such that $\mu_i: V_S \to [0,1]$ and $\nu_i: V_S \to [0,1]$ notate the degrees of membership and non-membership of the element $v_i \in V_S$ respectively.

And $0 \leq \mu_i(v_i) + \nu_i(v_i) \leq 1, \forall v_i \in V_S, (i = 1,2, \dots, n)$.

(2) $E_T \subset V_S \times V_S$ where $\mu_{ij}: V_S \times V_S \to [0,1]$ and $\nu_{ij}: V_S \times V_S \to [0,1] \ni \mu_{ij} \leq min\{\mu_i, \mu_j\}, \nu_{ij} \leq max\{\nu_i, \nu_j\}$ and $0 \leq \mu_{ij}(v_i, v_j) + \nu_{ij}(v_i, v_j) \leq 1\ \forall (v_i, v_j) \in E_T$.

The triplets (v_i, μ_i, ν_i) and $(e_{ij}, \mu_{ij}, \nu_{ij})$ notate levels of the membership and non-membership of vertex v_i and the relation between edges $e_{ij} = (v_i, v_j) on\ V_S \times V_S$.

Definition 1.9.

An Intuitionistic Fuzzy Euler Graph, Hamiltonian Graph and Petersen Graph, $E_{\hat{G}} = (V_S, E_T), H_{\hat{G}} = (V_S, E_T)\ and\ P_{\hat{G}} = (V_S, E_T)$ is called a Strong Intuitionistic Fuzzy Euler Graph, Hamiltonian Graph and Petersen Graph if $\mu_{ij} = \min\{\mu_i, \mu_j\}$ and $\nu_{ij} = \max\{\nu_i, \nu_j\}\ \forall (v_i, v_j) \in E_T;\ v_i, v_j \in V_s$.

2. Fuzzy Coloring and Total fuzzy Coloring in a Strong Intuitionistic Fuzzy - Euler Graph, Hamiltonian Graph, Petersen Graph

To proceed with the fuzzy coloring and total fuzzy coloring methodology to an IFG, some distinct fuzzy colors [4] are considered. Generally, a new color is obtained with the combination of any two colors. Mixing a white color with any of the distinct color, its reliability reduces, but the nature of the color doesn’t transform to another color. Here the term reliability is the fuzzy term. Generally, no two adjacent vertices are to be colored with the same distinct color in a graph. By reducing the solidity of the color, the adjacent vertices are colored using the alike distinct color if the edges are weak between the two vertices or there are no edges between them. This concept of coloring is called as fuzzy coloring. The color obtained by mixing white color with the distinct color is the fuzzy color. Now, considering a distinct color C_K and $\omega(\leq 1)$ color units C_K is mixed with $1-\omega$ white color units, that blend is called a quality blend of the color C_K and that consequent color what obtained is the fuzzy color of the color C_K with a membership value ω.

Definition 2.1.

The distinct colors [12] stated by $\hat{C} = \{c_1, c_2, \ldots, c_n\}, n \geq 1$. A fuzzy set $(\hat{C}, f)$, a fuzzy color set $f: \hat{C} \rightarrow [0,1]$, with $f(c_i)$, the membership value of the color c_i, amount of the color used per unit of quality blend. Tuning of a fuzzy color $\tilde{c}_i = (c_i, f(c_i))$ gives the distinct fuzzy color c_i with various solidaties. A fuzzy color has a membership value 1.

Fuzzy colors can be continued more in number with a single distinct color with different levels of white mix level giving outputs as different mild shades of that distinct color. For an instance, if red is to be the color, a "fuzzy red" color is easily formed by mingling a color red of 0.9 units with a color white of 0.1 unit. The "fuzzy red" color shall be noted as (red,0.9) or the red color can also be denoted as 'R' and in this case, it can be denoted as fuzzy red color as (R,0.9). Likewise, another fuzzy red color (R,0.8) is created by mingling a color red of 0.8 units and a color white of 0.2 units, and so forth, (red,1) or (R,1) represents the fuzzy red color. This fuzzy coloring can be generally denoted as (R,0.a),(R,0.b),(R,0.c) and so on.

To the strong intuitionistic fuzzy Euler graph, strong intuitionistic fuzzy Hamiltonian graph and a strong intuitionistic fuzzy Petersen graph, fuzzy coloring and total fuzzy coloring is done in the following two ways in consideration of the edges with all strong edges and no edges or there is weak edge in the strong intuitionistic fuzzy Euler graph, strong intuitionistic fuzzy Hamiltonian graph and strong intuitionistic fuzzy Petersen graph.

Case 1: Strong Intuitionistic Fuzzy - Euler Graph, Hamiltonian Graph, Petersen Graph with all Strong Edges

A strong intuitionistic fuzzy Euler graph, strong intuitionistic fuzzy Hamiltonian graph and a strong intuitionistic fuzzy Petersen graph with all strong edges, the fuzzy coloring to the adjacent vertices are fuzzy colored with two distinct fuzzy colors. For example, (G,1) and (R,1).

Case 2: Strong Intuitionistic Fuzzy - Euler Graph, Hamiltonian Graph, Petersen Graph with No Edges

In a strong intuitionistic fuzzy Euler graph, strong intuitionistic fuzzy Hamiltonian graph and a strong intuitionistic fuzzy Petersen graph, where there exists no edge connecting any two vertices, then choose any one of the vertex and color with a distinct fuzzy color (C,1), and thus, the adjacent vertices are colored with a fuzzy colors C_K by reducing the solidaties.

2.2 Chromatic Number in Fuzzy Coloring and Total Fuzzy Coloring in a Strong Intuitionistic Fuzzy - Euler Graph, Hamiltonian Graph, Petersen Graph

The least fuzzy colors required to color a strong intuitionistic fuzzy Euler graph, strong intuitionistic fuzzy Hamiltonian graph and a strong intuitionistic fuzzy Petersen graph is the fuzzy chromatic number [10]. The vertex chromatic number of a strong intuitionistic fuzzy Euler graph $E_{\hat{G}}$, strong intuitionistic fuzzy Hamiltonian graph $H_{\hat{G}}$ and strong intuitionistic fuzzy Peteresen graph $P_{\hat{G}}$ is given by the least colors required in vertex coloring of $E_{\hat{G}}$,$H_{\hat{G}}$ and $P_{\hat{G}}$ and is notated by $\chi(E_{\hat{G}})$,$\chi(H_{\hat{G}})$ and$\chi(P_{\hat{G}})$ and the least fuzzy colors used in vertex coloring gives the fuzzy vertex chromatic number notated by $\chi_F(E_{\hat{G}})$,$\chi_F(H_{\hat{G}})$ and $\chi_F(P_{\hat{G}})$ with fuzzy vertex chromatic index [2] denoted by $I[\chi_F(E_{\hat{G}})], I[\chi_F(H_{\hat{G}})]\ and\ I[\chi_F(P_{\hat{G}})]$. The edge chromatic number in a strong intuitionistic fuzzy Euler graph $E_{\hat{G}}$, strong intuitionistic fuzzy Hamiltonian graph $H_{\hat{G}}$ and strong intuitionistic fuzzy Petersen graph $P_{\hat{G}}$ is given by the least colors used in edge coloring of $E_{\hat{G}}$,$H_{\hat{G}}$ and $P_{\hat{G}}$ is notated by $\chi^{'}(E_{\hat{G}})$, $\chi^{'}(H_{\hat{G}})$ and $\chi^{'}(P_{\hat{G}})$ and the least fuzzy colors used in edge coloring gives the fuzzy edge chromatic number by ${\chi_F}^{'}(E_{\hat{G}})$,${\chi_F}^{'}(H_{\hat{G}})$ and ${\chi_F}^{'}(P_{\hat{G}})$ with fuzzy edge chromatic index given by $I\left[{\chi_F}^{'}(E_{\hat{G}})\right], I[{\chi_F}^{'}(H_{\hat{G}})]\ and\ I[{\chi_F}^{'}(P_{\hat{G}})\right]$ and total fuzzy chromatic number of a strong intuitionistic fuzzy Euler graph $E_{\hat{G}}$, strong intuitionistic fuzzy Hamiltonian graph $H_{\hat{G}}$ and strong intuitionistic fuzzy Petersen graph $P_{\hat{G}}$ is the least colors used in both the vertex and edge coloring of $E_{\hat{G}}$, $H_{\hat{G}}$ and $P_{\hat{G}}$ notated by $\chi^T(E_{\hat{G}})$,$\chi^T(H_{\hat{G}})$and $\chi^T(P_{\hat{G}})$ and the least fuzzy colors used in vertex and edge coloring gives the total fuzzy chromatic number by ${\chi_F}^T(E_{\hat{G}})$,${\chi_F}^T(H_{\hat{G}})$ and ${\chi_F}^T(P_{\hat{G}})$ with total fuzzy chromatic index denoted by $I[{\chi_F}^T(E_{\hat{G}})]$, $I[{\chi_F}^T(H_{\hat{G}})\ and\ I[{\chi_F}^T(P_{\hat{G}})]$.

Theorem 2.1.

In an IFG with every strong intuitionistic fuzzy bridge is strongly fuzzy colored.

Proof:

Let $\hat{G}$ be a strong IFG. The adjacent vertices of $\hat{G}$ are colored with distinct fuzzy colors if they are connected. In coloring of a graph, when all the edges are strong, no two adjacent vertices and edges are colored with same colors. And suppose, if two vertices v_1 and v_2 are considered in strong intuitionisitic fuzzy Euler, Hamiltonian and Petersen graphs, and when fuzzy coloring is applied, they are colored with any two distinct fuzzy colors. These two distinct fuzzy colors are given the maximum solidity at the nodes, for example, (R,1) and (G,1) representing fuzzy red and fuzzy green whose membership value is 1 which is the maximum. Since the complete solidity of the fuzzy color is applied, the nodes are strongly fuzzy colored with its maximum membership value of 1. Thus, in an IFG with every strong intuitionistic fuzzy bridge can be strongly fuzzy colored.

Theorem 2.2.

In an IFG with every weak or no intuitionistic fuzzy bridge is fuzzy colored.

Proof:

Let $\hat{G}$ be a strong IFG. In coloring of a graph, the adjacent vertices of $\hat{G}$ are colored with same colors if they are not connected. This edge disconnectivity can be considered as the case of weak edge or no edges. In strong intuitionistic fuzzy Euler graph, it covers all the edges once that are all connected and in strong intuitionistic fuzzy Hamiltonian graph, it covers all the vertices once that are all connected to each other. But, in the Euler graph, in the vertices v_6 and v_8, v_5 and v_7, there is a possibility for the edges to be drawn to connect them. In the Hamiltonian structure, between the vertices v_6 and v_8, v_5 and v_7 there are no edges. In the Petersen Graph structure, there are no edges between v_7 and v_8, v_9

and v_{10}. In this case, any one of the vertex is strongly fuzzy colored with a membership value 1 and the other vertex is colored with a fuzzy color less than the membership value 1. This enables to determine the path from one vertex to an another vertex if it is needed to identify the path to move on in the decreasing order of the solidity of the fuzzy color. The remaining all the other vertices are strongly fuzzy colored as they are connected to each other. Thus, in an IFG, when there is a weak bridge or no intuitionistic fuzzy bridge, it is fuzzy colored.

Theorem 2.3.

In an IFG with strong, weak or no intuitionistic fuzzy bridge, can be both strongly fuzzy colored or fuzzy colored.

Proof:

Let $\hat{G}$ to be a strong IFG. The adjacent vertices of $\hat{G}$ are strongly fuzzy colored with distinct fuzzy colors if they are connected to each other and the adjacent vertices are fuzzy colored obtained from a single distinct color if they are not connected to each other. All the edges are connected in strong intuitionisitic fuzzy Euler graph, so it can only be strongly fuzzy colored. In strong intuitionistic fuzzy Hamiltonian graph, all edges are connected except v_6 and v_8, v_5 and v_7, owing to the structure of the Hamiltonian graph and in strong intuitionistic fuzzy Petersen graph, all edges are connected except v_7 and v_8, v_9 and v_{10}, owing to the structure of the Petersen graph and there is a possibility to draw an edge if needed. So, fixing a single vertex, it can be colored with a strong fuzzy color, that is with the maximum solidity and the other one, adjacent to it can be colored with a fuzzy color and the same for the remaining possible vertices. Here, the strong intuitionistic fuzzy Hamiltonian graph and Petersen graph, comes in the case of, that it can be both strongly fuzzy colored and fuzzy colored. Thus, an IFG with strong, weak or no intuitionistic fuzzy bridge, can be both strongly fuzzy colored or fuzzy colored.

3. Examples of Strong Intuitionistic Fuzzy - Euler Graph, Hamiltonian Graph, Petersen Graph

Strong Intuitionistic Fuzzy Euler Graph:

Fig. 1 is an example for a strong intuitionistic fuzzy Euler graph [8] with a strong Eulerian path $v_1v_2v_3v_4v_5v_1v_4v_2v_5v_3v_1$ of 5 vertices and 10 strong edges, $V_s = \{v_1, v_2, \ldots, v_5\}$ and $E_T = \{e_1, e_2, \ldots, e_{10}\}$.

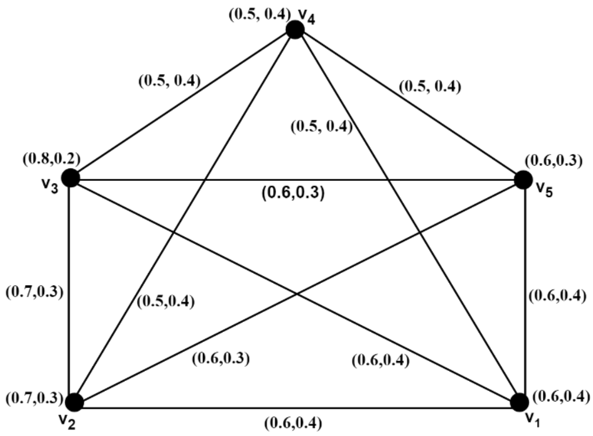

Fig. 1. Strong Intuitionistic Fuzzy Euler Graph

Let $\Gamma = \{\gamma_1, \gamma_2, \gamma_3, \gamma_4, \gamma_5\}$, be the family of intuitionistic fuzzy sets on V_S below:

$$V_S = \begin{cases} (0.6,0.4); s = 1 \\ (0.7,0.3); s = 2 \\ (0.8,0.2); s = 3 \\ (0.5,0.5); s = 4 \\ (0.6,0.3); s = 5 \end{cases} \qquad E_T = \begin{cases} (0.6,0.4); t = 12,35 \\ (0.7,0.1); t = 23 \\ (0.5,0.4); t = 34,51 \\ (0.4,0.4); t = 45 \\ (0.6,0.3); t = 51,27 \\ (0.5,0.3); t = 13 \\ (0.6,0.2); t = 25 \end{cases}$$

Strong Intuitionistic Fuzzy Hamiltonian Graph:

Fig. 2 is an example for strong intuitionistic fuzzy Hamiltonian graph [7] with a strong Hamiltonian path $v_1v_2v_7v_6v_3v_4v_5v_8v_1$ of 8 vertices and 12 strong edges, $V_s = \{v_1, v_2, \dots, v_8\}$ and $E_T = \{e_1, e_2, \dots, e_{12}\}$

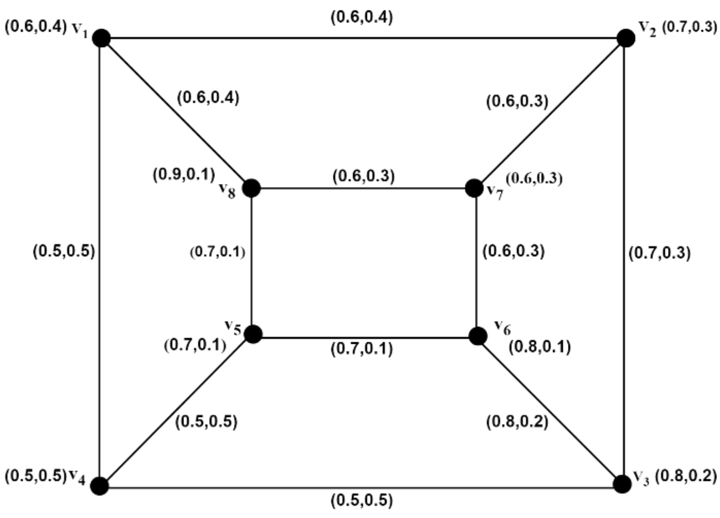

Fig. 2. Strong Intuitionistic Fuzzy Hamiltonian Graph

Let $\Gamma = \{\gamma_1, \gamma_2, \gamma_3, \gamma_4\}$, be the family of intuitionistic fuzzy sets on V_S below:

$$V_S = \begin{cases} (0.6,0.4); s = 1 \\ (0.7,0.3); s = 2 \\ (0.8,0.2); s = 3 \\ (0.5,0.5); s = 4 \\ (0.7,0.1); s = 5 \\ (0.8,0.1); s = 6 \\ (0.6,0.3); s = 7 \\ (0.9,0.1); s = 8 \end{cases} \qquad E_T = \begin{cases} (0.6,0.4); t = 12 \\ (0.7,0.3); t = 23 \\ (0.5,0.4); t = 34 \\ (0.5,0.5); t = 41 \\ (0.6,0.3); t = 18,27 \\ (0.8,0.1); t = 36 \\ (0.5,0.3); t = 45 \\ (0.7,0.1); t = 56,58 \\ (0.6,0.2); t = 67,78 \end{cases}$$

Strong Intuitionistic Fuzzy Petersen Graph:

Fig. 3 is an example for a strong intuitionistic fuzzy Petersen graph [3] with 10 vertices and 15 strong edges, $V_S = \{v_1, v_2, \dots, v_{10}\}$ and $E_T = \{e_1, e_2, \dots, e_{15}\}$

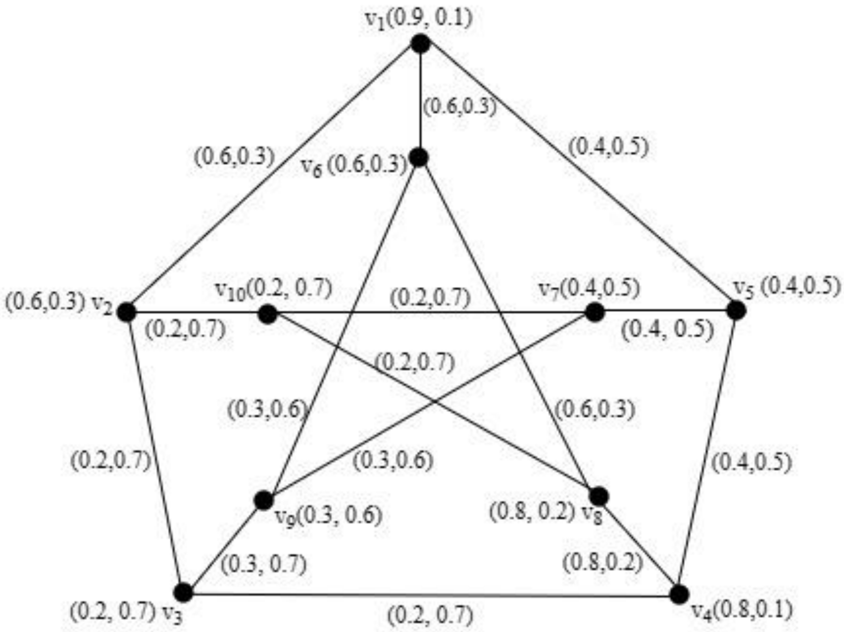

Fig. 3. Strong Intuitionistic Fuzzy Petersen Graph

Let $\Gamma = \{\gamma_1, \gamma_2, \gamma_3\}$, be the family of intuitionistic fuzzy sets on V_S as below:

$$\boldsymbol{V_S} = \begin{cases} (\mathbf{0.9, 0.1}); \boldsymbol{s} = \mathbf{1} \\ (\mathbf{0.8, 0.1}); \boldsymbol{s} = \mathbf{4} \\ (\mathbf{0.8, 0.2}); \boldsymbol{s} = \mathbf{8} \\ (\mathbf{0.3, 0.6}); \boldsymbol{s} = \mathbf{9} \\ (\mathbf{0.6, 0.3}); \boldsymbol{s} = \mathbf{2, 6} \\ (\mathbf{0.2, 0.7}); \boldsymbol{s} = \mathbf{3, 10} \\ (\mathbf{0.4, 0.5}); \boldsymbol{s} = \mathbf{5, 7} \end{cases} \quad \boldsymbol{E_T} = \begin{cases} (\mathbf{0.8, 0.2}); \boldsymbol{t} = \mathbf{48} \\ (\mathbf{0.3, 0.6}); \boldsymbol{t} = \mathbf{79, 69} \\ (\mathbf{0.6, 0.3}); \boldsymbol{t} = \mathbf{12, 16, 68} \\ (\mathbf{0.2, 0.7}); \boldsymbol{t} = \mathbf{34, 23, 39, 210, 710, 810} \\ (\mathbf{0.4, 0.5}); \boldsymbol{t} = \mathbf{25, 45, 57} \end{cases}$$

3.1 Fuzzy Vertex Coloring in Strong Intuitionistic Fuzzy - Euler Graph, Hamiltonian Graph, Petersen Graph

The family $\Gamma = \{\gamma_1, \dots . \gamma_k\}$ are sets which are intuitionistic fuzzy on V_S is a k - fuzzy vertex coloring of $E_{\hat{G}} = (V_s, E_T), H_{\hat{G}} = (V_s, E_T)$ and $P_{\hat{G}} = (V_s, E_T)$ when

i. $\vee \gamma_i(x) = V_S \forall x \in V_S$

ii. $\gamma_i \wedge \gamma_j = 0$

iii. $\forall xy$, strong edge of $E_{\hat{G}}, min\{\gamma_i(\mu_1(x)), \gamma_i(\mu_1(y))\} = 0; max\{\gamma_i(v_1(x)), \gamma_i(v_1(y))\} = 1, (1 \leq i \leq k)$.

The minimal value of k is chromatic number of $E_{\hat{G},} H_{\hat{G}}$ and $P_{\hat{G}}$ has a k - vertex fuzzy coloring [6] and the vertex chromatic number notated by $\chi(E_{\hat{G}}), \chi(H_{\hat{G}})$ and $\chi(P_{\hat{G}})$. For a strong intuitionistic fuzzy Euler graph, strong intuitionistic fuzzy Hamiltonian graph and strong intuitionistic fuzzy Petersen graph, $E_{\hat{G}} = (V_S, E_T), H_{\hat{G}} = (V_S, E_T)$ and $P_{\hat{G}} = (V_S, E_T)$ the fuzzy vertex chromatic number is, $\chi_F(E_{\hat{G}}), \chi_F(H_{\hat{G}})$ and $\chi_F(P_{\hat{G}})$ with fuzzy vertex chromatic index denoted by $I[\chi_F(E_{\hat{G}})], I[\chi_F(H_{\hat{G}})]$ and $I[\chi_F(P_{\hat{G}})] = \{x, (C_K, 1)\}$ where x denotes fuzzy vertex chromatic number.

Strong Intuitionistic Fuzzy Euler Graph:

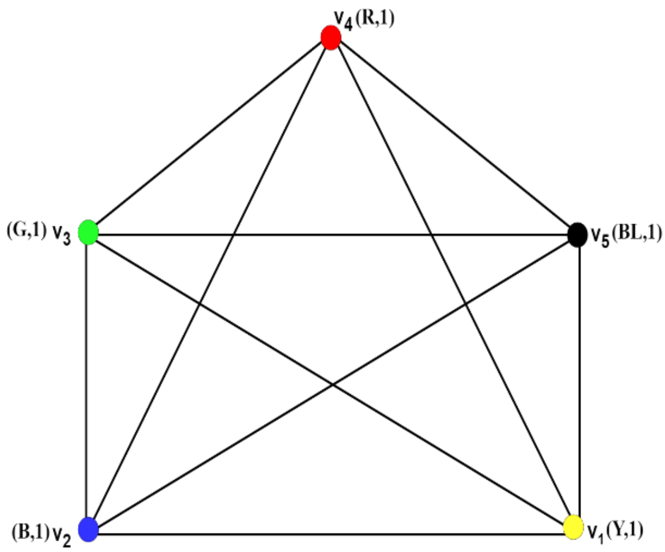

Fig. 4. Fuzzy Vertex Coloring in Strong Intuitionistic Fuzzy Euler Graph

$$(\gamma_1)V_S = \begin{cases}(0.6,0.4); s = 1\\(0,1); otherwise\end{cases} \quad (\gamma_2)V_S = \begin{cases}(0.7,0.3); s = 2\\(0,1); otherwise\end{cases} \quad (\gamma_3)V_S = \begin{cases}(0.8,0.2); s = 3\\(0,1); otherwise\end{cases}$$

$$(\gamma_4)V_S = \begin{cases}(0.5,0.4); s = 4\\(0,1); otherwise\end{cases} \quad (\gamma_5)V_S = \begin{cases}(0.6,0.3); s = 5\\(0,1); otherwise\end{cases}$$

Here, the family $\Gamma = \{\gamma_1, \gamma_2, \gamma_3, \gamma_4, \gamma_5\}$ in Fig. 4 satisfying the definition of the coloring of the fuzzy vertex of a strong intuitionistic fuzzy Euler graph with five members in the family satisfying the definition of minimum five numbers of distinct fuzzy colors and so, vertex chromatic number of $E_{\hat{G}}$ is $\chi(E_{\hat{G}}) = 5$ and fuzzy vertex chromatic number is $\chi_F(E_{\hat{G}}) = 5$ with the fuzzy vertex chromatic index $I[\chi_F(E_{\hat{G}})] = \{5, (R, 1)(G, 1), (B, 1), (Y, 1), (BL, 1)\}$.

Strong Intuitionistic Fuzzy Hamiltonian Graph:

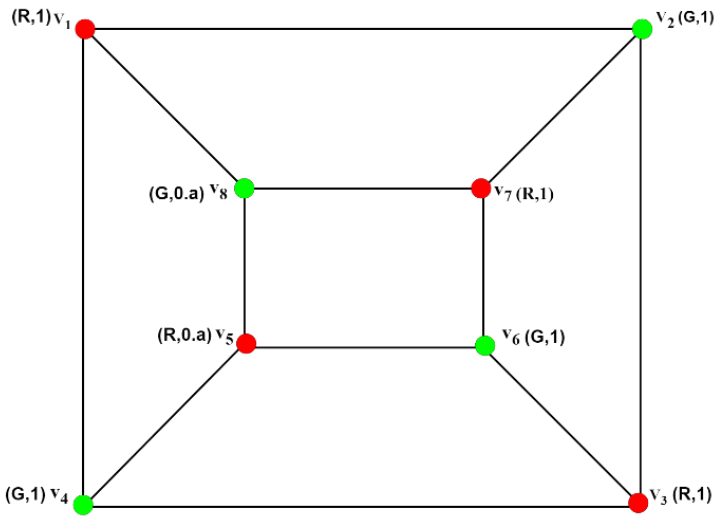

Fig. 5. Fuzzy Vertex Coloring in Strong Intuitionistic Fuzzy Hamiltonian Graph

$$(\gamma_1)V_S = \begin{cases} (0.6,0.4); s = 1 \\ (0.8,0.2); s = 3 \\ (0.7,0.1); s = 5 \\ (0.6,0.3); s = 7 \\ (0,1); otherwise \end{cases} \quad (\gamma_2)V_S = \begin{cases} (0.7,0.3); s = 2 \\ (0.5,0.5); s = 4 \\ (0.8,0.1); s = 6 \\ (0.9,0.1); s = 8 \\ (0,1); otherwise \end{cases}$$

Here, the family $\Gamma = \{\gamma_1, \gamma_2\}$ in Fig. 5 satisfying the definition in the coloring of the fuzzy vertex of a strong intuitionistic fuzzy Hamiltonian graph with two members in the family satisfying the definition of minimum two numbers of distinct fuzzy colors and so, the vertex chromatic number of $H_{\hat{G}}$ is given by $\chi(H_{\hat{G}}) = 2$ and the fuzzy vertex chromatic number is $\chi_F(H_{\hat{G}}) = 2$ with fuzzy vertex chromatic index $I[\chi_F(H_{\hat{G}})] = \{2, (R, 1)(G, 1)\}$.

Strong Intuitionistic Fuzzy Petersen Graph:

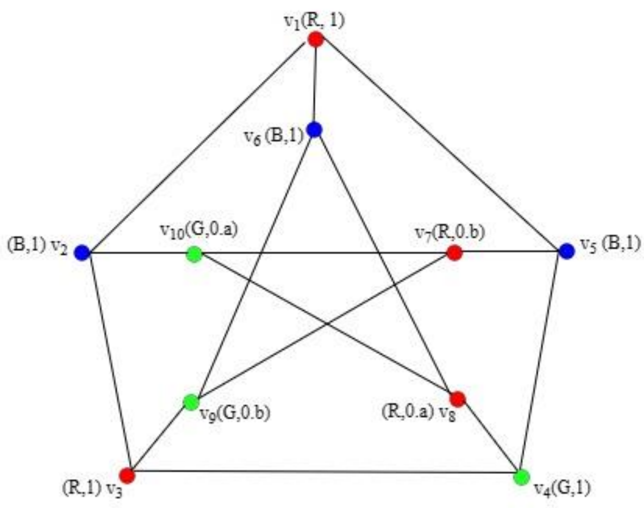

Fig. 6. Fuzzy Vertex Coloring in Strong Intuitionistic Fuzzy Petersen Graph

$$(\gamma_1)V_S = \begin{cases} (0.9,0.1); s = 1 \\ (0.2,0.7); s = 3 \\ (0.4,0.5); s = 7 \\ (0.8,0.2); s = 8 \\ (0,1); otherwise \end{cases} \quad (\gamma_2)V_S = \begin{cases} (0.6,0.3); s = 2,6 \\ (0.4,0.5); s = 5 \\ (0,1); otherwise \end{cases}$$

$$(\gamma_3)V_S = \begin{cases} (0.8,0.1); s = 4 \\ (0.3,0.6); s = 9 \\ (0.2,0.7); s = 10 \\ (0,1); otherwise \end{cases}$$

Here, the family $\Gamma = \{\gamma_1, \gamma_2, \gamma_3\}$ in Fig. 6 satisfying the definition of the coloring of the fuzzy vertex of a strong intuitionistic fuzzy Petersen graph with three members in the family satisfying the definition of minimum three numbers of distinct fuzzy colors and so, vertex chromatic number of $P_{\hat{G}}$ is $\chi(P_{\hat{G}})$= 3 and the fuzzy vertex chromatic number is $\chi_F(P_{\hat{G}}) = 3$ with the fuzzy vertex chromatic index $I[\chi_F(P_{\hat{G}})] = \{3, (R, 1)(G, 1), (B, 1)\}$.

3.2 Fuzzy Edge Coloring in Strong Intuitionistic Fuzzy - Euler Graph, Hamiltonian Graph, Petersen Graph

The family $\Gamma = \{\gamma_1, \ldots . \gamma_k\}$ are sets which are intuitionistic fuzzy on E_T is k - fuzzy edge coloring of $E_{\hat{G}} = (V_S, E_T), H_{\hat{G}} = (V_S, E_T)$ and $P_{\hat{G}} = (V_S, E_T)$, when

(1) $\vee \gamma_i(xy) = E_T \forall xy \in E_T$

(2) $\gamma_i \wedge \gamma_j = 0$

(3) $\forall xy, incident\ edges\ of E_T, min\{\gamma_i(\mu_2(xy))\} = 0; max\{\gamma_i(v_2(xy))\} = 1, (1 \leq i \leq k)$.

The minimal value of k - fuzzy edge coloring of $E_{\hat{G}}, H_{\hat{G}}$ and $P_{\hat{G}}$ notated by $\chi'(E_{\hat{G}}), \chi'(H_{\hat{G}}), \chi'(P_{\hat{G}})$ is the edge chromatic number. For a strong intuitionistic fuzzy Euler graph, strong intuitionistic fuzzy Hamiltonian graph and strong intuitionistic fuzzy Petersen graph, $E_{\hat{G}} = (V_S, E_T), H_{\hat{G}} = (V_S, E_T)$ and $P_{\hat{G}} = (V_S, E_T)$, the fuzzy edge chromatic number is, $\chi_F{}'(E_{\hat{G}}), \chi_F{}'(H_{\hat{G}})$ and $\chi_F{}'(P_{\hat{G}})$ and its fuzzy edge chromatic index is given by $I[\chi_F{}'(E_{\hat{G}})], I[\chi_F{}'(H_{\hat{G}})]$ and $I[\chi_F{}'(P_{\hat{G}})] = \{x', (C_K, 1)\}$ where x' denotes the fuzzy edge chromatic number.

Strong Intuitionistic Fuzzy Euler Graph:

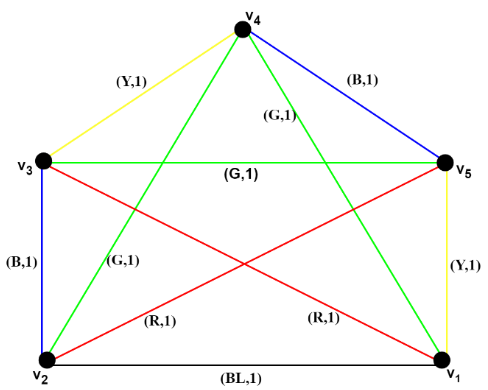

Fig. 7. Fuzzy Edge Coloring in Strong Intuitionistic Fuzzy Euler Graph

$$\gamma_1(V_iV_j) = \begin{cases} (0.5,0.3); ij = 13 \\ (0.5,0.3); ij = 25 \\ (0,1); otherwise \end{cases} \qquad \gamma_2(V_iV_j) = \begin{cases} (0.4,0.4); ij = 45 \\ (0.7,0.1); ij = 23 \\ (0,1); otherwise \end{cases}$$

$$\gamma_3(V_iV_j) = \begin{cases} (0.5,0.2); ij = 24 \\ (0.5,0.4); ij = 14 \\ (0.6,0.4); ij = 35 \\ (0,1); otherwise \end{cases} \qquad \gamma_4(V_iV_j) = \begin{cases} (0.5,0.4); ij = 34,15 \\ (0,1); otherwise \end{cases}$$

$$\gamma_5(V_iV_j) = \begin{cases} (0.6,0.4); ij = 12 \\ (0,1); otherwise \end{cases}$$

Here, the family $\Gamma = \{\gamma_1, \gamma_2, \gamma_3, \gamma_4, \gamma_5\}$ in Fig. 7 satisfying fuzzy edge coloring definition of a strong intuitionistic fuzzy Euler graph with five members in the family satisfying the definition of minimum five numbers of distinct fuzzy colors and so, the edge chromatic number of $E_{\hat{G}}$ is $\chi'(E_{\hat{G}}) = 5$ and fuzzy edge chromatic number is $\chi_F{}'(E_{\hat{G}}) = 5$ with fuzzy edge chromatic index $I[\chi_F{}'(E_{\hat{G}})] = \{5, (R, 1)(B, 1), (G, 1), (Y, 1), (BL, 1)\}$.

Strong Intuitionistic Fuzzy Hamiltonian Graph:

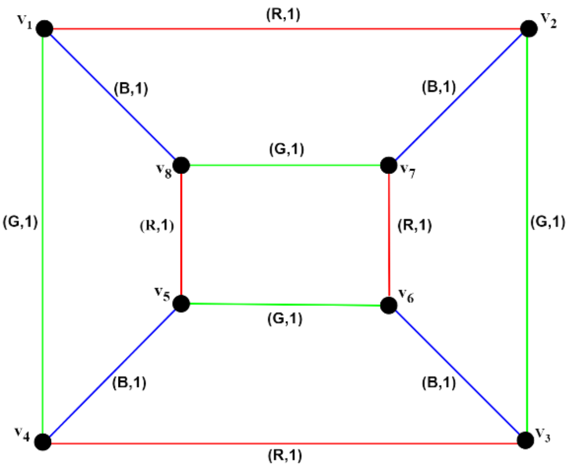

Fig. 8. Fuzzy Edge Coloring in Strong Intuitionistic Fuzzy Hamiltonian Graph

$$(V_iV_j) = \begin{cases} (0.6,0.4); ij = 12 \\ (0.7,0.3); ij = 43 \\ (0.7,0.1); ij = 58 \\ (0,1); otherwise \end{cases} \qquad \gamma_2(V_iV_j) = \begin{cases} (0.5,0.5); ij = 14 \\ (0.6,0.2); ij = 78 \\ (0.7,0.3); ij = 23 \\ (0.7,0.1); ij = 56 \\ (0,1); otherwise \end{cases}$$

$$\gamma_3(V_iV_j) = \begin{cases} (0.3,0.6); ij = 18 \\ (0.6,0.3); ij = 27 \\ (0.8,0.1); ij = 36 \\ (0.5,0.3); ij = 45 \\ (0,1); otherwise \end{cases}$$

Here, the family $\Gamma = \{\gamma_1, \gamma_2, \gamma_3\}$ in Fig. 8 satisfying fuzzy edge coloring definition of a strong intuitionistic fuzzy Hamiltonian graph with three members in the family satisfying the definition of minimum three numbers of distinct fuzzy colors and so, edge chromatic number of $H_{\hat{G}}$ is $\chi'(H_{\hat{G}}) = 3$ and the fuzzy chromatic number is $\chi_F{}'(H_{\hat{G}}) = 3$ with the fuzzy edge chromatic index $I[\chi_F{}'(H_{\hat{G}})] = \{3, (R, 1)(B, 1), (G, 1)\}$.

Strong Intuitionistic Fuzzy Petersen Graph:

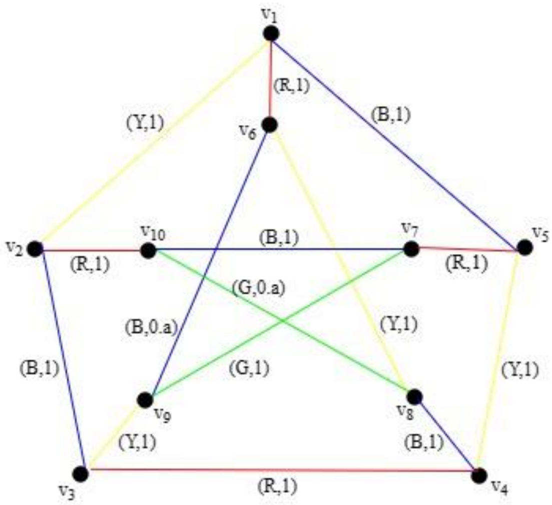

Fig. 9. Fuzzy Edge Coloring in Strong Intuitionistic Fuzzy Petersen Graph

$$\gamma_1(V_iV_j) = \begin{cases} (0.2,0.7); ij = 34,210 \\ (0.6,0.3); ij = 16 \\ (0.4,0.5); ij = 57 \\ (0,1); otherwise \end{cases} \quad \gamma_2(V_iV_j) = \begin{cases} (0.4,0.5); ij = 15 \\ (0.2,0.7); ij = 23,710 \\ (0.8,0.2); ij = 48 \\ (0.6,0.3); ij = 69 \\ (0,1); otherwise \end{cases}$$

$$\gamma_3(V_iV_j) = \begin{cases} (0.3,0.6); ij = 79 \\ (0.2,0.7); ij = 810 \\ (0,1); otherwise \end{cases} \quad \gamma_4(V_iV_j) = \begin{cases} (0.2,0.7); ij = 39 \\ (0.6,0.3); ij = 12,68 \\ (0.4,0.5); ij = 45 \\ (0,1); otherwise \end{cases}$$

Here, in the family, $\Gamma = \{\gamma_1, \gamma_2, \gamma_3, \gamma_4\}$ in Fig. 9 satisfying fuzzy edge coloring definition of a strong intuitionistic fuzzy Petersen graph with four members in the family satisfying the definition of minimum four numbers of distinct fuzzy colors and so, the edge chromatic number of $P_{\hat{G}}$ is $\chi'(P_{\hat{G}}) = 4$ and fuzzy edge chromatic number is $\chi_F'(P_{\hat{G}}) = 4$ with fuzzy edge chromatic index $I[\chi_F{}'(P_{\hat{G}})] = \{4, (R, 1)(B, 1), (G, 1), (Y, 1)\}$.

3.3 Total Fuzzy Coloring in Strong Intuitionistic Fuzzy - Euler Graph, Strong Hamiltonian Graph, Petersen Graph

The family $\Gamma = \{\gamma_1, \dots . \gamma_k\}$ of intuitionistic fuzzy sets on $V_S = \{v_1, v_2, \dots, v_5\}$ and $E_T = \{e_1, e_2, \dots, e_{10}\}$ is k - total fuzzy coloring [11] of $E_{\hat{G}} = (V_S, E_T), H_{\hat{G}} = (V_S, E_T)$ and $P_{\hat{G}} = (V_S, E_T)$, when

(1) $\vee \gamma_i(x) \vee \gamma_i(xy) = V_S \vee E_T \forall x \in V_S, xy \in E_T$

(2) $\gamma_i \wedge \gamma_j = 0$

(3) $\forall xy$, strong and incident vertices and edges of $E_{\hat{G}}, H_{\hat{G}}, P_{\hat{G}}$,

$$min\{\gamma_i(\mu_1(x)), \gamma_i(\mu_1(y)), \gamma_i(\mu_2(xy))\} = 0; max\{\gamma_i(v_1(x)), \gamma_i(v_1(y)), \gamma_i(v_2(xy))\} = 1, (1 \leq i \leq k).$$

Minimal value of k - total fuzzy coloring is total chromatic number of $E_{\hat{G}}, H_{\hat{G}}, P_{\hat{G}}$ notated as $\chi^T(E_{\hat{G}}), \chi^T(H_{\hat{G}}), \chi^T(P_{\hat{G}})$. For a strong intuitionistic fuzzy Euler graph, strong intuitionistic fuzzy Hamiltonian graph and a strong intuitionistic fuzzy Petersen graph $E_{\hat{G}} = (V_S, E_T), H_{\hat{G}} = (V_S, E_T)$ and $P_{\hat{G}} = (V_S, E_T)$, the total fuzzy chromatic number is notated by $\chi_F{}^T(E_{\hat{G}})$, $\chi_F{}^T(H_{\hat{G}})$ and $\chi_F{}^T(P_{\hat{G}})$ with total fuzzy chromatic index $I[\chi_F{}^T(E_{\hat{G}})], I[\chi_F{}^T(H_{\hat{G}})]$ and $I[\chi_F{}^T(P_{\hat{G}})] = \{x^T, (C_K, 1)\}$ where x^T denotes total fuzzy chromatic number.

Strong Intuitionistic Fuzzy Euler Graph:

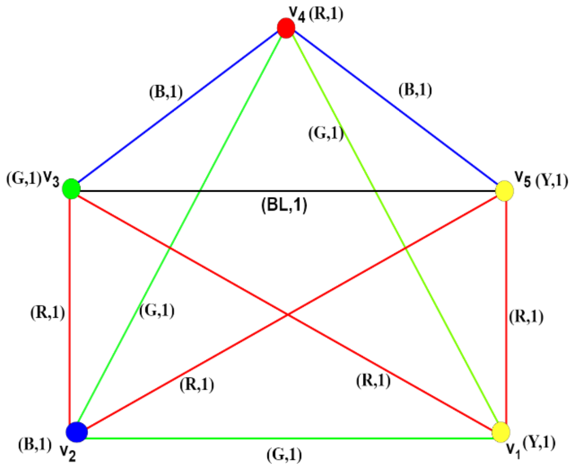

Fig. 10. Total Fuzzy Coloring in Strong Intuitionistic Fuzzy Euler Graph

$$(\gamma_1)\left(V_S, (V_i, V_j)\right) = \begin{cases} (0.7,0.3); s = 2 \\ (0.5,0.4); ij = 34 \\ (0,1); otherwise \end{cases}$$

$$(\gamma_2)\left(V_S, (V_i, V_j)\right) = \begin{cases} (0.8,0.2); s = 3 \\ (0.5,0.4); ij = 14 \\ (0.5,0.2); ij = 24 \\ (0.6,0.4); ij = 12 \\ (0,1); otherwise \end{cases} \quad (\gamma_3)\left(V_S, (V_i V_j)\right) = \begin{cases} (0.5,0.4); s = 4 \\ (0.5,0.3); ij = 13 \\ (0.6,0.2); ij = 25 \\ (0,1); otherwise \end{cases}$$

$$\gamma_4(V_i V_j) = \begin{cases} (0.6,0.4); ij = 34 \\ (0,1); otherwise \end{cases} \qquad \gamma_5(V_S) = \begin{cases} (0.6,0.4); s = 1 \\ (0,1); otherwise \end{cases}$$

Here, in the family $\Gamma = (\gamma_1, \gamma_2, \gamma_3, \gamma_4, \gamma_5)$ in Fig. 10 satisfying total fuzzy coloring definition of a strong intuitionistic fuzzy Euler graph with five members in the family satisfying the definition of minimum five numbers of distinct fuzzy colors and so, total chromatic number of $E_{\hat{G}}$ is $\chi^T(E_{\hat{G}}) = 5$ and total fuzzy chromatic number is $\chi_F{}^T(E_{\hat{G}}) = 5$ with the total fuzzy chromatic index $I[\chi_F{}^T(E_{\hat{G}})] = \{5, (R, 1)(B, 1), (G, 1), (Y, 1), (BL, 1)\}$.

Strong Intuitionistic Fuzzy Hamiltonian Graph:

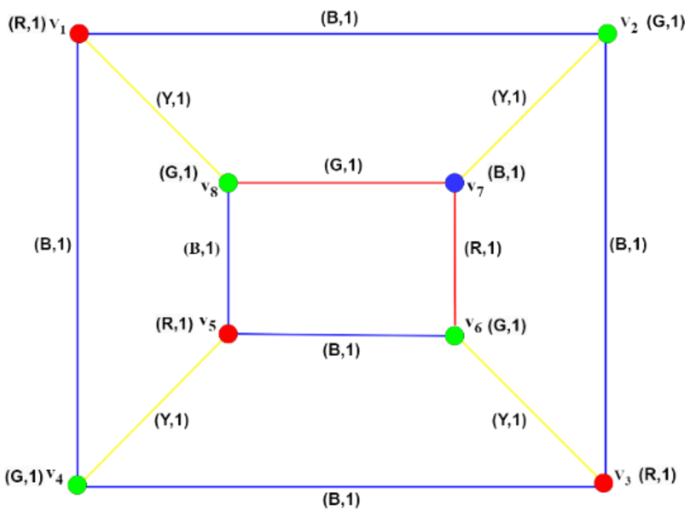

Fig. 11. Total Fuzzy Coloring in Strong Intuitionistic Fuzzy Hamiltonian Graph

$$(\gamma_1)\left(V_S,\left(V_i,V_j\right)\right)=\begin{cases}(0.6,0.4); s=1\\(0.8,0.2); s=3\\(0.7,0.1); s=5\\(0.6,0.2); ij=78,67\\(0,1); otherwise\end{cases}\quad(\gamma_2)\left(V_S,\left(V_i,V_j\right)\right)=\begin{cases}(0.6,0.3); s=7\\(0.7,0.1); ij=58\\(0.6,0.4); ij=12\\(0.7,0.1); s=56\\(0.5,0.4); ij=34\\(0.7,0.3); ij=23\\(0.5,0.5); ij=14\\(0,1); otherwise\end{cases}$$

$$(\gamma_3)(V_S)=\begin{cases}(0.7,0.3); s=2\\(0.9,0.1); s=8\\(0.8,0.1); s=6\\(0,1); otherwise\end{cases}\quad\gamma_4\left(V_iV_j\right)=\begin{cases}(0.6,0.3); ij=18,27\\(0.5,0.3); ij=45\\(0.8,0.1); ij=36\\(0,1); otherwise\end{cases}$$

Here, the family $\Gamma=(\gamma_1,\gamma_2,\gamma_3,\gamma_4)$ in Fig. 11 satisfying total fuzzy coloring definition of a strong intuitionistic fuzzy Hamiltonian graph with four members in the family satisfying the definition of minimum four numbers of distinct fuzzy colors and so, total chromatic number of $H_{\hat{G}}$ given by $\chi^T(H_{\hat{G}})=4$ and the total fuzzy chromatic number is $\chi_F{}^T(H_{\hat{G}})=4$ with the total fuzzy chromatic index $I[\chi_F{}^T(H_{\hat{G}})]=\{4,(R,1)(B,1),(G,1),(Y,1)\}$.

Strong Intuitionistic Fuzzy Petersen Graph:

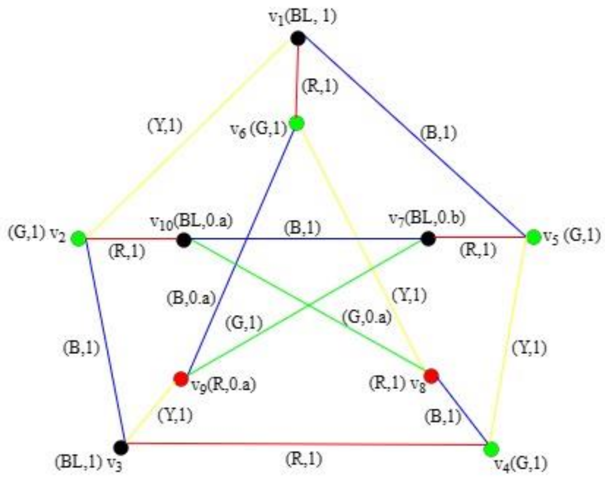

Fig. 12. Total Fuzzy Coloring in Strong Intuitionistic Fuzzy Petersen Graph

$$(\gamma_1))(V_s) = \begin{cases} (0.9, 0.1); s = 1 \\ (0.2,0.7); s = 3,10 \\ (0.4,0.5); s = 7 \\ (0.8,0.2); ij = 8 \\ (0.3,0.6); s = 9 \\ (0,1); \text{otherwise} \end{cases} \quad (\gamma_2)\left(V_s, (V_i, V_j)\right) = \begin{cases} (0.6,0.3); s = 2,6 \\ (0.8,0.1); s = 4 \\ (0.4,0.5); s = 5 \\ (0.3,0.6); ij = 79 \\ (0.2,0.7); ij = 810 \\ (0,1); otherwise \end{cases}$$

$$(\gamma_3)\left(V_S, (V_i, V_j)\right) = \begin{cases} (0.2,0.7); ij = 34,210 \\ (0.6,0.3); ij = 16 \\ (0.4,0.5); ij = 57 \\ (0,1); otherwise \end{cases} \quad \gamma_4(V_iV_j) = \begin{cases} (0.4,0.5); ij = 15 \\ (0.2,0.7); ij = 23,710 \\ (0.8,0.2); ij = 48 \\ (0.6,0.3); ij = 69 \\ (0,1); otherwise \end{cases}$$

$$(\gamma_5)(V_iV_j) = \begin{cases} (0.2,0.7); ij = 39 \\ (0.6,0.3); ij = 12,68 \\ (0.4,0.5); ij = 45 \\ (0,1); otherwise \end{cases}$$

Here in, the family $\Gamma = \{\gamma_1, \gamma_2, \gamma_3, \gamma_4, \gamma_5\}$ in Fig. 12 satisfying total fuzzy coloring definition of a strong intuitionistic fuzzy Petersen graph with five members in the family satisfying the definition of minimum five numbers of distinct fuzzy colors and so, total chromatic number of $P_{\hat{G}}$ is $\chi^T(P_{\hat{G}}) = 5$ and the total fuzzy chromatic number is ${\chi_F}^T(P_{\hat{G}}) = 5$ with the total fuzzy chromatic index $I[{\chi_F}^T(H_{\hat{G}})] = \{5, (R, 1)(B, 1), (G, 1), (Y, 1), (BL, 1)\}$.

4. Conclusion

Thus, fuzzy coloring and total fuzzy coloring ideas are well discussed to a strong intuitionistic fuzzy Euler graph $E_{\hat{G}}$, strong intuitionistic fuzzy Hamiltonian graph $H_{\hat{G}}$ and to a strong intuitionistic fuzzy Petersen graph $P_{\hat{G}}$ by discerning its chromatic number, fuzzy chromatic number with fuzzy chromatic indices of both the vertices and edges and the total chromatic number, total fuzzy chromatic number with total fuzzy chromatic index with an expository example. And, thus also the concept of fuzzy coloring paves in identifying the amount of the color (solidity) applied at the vertices and edges enabling the way for finding the continuity of the paths in various real-life applications in consideration of the solidity of the fuzzy color.

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