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Different types of products on Cohesive Fuzzy Graph

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Abstract: In this manuscript, we define the cartesian product, direct product, strong product and Semi-strong product of two cohesive fuzzy graphs, along with providing illustrative examples. Furthermore, we have established theorems using these products through rigorous proofs.

Keywords:
Cohesive Fuzzy graph, Cartesian product two cohesive fuzzy graphs, Direct product two cohesive fuzzy graphs, Strong product two cohesive fuzzy graphs, Semi strong product of two cohesive fuzzy graphs.

1. Introduction:

A. Zadeh [1] proposed fuzzy set theory in 1965 as a more effective decision-making technique for dealing with ambiguity in real-life scenarios. In 2002, D. Ramot [4] introduced the complex fuzzy set. The fuzzy graph to the theory of complex fuzzy graph was extended by Tamir et al. [9] in 2015. X. Xue [14] developed the notion of cohesive fuzzy sets in 2023. After that many researchers have given their ideas to developed the different types of complex fuzzy graphs. The cohesive fuzzy graph utilized in this communication is the greatest choice available under many favorable situations, and it may have a range in the extended unit circle of the complex plane. When there are numerous possibilities for positive conditions, a cohesive fuzzy graph helps us ignore unpleasant scenarios and helps us choose the optimal option in challenging situations. Without a doubt, this model will save us our time and effort. This paper is a continuation of cohesive fuzzy graph and its properties [16] and this paper contains the different types of product on cohesive fuzzy graph with some theorems.

2. Definitions of Terminologies Used in the Study:

The following is a list of some of the definitions used in the current study

Definition 2.1: [**1**] Let a non-empty set be $\mathbb{S}$ and a is the element in $\mathbb{S}$. Then the fuzzy set $\mathbb{P}$ on $\mathbb{S}$ is the set of ordered pairs,

$$\mathbb{P} = \{< a, \lambda_{\mathbb{P}}(a) >: a \in \mathbb{S}\}$$

where $\lambda_{\mathbb{P}}(a) : \mathbb{S} \to [0, 1]$ is a membership function.

Definition 2.2: [**15**] Let a fuzzy set be C that is defined on S, where S is a fixed universal discourse; According to the function h, a cohesive fuzzy set on C, when applied on S that yields the result in a unit circle subset. That is,

$$S_1 = \{< a, h_C(a) >: a \in S\}.$$

here h_C is a complex valued set which is contained in the complex plane of circle with unit, meaning the membership degrees of elements $a \in S$ to $C \subset S$. And, h_C is represented as $\gamma_C(a) \exp^{i\Omega_C(a)}$, where $i = \sqrt{-1}, \gamma_C(a)$ and $\Omega_C(a)$ both are take real worth values and $\gamma_C(a) \in [0, 1]$.

Example 2.3: [**15**] The basic representation of cohesive fuzzy set, let $P = \{y_1, y_2, y_3\}$ be the reference set. Suppose

$$h_{C_1}(y_1) = \{0.5 \exp \pi, 0.8 \exp \tfrac{\pi}{2}, 0.7 \exp \tfrac{\pi}{2}\},$$
$$h_{C_2}(y_2) = \{0.6 \exp \pi, 0.9 \exp \pi, 0.7 \exp \tfrac{\pi}{4}\},$$
$$\text{and } h_{C_3}(y_3) = \{0.5 \exp \pi, 0.7 \exp \tfrac{\pi}{2}, 0.7 \exp \pi\},$$

are the membership set of $y_k (1 \leq k \leq 3)$ to C. Then, it's cohesive fuzzy set can be written as

$$C = \{< y_1, \{0.5 \exp \pi, 0.8 \exp \tfrac{\pi}{2}, 0.7 \exp \tfrac{\pi}{2}\} >, < y_2, \{0.6 \exp \pi, 0.9 \exp \pi, 0.7 \exp \tfrac{\pi}{4}\} >,$$
$$< y_3, \{0.5 \exp \pi, 0.7 \exp \tfrac{\pi}{2}, 0.7 \exp \pi\} >\}.$$

Definition 2.4: **[16]** Let an underlying set be U. A Cohesive fuzzy graph with U is an ordered pair $\mathbb{G} = (\mathbb{P}, \mathbb{Q})$ where,

$$\mathbb{P} = (a, \lambda_{\mathbb{P}} e^{i\alpha_{\mathbb{P}}}, \gamma_{\mathbb{P}} e^{i\beta_{\mathbb{P}}}, \delta_{\mathbb{P}} e^{i\vartheta_{\mathbb{P}}}) \text{ is a cohesive fuzzy set on a vertex set } V.$$
$$\mathbb{Q} = (ab, \lambda_{\mathbb{Q}} e^{i\alpha_{\mathbb{Q}}}, \gamma_{\mathbb{Q}} e^{i\beta_{\mathbb{Q}}}, \delta_{\mathbb{Q}} e^{i\vartheta_{\mathbb{Q}}}) \text{ is a cohesive fuzzy set on a edge set } E$$

with $\mathbb{P} : V \to D_u\{\chi \in C : |\chi| \leq 1\}$ and $\mathbb{Q} : E \to D_u\{\chi \in C : |\chi| \leq 1\}$, Where $D_u\{\chi \in C : |\chi| \leq 1\}$ is the set of all the unit disc's finite subsets, s.t

$$\lambda_{\mathbb{Q}}(ab) e^{i\alpha_{\mathbb{Q}}(ab)} \leq min\{\lambda_{\mathbb{P}}(a), \lambda_{\mathbb{P}}(b)\} e^{i \min\{\alpha_{\mathbb{P}}(a), \alpha_{\mathbb{P}}(b)\}}$$
$$\gamma_{\mathbb{Q}}(ab) e^{i\beta_{\mathbb{Q}}(ab)} \leq min\{\gamma_{\mathbb{P}}(a), \gamma_{\mathbb{P}}(b)\} e^{i \min\{\beta_{\mathbb{P}}(a), \beta_{\mathbb{P}}(b)\}}$$
$$\delta_{\mathbb{Q}}(ab) e^{i\omega_{\mathbb{Q}}(ab)} \leq min\{\delta_{\mathbb{P}}(a), \delta_{\mathbb{P}}(b)\} e^{i \min\{\omega_{\mathbb{P}}(a), \omega_{\mathbb{P}}(b)\}}$$

for all $a, b \in \mathbb{P}$.

3. Some products on cohesive fuzzy graph:

This section, contains four different types of products on cohesive fuzzy graphs that is cartesian product, direct product, strong product and semi-strong product.

3.1 Cartesian product of two cohesive fuzzy graph:

A pair $\mathbb{G}' \times \mathbb{G}'' = (\mathbb{P}_1 \times \mathbb{P}_2, \mathbb{Q}_1 \times \mathbb{Q}_2)$ is the cartesian product of two cohesive fuzzy graphs, such that:

(i). $\mu_{\mathbb{P}_1 \times \mathbb{P}_2}(a_1, a_2) e^{i\alpha_{\mathbb{P}_1 \times \mathbb{P}_2}(a_1, a_2)} = \min\{\mu_{\mathbb{P}_1}(a_1), \mu_{\mathbb{P}_2}(a_2)\} e^{i \min\{\alpha_{\mathbb{P}_1}(a_1), \alpha_{\mathbb{P}_2}(a_2)\}}$
$\gamma_{\mathbb{P}_1 \times \mathbb{P}_2}(a_1, a_2) e^{i\beta_{\mathbb{P}_1 \times \mathbb{P}_2}(a_1, a_2)} = \min\{\gamma_{\mathbb{P}_1}(a_1), \gamma_{\mathbb{P}_2}(a_2)\} e^{i \min\{\beta_{\mathbb{P}_1}(a_1), \beta_{\mathbb{P}_2}(a_2)\}}$
$\delta_{\mathbb{P}_1 \times \mathbb{P}_2}(a_1, a_2) e^{i\omega_{\mathbb{P}_1 \times \mathbb{P}_2}(a_1, a_2)} = \min\{\delta_{\mathbb{P}_1}(a_1), \delta_{\mathbb{P}_2}(a_2)\} e^{i \min\{\omega_{\mathbb{P}_1}(a_1), \omega_{\mathbb{P}_2}(a_2)\}}$
for all $(a_1, a_2) \in (\mathbb{P}_1 \times \mathbb{P}_2)$
(ii). $\mu_{\mathbb{Q}_1 \times \mathbb{Q}_2}((c, a_2)(c, b_2)) e^{i\alpha_{\mathbb{Q}_1 \times \mathbb{Q}_2}((c, a_2)(c, b_2))} = \min\{\mu_{\mathbb{P}_1}(c), \mu_{\mathbb{Q}_2}(a_2 b_2)\} e^{i \min\{\alpha_{\mathbb{P}_1}(c), \alpha_{\mathbb{Q}_2}(a_2 b_2)\}}$
$\gamma_{\mathbb{Q}_1 \times \mathbb{Q}_2}((c, a_2)(c, b_2)) e^{i\beta_{\mathbb{Q}_1 \times \mathbb{Q}_2}((c, a_2)(c, b_2))} = \min\{\gamma_{\mathbb{P}_1}(c), \gamma_{\mathbb{Q}_2}(a_2 b_2)\} e^{i \min\{\beta_{\mathbb{P}_1}(c), \beta_{\mathbb{Q}_2}(a_2 b_2)\}}$
$\delta_{\mathbb{Q}_1 \times \mathbb{Q}_2}((c, a_2)(c, b_2)) e^{i\omega_{\mathbb{Q}_1 \times \mathbb{Q}_2}((c, a_2)(c, b_2))} = \min\{\delta_{\mathbb{P}_1}(c), \delta_{\mathbb{Q}_2}(a_2 b_2)\} e^{i \min\{\omega_{\mathbb{P}_1}(c), \omega_{\mathbb{Q}_2}(a_2 b_2)\}}$
for all $c \in \mathbb{P}_1, a_2 b_2 \in \mathbb{Q}_2$
(iii). $\mu_{\mathbb{Q}_1 \times \mathbb{Q}_2}((a_1, d)(b_1, d)) e^{i\alpha_{\mathbb{Q}_1 \times \mathbb{Q}_2}((a_1, d)(b_1, d))} = \min\{\mu_{\mathbb{Q}_1}(a_1 b_1), \mu_{\mathbb{P}_2}(d)\} e^{i \min\{\alpha_{\mathbb{Q}_1}(a_1 b_1), \alpha_{\mathbb{P}_2}(d)\}}$
$\gamma_{\mathbb{Q}_1 \times \mathbb{Q}_2}((a_1, d)(b_1, d)) e^{i\beta_{\mathbb{Q}_1 \times \mathbb{Q}_2}((a_1, d)(b_1, d))} = \min\{\gamma_{\mathbb{Q}_1}(a_1 b_1), \gamma_{\mathbb{P}_2}(d)\} e^{i \min\{\beta_{\mathbb{Q}_1}(a_1 b_1), \beta_{\mathbb{P}_2}(d)\}}$
$\delta_{\mathbb{Q}_1 \times \mathbb{Q}_2}((a_1, d)(b_1, d)) e^{i\omega_{\mathbb{Q}_1 \times \mathbb{Q}_2}((a_1, d)(b_1, d))} = \min\{\delta_{\mathbb{Q}_1}(a_1 b_1), \delta_{\mathbb{P}_2}(d)\} e^{i \min\{\omega_{\mathbb{Q}_1}(a_1 b_1), \omega_{\mathbb{P}_2}(d)\}}$
for all $d \in \mathbb{P}_2, a_1 b_1 \in \mathbb{Q}_1$

Example 3.1.1:

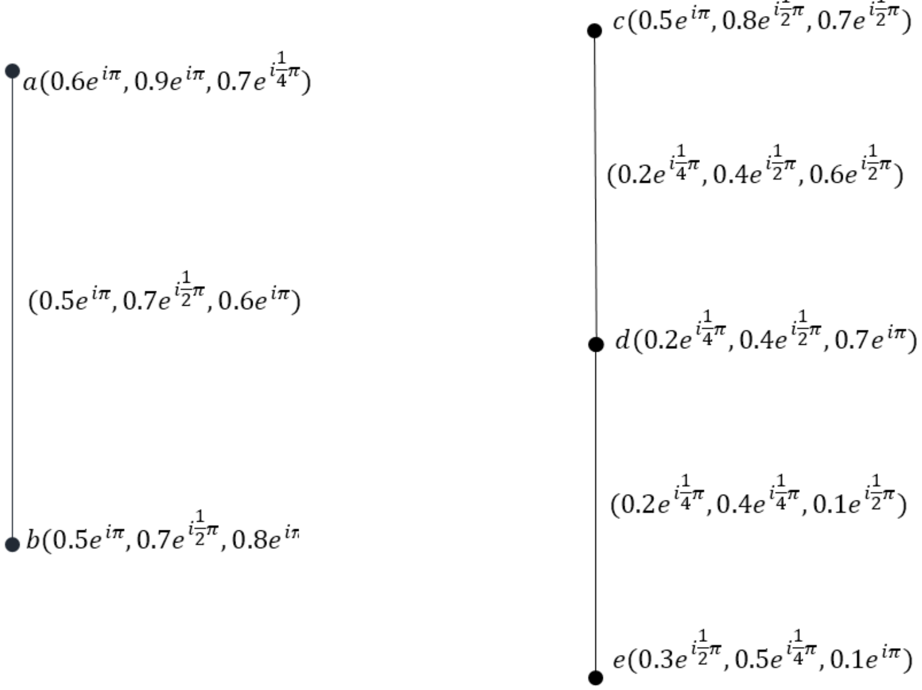

Figure 1: $\mathbb{G}'$ and $\mathbb{G}''$

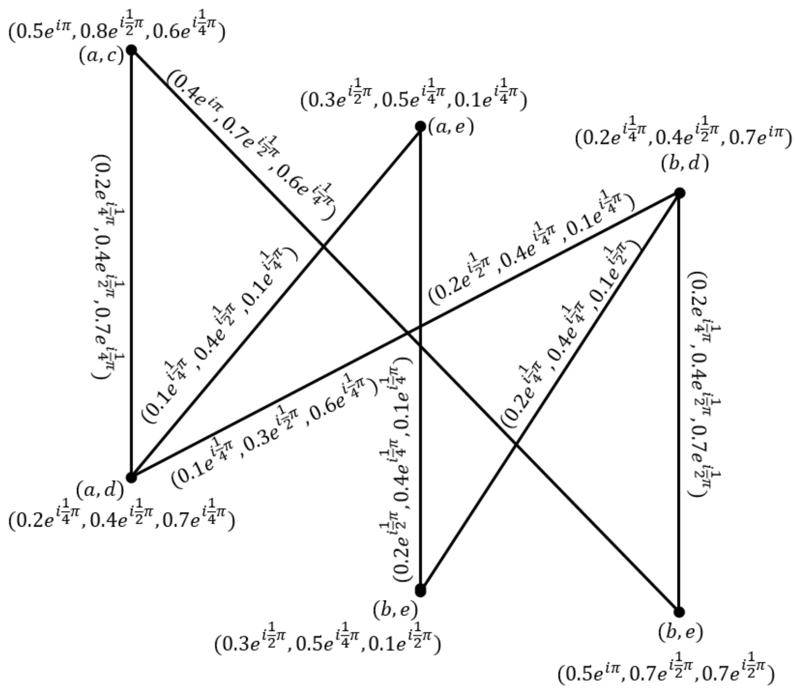

Figure 2: $\mathbb{G} = \mathbb{G}' \times \mathbb{G}''$

Proposition 3.1.2:
Let $\mathbb{G}' = (\mathbb{P}_1, \mathbb{Q}_1)$ and $\mathbb{G}'' = (\mathbb{P}_2, \mathbb{Q}_2)$ be two cohesive fuzzy graphs, where $\mathbb{P}$ is a vertex set and $\mathbb{Q}$ is a edge set respectively. Then there is a cartesian product between them $\mathbb{G}' \times \mathbb{G}''$ of $\mathbb{G}' = (\mathbb{P}_1, \mathbb{Q}_1)$ and $\mathbb{G}'' = (\mathbb{P}_2, \mathbb{Q}_2)$ is a cohesive fuzzy graph.
Proof:
Suppose that $\mathbb{G}' = (\mathbb{P}_1, \mathbb{Q}_1)$ and $\mathbb{G}'' = (\mathbb{P}_2, \mathbb{Q}_2)$ are two cohesive fuzzy graphs, where $\mathbb{P}$ is a vertex set and $\mathbb{Q}$ is a edge set respectively. Then cartesian product $\mathbb{G}' \times \mathbb{G}''$ of $\mathbb{G}' = (\mathbb{P}_1, \mathbb{Q}_1)$ and $\mathbb{G}'' = (\mathbb{P}_2, \mathbb{Q}_2)$ can be proved. Let $(a_1, a_2)(b_1, b_2) \in \mathbb{Q}_1 \times \mathbb{Q}_2$. For proving this we have three cases.
case 1: It is trivial.
case 2: If $a_1 = b_1 = c$ and Let $c \in \mathbb{P}_1$ and $a_2b_2 \in \mathbb{Q}_2$

$$\mu_{\mathbb{Q}_1\times\mathbb{Q}_2}((c,a_2)(c,b_2))e^{i\alpha_{\mathbb{Q}_1\times\mathbb{Q}_2}((c,a_2)(c,b_2))}$$
$$= \min\{\mu_{\mathbb{P}_1}(c), \mu_{\mathbb{Q}_2}(a_2b_2)\}e^{i\min\{\alpha_{\mathbb{P}_1}(c),\alpha_{\mathbb{Q}_2}(a_2b_2)\}}$$
$$\leq \min\{\mu_{\mathbb{P}_1}(c), \min\{\mu_{\mathbb{P}_2}(a_2), \mu_{\mathbb{P}_2}(b_2)\}\}e^{i\min\{\alpha_{\mathbb{P}_1}(c),\min\{\alpha_{\mathbb{P}_2}(a_2),\alpha_{\mathbb{P}_2}(b_2)\}\}}$$
$$= \min\{\min\{\mu_{\mathbb{P}_1}(c), \mu_{\mathbb{P}_2}(a_2)\}, \min\{\mu_{\mathbb{P}_1}(c), \mu_{\mathbb{P}_2}(b_2)\}\}e^{i\min\{\min\{\alpha_{\mathbb{P}_1}(c),\alpha_{\mathbb{P}_2}(a_2)\},\min\{\alpha_{\mathbb{P}_1}(c),\alpha_{\mathbb{P}_2}(b_2)\}\}}$$
$$= \min\{\mu_{\mathbb{P}_1\times\mathbb{P}_2}(c,a_2), \mu_{\mathbb{P}_1\times\mathbb{P}_2}(c,b_2)\}e^{i\min\{\alpha_{\mathbb{P}_1\times\mathbb{P}_2}(c,a_2),\alpha_{\mathbb{P}_1\times\mathbb{P}_2}(c,b_2)\}}$$

$$\gamma_{\mathbb{Q}_1\times\mathbb{Q}_2}((c,a_2)(c,b_2))e^{i\beta_{\mathbb{Q}_1\times\mathbb{Q}_2}((c,a_2)(c,b_2))}$$
$$= \min\{\gamma_{\mathbb{P}_1}(c), \mu_{\mathbb{Q}_2}(a_2b_2)\}e^{i\min\{\beta_{\mathbb{P}_1}(c),\beta_{\mathbb{Q}_2}(a_2b_2)\}}$$
$$\leq \min\{\gamma_{\mathbb{P}_1}(c), \min\{\gamma_{\mathbb{P}_2}(a_2), \gamma_{\mathbb{P}_2}(b_2)\}\}e^{i\min\{\beta_{\mathbb{P}_1}(c),\min\{\beta_{\mathbb{P}_2}(a_2),\beta_{\mathbb{P}_2}(b_2)\}\}}$$
$$= \min\{\min\{\gamma_{\mathbb{P}_1}(c), \gamma_{\mathbb{P}_2}(a_2)\}, \min\{\gamma_{\mathbb{P}_1}(c), \gamma_{\mathbb{P}_2}(b_2)\}\}e^{i\min\{\min\{\beta_{\mathbb{P}_1}(c),\beta_{\mathbb{P}_2}(a_2)\},\min\{\beta_{\mathbb{P}_1}(c),\beta_{\mathbb{P}_2}(b_2)\}\}}$$
$$= \min\{\gamma_{\mathbb{P}_1\times\mathbb{P}_2}(c,a_2), \mu_{\mathbb{P}_1\times\mathbb{P}_2}(c,b_2)\}e^{i\min\{\beta_{\mathbb{P}_1\times\mathbb{P}_2}(c,a_2),\beta_{\mathbb{P}_1\times\mathbb{P}_2}(c,b_2)\}}$$

$$\delta_{\mathbb{Q}_1\times\mathbb{Q}_2}((c,a_2)(c,b_2))e^{i\omega_{\mathbb{Q}_1\times\mathbb{Q}_2}((c,a_2)(c,b_2))}$$
$$= \min\{\delta_{\mathbb{P}_1}(c), \delta_{\mathbb{Q}_2}(a_2b_2)\}e^{i\min\{\omega_{\mathbb{P}_1}(c),\omega_{\mathbb{Q}_2}(a_2b_2)\}}$$
$$\leq \min\{\delta_{\mathbb{P}_1}(c), \min\{\delta_{\mathbb{P}_2}(a_2), \delta_{\mathbb{P}_2}(b_2)\}\}e^{i\min\{\omega_{\mathbb{P}_1}(c),\min\{\omega_{\mathbb{P}_2}(a_2),\omega_{\mathbb{P}_2}(b_2)\}\}}$$
$$= \min\{\min\{\delta_{\mathbb{P}_1}(c), \delta_{\mathbb{P}_2}(a_2)\}, \min\{\delta_{\mathbb{P}_1}(c), \delta_{\mathbb{P}_2}(b_2)\}\}e^{i\min\{\min\{\omega_{\mathbb{P}_1}(c),\omega_{\mathbb{P}_2}(a_2)\},\min\{\omega_{\mathbb{P}_1}(c),\omega_{\mathbb{P}_2}(b_2)\}\}}$$
$$= \min\{\delta_{\mathbb{P}_1\times\mathbb{P}_2}(c,a_2), \delta_{\mathbb{P}_1\times\mathbb{P}_2}(c,b_2)\}e^{i\min\{\omega_{\mathbb{P}_1\times\mathbb{P}_2}(c,a_2),\omega_{\mathbb{P}_1\times\mathbb{P}_2}(c,b_2)\}}$$

case 3: If $a_2 = b_2 = d$ and Let $d \in \mathbb{P}_2$ and $a_1b_1 \in \mathbb{Q}_1$

$$\mu_{\mathbb{Q}_1\times\mathbb{Q}_2}((a_1,d)(b_1,d))e^{i\alpha_{\mathbb{Q}_1\times\mathbb{Q}_2}((a_1,d)(b_1,d))}$$
$$= \min\{\mu_{\mathbb{Q}_1}(a_1b_1), \mu_{\mathbb{P}_2}(d)\}e^{i\min\{\alpha_{\mathbb{Q}_1}(a_1b_1),\alpha_{\mathbb{P}_2}(d)\}}$$
$$\leq \min\{\min\{\mu_{\mathbb{P}_1}(a_1), \mu_{\mathbb{P}_1}(b_1)\}, \mu_{\mathbb{P}_2}(d)\}e^{i\min\{\min\{\alpha_{\mathbb{P}_1}(a_1),\alpha_{\mathbb{P}_1}(b_1)\},\alpha_{\mathbb{P}_2}(d)\}}$$
$$= \min\{\min\{\mu_{\mathbb{P}_1}(a_1), \mu_{\mathbb{P}_2}(d)\}, \min\{\mu_{\mathbb{P}_1}(b_1), \mu_{\mathbb{P}_2}(d)\}\}e^{i\min\{\min\{\alpha_{\mathbb{P}_1}(a_1),\alpha_{\mathbb{P}_2}(d)\},\min\{\alpha_{\mathbb{P}_2}(b_1),\alpha_{\mathbb{P}_2}(d)\}\}}$$
$$= \min\{\mu_{\mathbb{P}_1\times\mathbb{P}_2}(a_1,d), \mu_{\mathbb{P}_1\times\mathbb{P}_2}(b_1,d)\}e^{i\min\{\alpha_{\mathbb{P}_1\times\mathbb{P}_2}(a_1,d),\alpha_{\mathbb{P}_1\times\mathbb{P}_2}(b_1,d)\}}$$

$$\gamma_{\mathbb{Q}_1\times\mathbb{Q}_2}((a_1,d)(b_1,d))e^{i\beta_{\mathbb{Q}_1\times\mathbb{Q}_2}((a_1,d)(b_1,d))}$$
$$= \min\{\gamma_{\mathbb{Q}_1}(a_1b_1), \gamma_{\mathbb{P}_2}(d)\}e^{i\min\{\beta_{\mathbb{Q}_1}(a_1b_1),\beta_{\mathbb{P}_2}(d)\}}$$

$\leq \min\{\min\{\gamma_{\mathbb{P}_1}(a_1), \gamma_{\mathbb{P}_1}(b_1)\}, \gamma_{\mathbb{P}_2}(d)\}e^{i\min\{\min\{\beta_{\mathbb{P}_1}(a_1),\beta_{\mathbb{P}_1}(b_1)\},\beta_{\mathbb{P}_2}(d)\}}$
$= \min\{\min\{\gamma_{\mathbb{P}_1}(a_1), \gamma_{\mathbb{P}_2}(d)\}, \min\{\gamma_{\mathbb{P}_1}(b_1), \gamma_{\mathbb{P}_2}(d)\}\}e^{i\min\{\min\{\beta_{\mathbb{P}_1}(a_1),\beta_{\mathbb{P}_2}(d)\},\min\{\beta_{\mathbb{P}_2}(b_1),\beta_{\mathbb{P}_2}(d)\}\}}$
$= \min\{\gamma_{\mathbb{P}_1\times\mathbb{P}_2}(a_1, d), \gamma_{\mathbb{P}_1\times\mathbb{P}_2}(b_1, d)\}e^{i\min\{\beta_{\mathbb{P}_1\times\mathbb{P}_2}(a_1,d),\beta_{\mathbb{P}_1\times\mathbb{P}_2}(b_1,d)\}}$

$\delta_{\mathbb{Q}_1\times\mathbb{Q}_2}((a_1, d)(b_1, d))e^{i\omega_{\mathbb{Q}_1\times\mathbb{Q}_2}((a_1,d)(b_1,d))}$
$= \min\{\delta_{\mathbb{Q}_1}(a_1b_1), \delta_{\mathbb{P}_2}(d)\}e^{i\min\{\omega_{\mathbb{Q}_1}(a_1b_1),\omega_{\mathbb{P}_2}(d)\}}$
$\leq \min\{\min\{\delta_{\mathbb{P}_1}(a_1), \delta_{\mathbb{P}_1}(b_1)\}, \delta_{\mathbb{P}_2}(d)\}e^{i\min\{\min\{\omega_{\mathbb{P}_1}(a_1),\omega_{\mathbb{P}_1}(b_1)\},\omega_{\mathbb{P}_2}(d)\}}$
$= \min\{\min\{\delta_{\mathbb{P}_1}(a_1), \delta_{\mathbb{P}_2}(d)\}, \min\{\delta_{\mathbb{P}_1}(b_1), \delta_{\mathbb{P}_2}(d)\}\}e^{i\min\{\min\{\omega_{\mathbb{P}_1}(a_1),\omega_{\mathbb{P}_2}(d)\},\min\{\omega_{\mathbb{P}_2}(b_1),\omega_{\mathbb{P}_2}(d)\}\}}$
$= \min\{\delta_{\mathbb{P}_1\times\mathbb{P}_2}(a_1, d), \delta_{\mathbb{P}_1\times\mathbb{P}_2}(b_1, d)\}e^{i\min\{\omega_{\mathbb{P}_1\times\mathbb{P}_2}(a_1,d),\omega_{\mathbb{P}_1\times\mathbb{P}_2}(b_1,d)\}}$

Hence the cartesian product $\mathbb{G}' \times \mathbb{G}''$ of two cohesive fuzzy graph is a cohesive fuzzy graph.

3.2. Direct product of two cohesive fuzzy graph:

The Direct product of two cohesive fuzzy graphs $\mathbb{G}'$ and $\mathbb{G}''$ is defined as a pair $\mathbb{G}' \bullet \mathbb{G}'' = (\mathbb{P}_1 \bullet \mathbb{P}_2, \mathbb{Q}_1 \bullet \mathbb{Q}_2)$, such that:

(i). $\mu_{\mathbb{P}_1\bullet\mathbb{P}_2}(a_1, a_2)e^{i\alpha_{\mathbb{P}_1\bullet\mathbb{P}_2}(a_1,a_2)} = \min\{\mu_{\mathbb{P}_1}(a_1), \mu_{\mathbb{P}_2}(a_2)\}e^{i\min\{\alpha_{\mathbb{P}_1}(a_1),\alpha_{\mathbb{P}_2}(a_2)\}}$
$\gamma_{\mathbb{P}_1\bullet\mathbb{P}_2}(a_1, a_2)e^{i\beta_{\mathbb{P}_1\bullet\mathbb{P}_2}(a_1,a_2)} = \min\{\gamma_{\mathbb{P}_1}(a_1), \gamma_{\mathbb{P}_2}(a_2)\}e^{i\min\{\beta_{\mathbb{P}_1}(a_1),\beta_{\mathbb{P}_2}(a_2)\}}$
$\delta_{\mathbb{P}_1\bullet\mathbb{P}_2}(a_1, a_2)e^{i\omega_{\mathbb{P}_1\bullet\mathbb{P}_2}(a_1,a_2)} = \min\{\delta_{\mathbb{P}_1}(a_1), \delta_{\mathbb{P}_2}(a_2)\}e^{i\min\{\omega_{\mathbb{P}_1}(a_1),\omega_{\mathbb{P}_2}(a_2)\}}$
for all $(a_1, a_2) \in \mathbb{P}_1 \times \mathbb{P}_2$
(ii). $\mu_{\mathbb{Q}_1\bullet\mathbb{Q}_2}((a_1, a_2)(b_1, b_2))e^{i\alpha_{\mathbb{Q}_1\bullet\mathbb{Q}_2}((a_1,a_2)(b_1,b_2))} = \min\{\mu_{\mathbb{Q}_1}(a_1b_1), \mu_{\mathbb{Q}_2}(a_2b_2)\}e^{i\min\{\alpha_{\mathbb{Q}_1}(a_1b_1),\alpha_{\mathbb{Q}_2}(a_2b_2)\}}$
$\gamma_{\mathbb{Q}_1\bullet\mathbb{Q}_2}((a_1, a_2)(b_1, b_2))e^{i\beta_{\mathbb{Q}_1\bullet\mathbb{Q}_2}((a_1,a_2)(b_1,b_2))} = \min\{\gamma_{\mathbb{Q}_1}(a_1b_1), \gamma_{\mathbb{Q}_2}(a_2b_2)\}e^{i\min\{\beta_{\mathbb{Q}_1}(a_1b_1),\beta_{\mathbb{Q}_2}(a_2b_2)\}}$,
$\delta_{\mathbb{Q}_1\bullet\mathbb{Q}_2}((a_1, a_2)(b_1, b_2))e^{i\omega_{\mathbb{Q}_1\bullet\mathbb{Q}_2}((a_1,a_2)(b_1,b_2))} = \min\{\delta_{\mathbb{Q}_1}(a_1b_1), \delta_{\mathbb{Q}_2}(a_2b_2)\}e^{i\min\{\omega_{\mathbb{Q}_1}(a_1b_1),\omega_{\mathbb{Q}_2}(a_2b_2)\}}$
for all $(a_1b_1) \in \mathbb{Q}_1$ and for all $(a_2b_2) \in \mathbb{Q}_2$

Example 3.2.1:

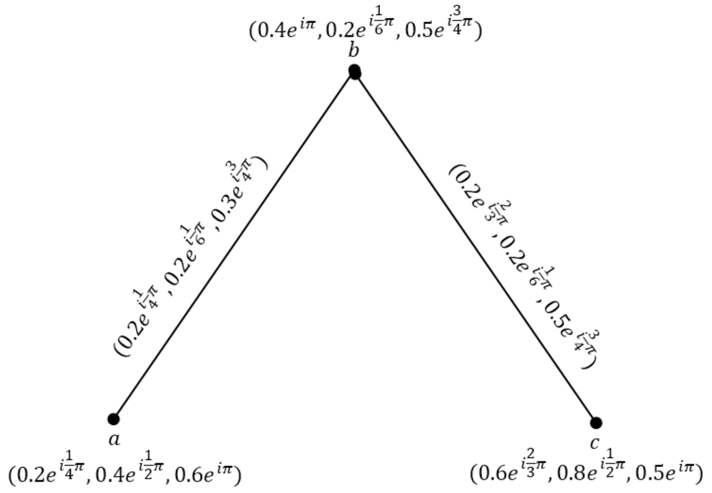

Figure 3: $\mathbb{G}'$

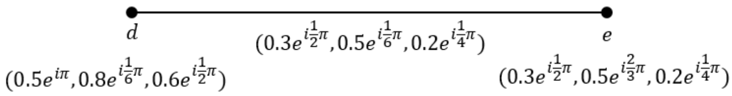

Figure 4: $\mathbb{G}''$

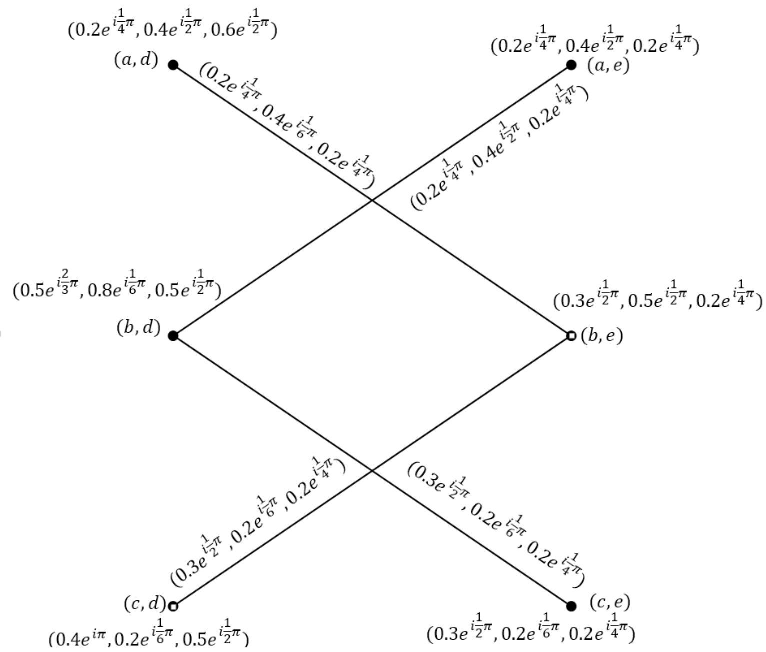

Figure 5: $\mathbb{G} = \mathbb{G}' \times \mathbb{G}''$

Proposition 3.2.2:
Let $\mathbb{G}' = (\mathbb{P}_1, \mathbb{Q}_1)$ and $\mathbb{G}'' = (\mathbb{P}_2, \mathbb{Q}_2)$ be two cohesive fuzzy graphs, where $\mathbb{P}$ is a vertex set and $\mathbb{Q}$ is a edge set respectively. Then there is the direct product between them is a cohesive fuzzy graph.

Proof:
Suppose that $\mathbb{G}' = (\mathbb{P}_1, \mathbb{Q}_1)$ and $\mathbb{G}'' = (\mathbb{P}_2, \mathbb{Q}_2)$ are two cohesive fuzzy graphs, where $\mathbb{P}$ is a vertex set and $\mathbb{Q}$ is a edge set respectively. Then, the direct product $\mathbb{G}' \bullet \mathbb{G}''$ of $\mathbb{G}' = (\mathbb{P}_1, \mathbb{Q}_1)$ and $\mathbb{G}'' = (\mathbb{P}_2, \mathbb{Q}_2)$ can be proved. Let $(a_1, a_2)(b_1, b_2) \in \mathbb{Q}_1 \times \mathbb{Q}_2$. For proving this we have three cases.

case 1: It is trivial.

case 2: If $a_1b_1 \in \mathbb{Q}_1$ and $a_2b_2 \in \mathbb{Q}_2$

$\mu_{\mathbb{Q}_1 \bullet \mathbb{Q}_2}((a_1, b_1)(a_2, b_2))e^{i\alpha_{\mathbb{Q}_1 \bullet \mathbb{Q}_2}((a_1,b_1)(a_2,b_2))}$

$= \min\{\mu_{\mathbb{Q}_1}(a_1b_1), \mu_{\mathbb{Q}_2}(a_2b_2)\}e^{i\min\{\alpha_{\mathbb{Q}_1}((a_1b_1), \alpha_{\mathbb{Q}_2}(a_2b_2)\}}$

$\leq \min\{\min\{\mu_{\mathbb{P}_1}(a_1), \mu_{\mathbb{P}_2}(a_2)\}, \mu_{\mathbb{P}_1}(b_1), \mu_{\mathbb{P}_2}(b_2)\}e^{i\min\{\min\{\alpha_{\mathbb{P}_1}(a_1), \alpha_{\mathbb{P}_2}(a_2)\}, \alpha_{\mathbb{P}_1}(b_1), \alpha_{\mathbb{P}_2}(b_2)\}}$

$= \min\{\mu_{\mathbb{P}_1 \bullet \mathbb{P}_2}(a_1, a_2), \mu_{\mathbb{P}_1 \bullet \mathbb{P}_2}(b_1, b_2)\}e^{i\min\{\alpha_{\mathbb{P}_1 \bullet \mathbb{P}_2}(a_1,a_2), \alpha_{\mathbb{P}_1 \bullet \mathbb{P}_2}(b_1,b_2)\}}$

$$\gamma_{\mathbb{Q}_1\bullet\mathbb{Q}_2}((a_1b_1)(a_2b_2))e^{i\beta_{\mathbb{Q}_1\bullet\mathbb{Q}_2}((a_1,b_1)(a_2,b_2))}$$
$$=\min\{\gamma_{\mathbb{Q}_1}(a_1b_1),\gamma_{\mathbb{Q}_2}(a_2b_2)\}e^{i\min\{\beta_{\mathbb{Q}_1}((a_1b_1),\beta_{\mathbb{Q}_2}(a_2b_2)\}}$$
$$\leq \min\{\min\{\gamma_{\mathbb{P}_1}(a_1),\gamma_{\mathbb{P}_2}(a_2)\},\gamma_{\mathbb{P}_1}(b_1),\gamma_{\mathbb{P}_2}(b_2)\}e^{i\min\{\min\{\beta_{\mathbb{P}_1}(a_1),\beta_{\mathbb{P}_2}(a_2)\},\beta_{\mathbb{P}_1}(b_1),\beta_{\mathbb{P}_2}(b_2)\}}$$
$$=\min\{\gamma_{\mathbb{P}_1\bullet\mathbb{P}_2}(a_1,a_2),\gamma_{\mathbb{P}_1\bullet\mathbb{P}_2}(b_1,b_2)\}e^{i\min\{\beta_{\mathbb{P}_1\bullet\mathbb{P}_2}(a_1,a_2),\beta_{\mathbb{P}_1\bullet\mathbb{P}_2}(b_1,b_2)\}}$$

$$\delta_{\mathbb{Q}_1\bullet\mathbb{Q}_2}((a_1b_1)(a_2b_2))e^{i\omega_{\mathbb{Q}_1\bullet\mathbb{Q}_2}((a_1,b_1)(a_2,b_2))}$$
$$=\min\{\delta_{\mathbb{Q}_1}(a_1b_1),\delta_{\mathbb{Q}_2}(a_2b_2)\}e^{i\min\{\omega_{\mathbb{Q}_1}((a_1b_1),\omega_{\mathbb{Q}_2}(a_2b_2)\}}$$
$$\leq \min\{\min\{\delta_{\mathbb{P}_1}(a_1),\delta_{\mathbb{P}_2}(a_2)\},\delta_{\mathbb{P}_1}(b_1),\delta_{\mathbb{P}_2}(b_2)\}e^{i\min\{\min\{\omega_{\mathbb{P}_1}(a_1),\omega_{\mathbb{P}_2}(a_2)\},\omega_{\mathbb{P}_1}(b_1),\omega_{\mathbb{P}_2}(b_2)\}}$$
$$=\min\{\delta_{\mathbb{P}_1\bullet\mathbb{P}_2}(a_1,a_2),\delta_{\mathbb{P}_1\bullet\mathbb{P}_2}(b_1,b_2)\}e^{i\min\{\omega_{\mathbb{P}_1\bullet\mathbb{P}_2}(a_1,a_2),\omega_{\mathbb{P}_1\bullet\mathbb{P}_2}(b_1,b_2)\}}$$

Definition 3.3:
The strong product $\mathbb{G}'\otimes\mathbb{G}''$ of two cohesive fuzzy graph is defined as a pair $\mathbb{G}'\otimes\mathbb{G}''=(\mathbb{P}_1\otimes\mathbb{P}_2,\mathbb{Q}_1\otimes\mathbb{Q}_2)$, such that

(i). $\mu_{\mathbb{P}_1\otimes\mathbb{P}_2}(a_1,a_2)e^{i\alpha_{\mathbb{P}_1\otimes\mathbb{P}_2}(a_1,a_2)}=\min\{\mu_{\mathbb{P}_1}(a_1),\mu_{\mathbb{P}_2}(a_2)\}e^{i\min\{\alpha_{\mathbb{P}_1}(a_1),\alpha_{\mathbb{P}_2}(a_2)\}}$
$\gamma_{\mathbb{P}_1\otimes\mathbb{P}_2}(a_1,a_2)e^{i\beta_{\mathbb{P}_1\otimes\mathbb{P}_2}(a_1,a_2)}=\min\{\gamma_{\mathbb{P}_1}(a_1),\gamma_{\mathbb{P}_2}(a_2)\}e^{i\min\{\beta_{\mathbb{P}_1}(a_1),\beta_{\mathbb{P}_2}(a_2)\}}$
$\delta_{\mathbb{P}_1\otimes\mathbb{P}_2}(a_1,a_2)e^{i\omega_{\mathbb{P}_1\otimes\mathbb{P}_2}(a_1,a_2)}=\min\{\delta_{\mathbb{P}_1}(a_1),\delta_{\mathbb{P}_2}(a_2)\}e^{i\min\{\omega_{\mathbb{P}1}(a_1),\omega_{\mathbb{P}_2}(a_2)\}}$
for all $(a_1,a_2)\in(\mathbb{P}_1\times\mathbb{P}_2)$

(ii). $\mu_{\mathbb{Q}_1\otimes\mathbb{Q}_2}((c,a_2)(c,b_2))e^{i\alpha_{\mathbb{Q}_1\otimes\mathbb{Q}_2}((c,a_2)(c,b_2))}=\min\{\mu_{\mathbb{P}_1}(c),\mu_{\mathbb{Q}_2}(a_2b_2)\}e^{i\min\{\alpha_{\mathbb{P}_1}(c),\alpha_{\mathbb{Q}_2}(a_2b_2)\}}$
$\gamma_{\mathbb{Q}_1\otimes\mathbb{Q}_2}((c,a_2)(c,b_2))e^{i\beta_{\mathbb{Q}_1\otimes\mathbb{Q}_2}((c,a_2)(c,b_2))}=\min\{\gamma_{\mathbb{P}_1}(c),\gamma_{\mathbb{Q}_2}(a_2b_2)\}e^{i\min\{\beta_{\mathbb{P}_1}(c),\beta_{\mathbb{Q}_2}(a_2b_2)\}}$
$\delta_{\mathbb{Q}_1\otimes\mathbb{Q}_2}((c,a_2)(c,b_2))e^{i\omega_{\mathbb{Q}_1\otimes\mathbb{Q}_2}((c,a_2)(c,b_2))}=\min\{\delta_{\mathbb{P}_1}(c),\delta_{\mathbb{Q}_2}(a_2b_2)\}e^{i\min\{\omega_{\mathbb{P}_1}(c),\omega_{\mathbb{Q}_2}(a_2b_2)\}}$
for all $c\in\mathbb{P}_1, a_2b_2\in\mathbb{Q}_2$

(iii). $\mu_{\mathbb{Q}_1\otimes\mathbb{Q}_2}((a_1,d)(b_1,d))e^{i\alpha_{\mathbb{Q}_1\otimes\mathbb{Q}_2}((a_1,d)(b_1,d))}=\min\{\mu_{\mathbb{Q}_1}(a_1b_1),\mu_{\mathbb{P}_2}(d)\}e^{i\min\{\alpha_{\mathbb{Q}_1}(a_1b_1),\alpha_{\mathbb{P}_2}(d)\}}$
$\gamma_{\mathbb{Q}_1\otimes\mathbb{Q}_2}((a_1,d)(b_1,d))e^{i\beta_{\mathbb{Q}_1\otimes\mathbb{Q}_2}((a_1,d)(b_1,d))}=\min\{\gamma_{\mathbb{Q}_1}(a_1b_1),\gamma_{\mathbb{P}_2}(d)\}e^{i\min\{\beta_{\mathbb{Q}_1}(a_1b_1),\beta_{\mathbb{P}_2}(d)\}}$
$\delta_{\mathbb{Q}_1\otimes\mathbb{Q}_2}((a_1,d)(b_1,d))e^{i\omega_{\mathbb{Q}_1\otimes\mathbb{Q}_2}((a_1,d)(b_1,d))}=\min\{\delta_{\mathbb{Q}_1}(a_1b_1),\delta_{\mathbb{P}_2}(d)\}e^{i\min\{\omega_{\mathbb{Q}_1}(a_1b_1),\omega_{\mathbb{P}_2}(d)\}}$
for all $d\in\mathbb{P}_2, a_1b_1\in\mathbb{Q}_1$

(iv). $\mu_{\mathbb{Q}_1\otimes\mathbb{Q}_2}((a_1,a_2)(b_1,b_2))e^{i\alpha_{\mathbb{Q}_1\otimes\mathbb{Q}_2}((a_1,a_2)(b_1,b_2))}=\min\{\mu_{\mathbb{Q}_1}(a_1b_1),\mu_{\mathbb{Q}_2}(a_2b_2)\}e^{i\min\{\alpha_{\mathbb{Q}_1}(a_1b_1),\alpha_{\mathbb{Q}_2}(a_2b_2)\}}$
$\gamma_{\mathbb{Q}_1\otimes\mathbb{Q}_2}((a_1,a_2)(b_1,b_2))e^{i\beta_{Q_1\otimes\mathbb{Q}_2}((a_1,a_2)(b_1,b_2))}=\min\{\gamma_{\mathbb{Q}_1}(a_1b_1),\gamma_{\mathbb{Q}_2}(a_2b_2)\}e^{i\min\{\beta_{\mathbb{Q}_1}(a_1b_1),\beta_{\mathbb{Q}_2}(a_2b_2)\}}$,
$\delta_{\mathbb{Q}_1\otimes\mathbb{Q}_2}((a_1,a_2)(b_1,b_2))e^{i\omega_{\mathbb{Q}_1\otimes\mathbb{Q}_2}((a_1,a_2)(b_1,b_2))}=\min\{\delta_{\mathbb{Q}_1}(a_1b_1),\delta_{\mathbb{Q}_2}(a_2b_2)\}e^{i\min\{\omega_{\mathbb{Q}_1}(a_1b_1),\omega_{\mathbb{Q}_2}(a_2b_2)\}}$
for all $a_1b_1\in\mathbb{Q}_1$ and for all $a_2b_2\in\mathbb{Q}_2$

Example 3.3.1:

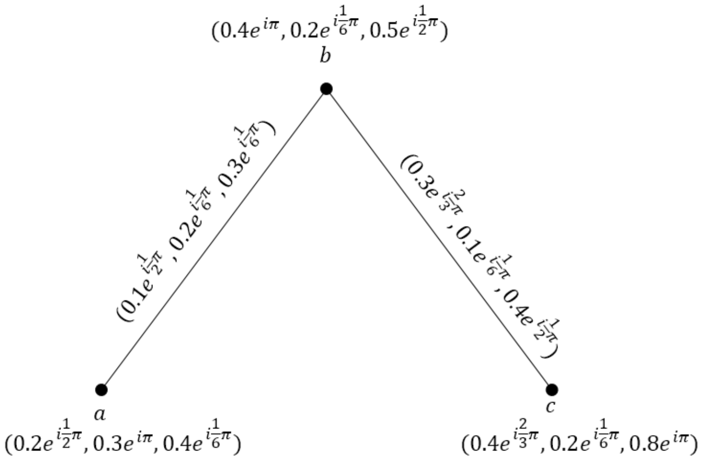

Figure 6: $\mathbb{G}'$

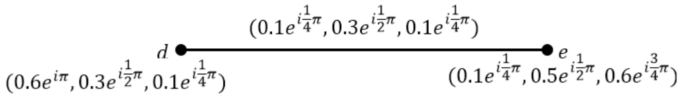

Figure 7: $\mathbb{G}''$

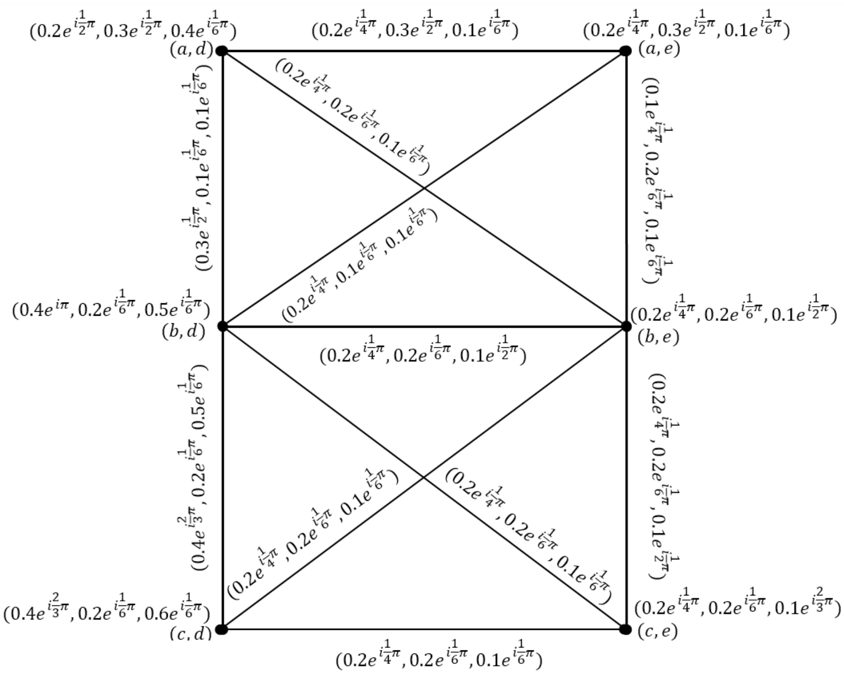

Figure 8: $\mathbb{G} = \mathbb{G}' \otimes \mathbb{G}''$

Proposition 3.3.2:
Let $\mathbb{G}' = (\mathbb{P}_1, \mathbb{Q}_1)$ and $\mathbb{G}'' = (\mathbb{P}_2, \mathbb{Q}_2)$ be two cohesive fuzzy graphs, where $\mathbb{P}$ is a vertex set and $\mathbb{Q}$ is a edge set respectively. Then there is a strong product between them $\mathbb{G}' \otimes \mathbb{G}''$ of $\mathbb{G}' = (\mathbb{P}_1, \mathbb{Q}_1)$ and $\mathbb{G}'' = (\mathbb{P}_2, \mathbb{Q}_2)$ is a cohesive fuzzy graph.
Proof:
Suppose that $\mathbb{G}' = (\mathbb{P}_1, \mathbb{Q}_1)$ and $\mathbb{G}'' = (\mathbb{P}_2, \mathbb{Q}_2)$ are two cohesive fuzzy graphs, where $\mathbb{P}$ is a vertex set and $\mathbb{Q}$ is a edge set respectively. Then, the strong product $\mathbb{G}' \otimes \mathbb{G}''$ of $\mathbb{G}' = (\mathbb{P}_1, \mathbb{Q}_1)$ and $\mathbb{G}'' = (\mathbb{P}_2, \mathbb{Q}_2)$ can be proved. Let $(a_1, a_2)(b_1, b_2) \in \mathbb{Q}_1 \times \mathbb{Q}_2$. For proving this we have three cases.
case 1: It is trivial.
case 2: If $a_1 = b_1 = c$ and Let $a \in \mathbb{P}_1$ and $a_2b_2 \in \mathbb{Q}_2$

$$\mu_{\mathbb{Q}_1\otimes\mathbb{Q}_2}((c,a_2)(c,b_2))e^{i\alpha_{\mathbb{Q}_1\otimes\mathbb{Q}_2}((c,a_2)(c,b_2))}$$
$$= \min\{\mu_{\mathbb{P}_1}(c), \mu_{\mathbb{Q}_2}(a_2b_2)\}e^{i\min\{\alpha_{\mathbb{P}_1}(c),\alpha_{\mathbb{Q}_2}(a_2b_2)\}}$$
$$\leq \min\{\mu_{\mathbb{P}_1}(c), \min\{\mu_{\mathbb{P}_2}(a_2), \mu_{\mathbb{P}_2}(b_2)\}\}e^{i\min\{\alpha_{\mathbb{P}_1}(c),\min\{\alpha_{\mathbb{P}_2}(a_2),\alpha_{\mathbb{P}_2}(b_2)\}\}},$$
$$= \min\{\min\{\mu_{\mathbb{P}_1}(c), \mu_{\mathbb{P}_2}(a_2)\}, \min\{\mu_{\mathbb{P}_1}(c), \mu_{\mathbb{P}_2}(b_2)\}\}e^{i\min\{\min\{\alpha_{\mathbb{P}_1}(c),\alpha_{\mathbb{P}_2}(a_2)\},\min\{\alpha_{\mathbb{P}_1}(c),\alpha_{\mathbb{P}_2}(b_2)\}\}}$$
$$= \min\{\mu_{\mathbb{P}_1\otimes\mathbb{P}_2}(c,a_2), \mu_{\mathbb{P}_1\otimes\mathbb{P}_2}(c,b_2)\}e^{i\min\{\alpha_{\mathbb{P}_1\otimes\mathbb{P}_2}(c,a_2),\alpha_{\mathbb{P}_1\otimes\mathbb{P}_2}(c,b_2)\}}$$

$$\gamma_{\mathbb{Q}_1\otimes\mathbb{Q}_2}((c,a_2)(c,b_2))e^{i\beta_{\mathbb{Q}_1\otimes\mathbb{Q}_2}((c,a_2)(c,b_2))}$$
$$= \min\{\gamma_{\mathbb{P}_1}(c), \gamma_{\mathbb{Q}_2}(a_2b_2)\}e^{i\min\{\beta_{\mathbb{P}_1}(c),\beta_{\mathbb{Q}_2}(a_2b_2)\}}$$
$$\leq \min\{\gamma_{\mathbb{P}_1}(c), \min\{\gamma_{\mathbb{P}_2}(a_2), \gamma_{\mathbb{P}_2}(b_2)\}\}e^{i\min\{\beta_{\mathbb{P}_1}(c),\min\{\beta_{\mathbb{P}_2}(a_2),\beta_{\mathbb{P}_2}(b_2)\}\}},$$
$$= \min\{\min\{\gamma_{\mathbb{P}_1}(c), \gamma_{\mathbb{P}_2}(a_2)\}, \min\{\gamma_{\mathbb{P}_1}(c), \gamma_{\mathbb{P}_2}(b_2)\}\}e^{i\min\{\min\{\beta_{\mathbb{P}_1}(c),\beta_{\mathbb{P}_2}(a_2)\},\min\{\beta_{\mathbb{P}_1}(c),\beta_{\mathbb{P}_2}(b_2)\}\}}$$
$$= \min\{\gamma_{\mathbb{P}_1\otimes\mathbb{P}_2}(c,a_2), \gamma_{\mathbb{P}_1\otimes\mathbb{P}_2}(c,b_2)\}e^{i\min\{\beta_{\mathbb{P}_1\otimes\mathbb{P}_2}(c,a_2),\beta_{\mathbb{P}_1\otimes\mathbb{P}_2}(c,b_2)\}}$$

$$\delta_{\mathbb{Q}_1\otimes\mathbb{Q}_2}((c,a_2)(c,b_2))e^{i\omega_{\mathbb{Q}_1\otimes\mathbb{Q}_2}((c,a_2)(c,b_2))}$$
$$= \min\{\delta_{\mathbb{P}_1}(c), \delta_{\mathbb{Q}_2}(a_2b_2)\}e^{i\min\{\omega_{\mathbb{P}_1}(c),\omega_{\mathbb{Q}_2}(a_2b_2)\}}$$
$$\leq \min\{\delta_{\mathbb{P}_1}(c), \min\{\delta_{\mathbb{P}_2}(a_2), \delta_{\mathbb{P}_2}(b_2)\}\}e^{i\min\{\omega_{\mathbb{P}_1}(c),\min\{\omega_{\mathbb{P}_2}(a_2),\omega_{\mathbb{P}_2}(b_2)\}\}},$$
$$= \min\{\min\{\delta_{\mathbb{P}_1}(c), \delta_{\mathbb{P}_2}(a_2)\}, \min\{\delta_{\mathbb{P}_1}(c), \delta_{\mathbb{P}_2}(b_2)\}\}e^{i\min\{\min\{\omega_{\mathbb{P}_1}(c),\omega_{\mathbb{P}_2}(a_2)\},\min\{\omega_{\mathbb{P}_1}(c),\omega_{\mathbb{P}_2}(b_2)\}\}}$$
$$= \min\{\delta_{\mathbb{P}_1\otimes\mathbb{P}_2}(c,a_2), \delta_{\mathbb{P}_1\otimes\mathbb{P}_2}(c,b_2)\}e^{i\min\{\omega_{\mathbb{P}_1\otimes\mathbb{P}_2}(c,a_2),\omega_{\mathbb{P}_1\otimes\mathbb{P}_2}(c,b_2)\}}$$

case 3: If $a_2 = b_2 = d$ and Let $d \in \mathbb{P}_2$ and $a_1b_1 \in \mathbb{Q}_1$

$$\mu_{\mathbb{Q}_1\otimes\mathbb{Q}_2}((a_1,d)(b_1,d))e^{i\alpha_{\mathbb{Q}_1\otimes\mathbb{Q}_2}((a_1,d)(b_1,d))}$$
$$= \min\{\mu_{\mathbb{Q}_1}(a_1b_1), \mu_{\mathbb{P}_2}(d)\}e^{i\min\{\alpha_{\mathbb{Q}_1}(a_1b_1),\alpha_{\mathbb{P}_2}(d)\}}$$
$$\leq \min\{\min\{\mu_{\mathbb{P}_1}(a_1), \mu_{\mathbb{P}_1}(b_1)\}, \mu_{\mathbb{P}_2}(d)\}e^{i\min\{\min\{\alpha_{\mathbb{P}_1}(a_1),\alpha_{\mathbb{P}_1}(b_1)\},\alpha_{\mathbb{P}_2}(d)\}}$$
$$= \min\{\min\{\mu_{\mathbb{P}_1}(a_1), \mu_{\mathbb{P}_2}(d)\}, \min\{\mu_{\mathbb{P}_1}(b_1), \mu_{\mathbb{P}_2}(d)\}\}e^{i\min\{\min\{\alpha_{\mathbb{P}_1}(a_1),\alpha_{\mathbb{P}_2}(d)\},\min\{\alpha_{\mathbb{P}_2}(b_1),\alpha_{\mathbb{P}_2}(d)\}\}}$$
$$= \min\{\mu_{\mathbb{P}_1\otimes\mathbb{P}_2}(a_1,d), \mu_{\mathbb{P}_1\otimes\mathbb{P}2}(b_1,d)\}e^{i\min\{\alpha_{\mathbb{P}_1\otimes\mathbb{P}_2}(a_1,d),\alpha_{\mathbb{P}_1\otimes\mathbb{P}_2}(b_1,d)\}}$$

$$\gamma_{\mathbb{Q}_1\otimes\mathbb{Q}_2}((a_1,d)(b_1,d))e^{i\beta_{\mathbb{Q}_1\otimes\mathbb{Q}_2}((a_1,d)(b_1,d))}$$
$$= \min\{\gamma_{\mathbb{Q}_1}(a_1b_1), \gamma_{\mathbb{P}_2}(d)\}e^{i\min\{\beta_{\mathbb{Q}_1}(a_1b_1),\beta_{\mathbb{P}_2}(d)\}}$$

$\leq \min\{\min\{\gamma_{\mathbb{P}_1}(a_1), \gamma_{\mathbb{P}_1}(b_1)\}, \gamma_{\mathbb{P}_2}(d)\}e^{i\min\{\min\{\beta_{\mathbb{P}_1}(a_1),\beta_{\mathbb{P}_1}(b_1)\},\beta_{\mathbb{P}_2}(d)\}}$
$= \min\{\min\{\gamma_{\mathbb{P}_1}(a_1), \gamma_{\mathbb{P}_2}(d)\}, \min\{\gamma_{\mathbb{P}_1}(b_1), \gamma_{\mathbb{P}_2}(d)\}\}e^{i\min\{\min\{\beta_{\mathbb{P}_1}(a_1),\beta_{\mathbb{P}_2}(d)\},\min\{\beta_{\mathbb{P}_2}(b_1),\beta_{\mathbb{P}_2}(d)\}\}}$
$= \min\{\gamma_{\mathbb{P}_1\otimes\mathbb{P}_2}(a_1, d), \gamma_{\mathbb{P}_1\otimes\mathbb{P}_2}(b_1, d)\}e^{i\min\{\beta_{\mathbb{P}_1\otimes\mathbb{P}_2}(a_1,d),\beta_{\mathbb{P}_1\otimes\mathbb{P}_2}(b_1,d)\}}$

$\delta_{\mathbb{Q}_1\otimes\mathbb{Q}_2}((a_1, d)(b_1, d))e^{i\omega_{\mathbb{Q}_1\otimes\mathbb{Q}_2}((a_1,d)(b_1,d))}$
$= \min\{\delta_{\mathbb{Q}_1}(a_1b_1), \delta_{\mathbb{P}_2}(d)\}e^{i\min\{\omega_{\mathbb{Q}_1}(a_1b_1),\omega_{\mathbb{P}_2}(d)\}}$
$\leq \min\{\min\{\delta_{\mathbb{P}_1}(a_1), \delta_{\mathbb{P}_1}(b_1)\}, \delta_{\mathbb{P}_2}(d)\}e^{i\min\{\min\{\omega_{\mathbb{P}_1}(a_1),\omega_{\mathbb{P}_1}(b_1)\},\omega_{\mathbb{P}_2}(d)\}}$
$= \min\{\min\{\delta_{\mathbb{P}_1}(a_1), \delta_{\mathbb{P}_2}(d)\}, \min\{\delta_{\mathbb{P}_1}(b_1), \delta_{\mathbb{P}_2}(d)\}\}e^{i\min\{\min\{\omega_{\mathbb{P}_1}(a_1),\omega_{\mathbb{P}_2}(d)\},\min\{\omega_{\mathbb{P}_1}(b_1),\omega_{\mathbb{P}_2}(d)\}\}}$
$= \min\{\delta_{\mathbb{P}_1\otimes\mathbb{P}_2}(a_1, d), \delta_{\mathbb{P}_1\otimes\mathbb{P}_2}(b_1, d)\}e^{i\min\{\omega_{\mathbb{P}_1\otimes\mathbb{P}_2}(a_1,d),\omega_{\mathbb{P}_1\otimes\mathbb{P}_2}(b_1,d)\}}$

case 4: If $a_1 \neq b_1$ and $a_2 \neq b_2$, Let $a_1b_1 \in \mathbb{Q}_1$ and $a_2b_2 \in \mathbb{Q}_2$
$\mu_{\mathbb{Q}_1\otimes\mathbb{Q}_2}((a_1, a_2)(b_1, b_2))e^{i\alpha_{\mathbb{Q}_1\otimes\mathbb{Q}_2}((a_1,a_2)(b_1,b_2))}$
$= \min\{\mu_{\mathbb{Q}_1}(a_1b_1), \mu_{\mathbb{Q}_2}(a_2b_2)e^{i\min\{\alpha_{\mathbb{Q}_1}(a_1b_1),\alpha_{\mathbb{Q}_2}(a_2b_2)\}}$
$\leq \min\{\min\{\mu_{\mathbb{P}_1}(a_1), \mu_{\mathbb{P}_1}(b_1)\}, \{\mu_{\mathbb{P}_2}(a_2), \mu_{\mathbb{P}_2}(b_2)\}\}e^{i\min\{\min\{\alpha_{\mathbb{P}_1}(a_1),\alpha_{\mathbb{P}_1}(b_1)\},\{\alpha_{\mathbb{P}_2}(a_2),\alpha_{\mathbb{P}_2}(b_2)\}\}}$
$= \min\{\min\{\mu_{\mathbb{P}_1}(a_1), \mu_{\mathbb{P}_2}(a_2)\}, \min\{\mu_{\mathbb{P}_1}(b_1), \mu_{\mathbb{P}_2}(b_2)\}\}e^{i\min\{\min\{\alpha_{\mathbb{P}_1}(a_1),\alpha_{\mathbb{P}_2}(a_2)\},\min\{\alpha_{\mathbb{P}_1}(b_1),\alpha_{\mathbb{P}_2}(b_2)\}\}}$
$= \min\{\mu_{\mathbb{P}_1\otimes\mathbb{P}_2}(a_1, a_2), \mu_{\mathbb{P}_1\otimes\mathbb{P}_2}(b_1, b_2)\}e^{i\min\{\alpha_{\mathbb{P}_1\otimes\mathbb{P}_2}(a_1,a_2),\alpha_{\mathbb{P}_1\otimes\mathbb{P}_2}(b_1,b_2)\}}$
Similar way we can prove for γ and δ.
Hence the strong product $\mathbb{G}' \otimes \mathbb{G}''$ of two cohesive fuzzy graphs is a cohesive fuzzy graph.

Definition 3.4:
The semi-strong product $\mathbb{G}' \odot \mathbb{G}''$ of two cohesive fuzzy graph is defined as a pair $\mathbb{G}' \odot \mathbb{G}'' = (\mathbb{P}_1 \odot \mathbb{P}_2, \mathbb{Q}_1 \odot \mathbb{Q}_2)$, such that
(i). $\mu_{\mathbb{P}_1\odot\mathbb{P}_2}(a_1, a_2)e^{i\alpha_{\mathbb{P}_1\odot\mathbb{P}_2}(a_1,a_2)} = \min\{\mu_{\mathbb{P}_1}(a_1), \mu_{\mathbb{P}_2}(a_2)\}e^{i\min\{\alpha_{\mathbb{P}_1}(a_1),\alpha_{\mathbb{P}_2}(a_2)\}}$
$\gamma_{\mathbb{P}_1\odot\mathbb{P}_2}(a_1, a_2)e^{i\beta_{\mathbb{P}_1\odot\mathbb{P}_2}(a_1,a_2)} = \min\{\gamma_{\mathbb{P}_1}(a_1), \gamma_{\mathbb{P}_2}(a_2)\}e^{i\min\{\beta_{\mathbb{P}_1}(a_1),\beta_{\mathbb{P}_2}(a_2)\}}$
$\delta_{\mathbb{P}_1\odot\mathbb{P}_2}(a_1, a_2)e^{i\omega_{\mathbb{P}_1\odot\mathbb{P}_2}(a_1,a_2)} = \min\{\delta_{\mathbb{P}_1}(a_1), \delta_{\mathbb{P}_2}(a_2)\}e^{i\min\{\omega_{\mathbb{P}_1}(a_1),\omega_{\mathbb{P}_2}(a_2)\}}$
for all $(a_1, a_2) \in (\mathbb{P}_1 \times \mathbb{P}_2)$
(ii). $\mu_{\mathbb{Q}_1\odot\mathbb{Q}_2}((c, a_2)(c, b_2))e^{i\beta_{\mathbb{Q}_1\odot\mathbb{Q}_2}((c,a_2)(c,b_2))} = \min\{\mu_{\mathbb{P}_1}(c), \mu_{\mathbb{Q}_2}(a_2b_2)\}e^{i\min\{\alpha_{\mathbb{P}_1}(c),\alpha_{\mathbb{Q}_2}(a_2b_2)\}}$
$\gamma_{\mathbb{Q}_1\odot\mathbb{Q}_2}((c, a_2)(c, b_2))e^{i\beta_{\mathbb{Q}_1\odot\mathbb{Q}_2}((c,a_2)(c,b_2))} = \min\{\gamma_{\mathbb{P}_1}(c), \gamma_{\mathbb{Q}_2}(a_2b_2)\}e^{i\min\{\beta_{\mathbb{P}_1}(c),\beta_{\mathbb{Q}_2}(a_2b_2)\}}$
$\delta_{\mathbb{Q}_1\odot\mathbb{Q}_2}((c, a_2)(c, b_2))e^{i\omega_{\mathbb{Q}_1\odot\mathbb{Q}_2}((c,a_2)(c,b_2))} = \min\{\delta_{\mathbb{P}_1}(c), \delta_{\mathbb{Q}_2}(a_2b_2)\}e^{i\min\{\omega_{\mathbb{P}_1}(c),\omega_{\mathbb{Q}_2}(a_2b_2)\}}$
for all $c \in \mathbb{P}_1, a_2b_2 \in \mathbb{Q}_2$
(iii). $\mu_{\mathbb{Q}_1\odot\mathbb{Q}_2}((a_1, a_2)(b_1, b_2))e^{i\alpha_{\mathbb{Q}_1\odot\mathbb{Q}_2}((a_1,a_2)(b_1,b_2))} = \min\{\mu_{\mathbb{Q}_1}(a_1b_1), \mu_{\mathbb{Q}_2}(a_2b_2)\}e^{i\min\{\alpha_{\mathbb{Q}_1}(a_1b_1),\alpha_{\mathbb{Q}_2}(a_2b_2)\}}$
$\gamma_{\mathbb{Q}_1\odot\mathbb{Q}_2}((a_1, a_2)(b_1, b_2))e^{i\beta_{\mathbb{Q}_1\odot\mathbb{Q}_2}((a_1,a_2)(b_1,b_2))} = \min\{\gamma_{\mathbb{Q}_1}(a_1b_1), \gamma_{\mathbb{Q}_2}(a_2b_2)\}e^{i\min\{\beta_{\mathbb{Q}_1}(a_1b_1),\beta_{\mathbb{Q}_2}(a_2b_2)\}}$
$\delta_{\mathbb{Q}_1\odot\mathbb{Q}_2}((a_1, a_2)(b_1, b_2))e^{i\omega_{\mathbb{Q}_1\odot\mathbb{Q}_2}((a_1,a_2)(b_1,b_2))} = \min\{\delta_{\mathbb{Q}_1}(a_1b_1), \delta_{\mathbb{Q}_2}(a_2b_2)\}e^{i\min\{\omega_{\mathbb{Q}_1}(a_1b_1),\omega_{\mathbb{Q}_2}(a_2b_2)\}}$

Example 3.4.1:

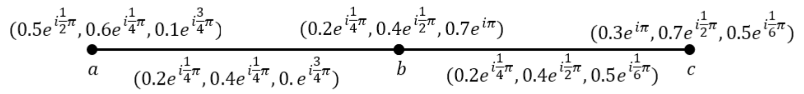

Figure 9: $\mathbb{G}'$

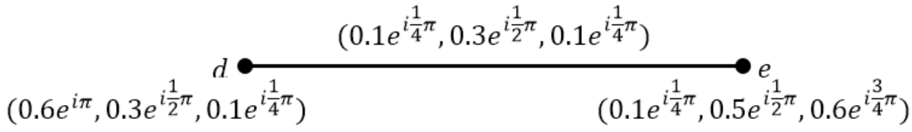

Figure 10: $\mathbb{G}''$

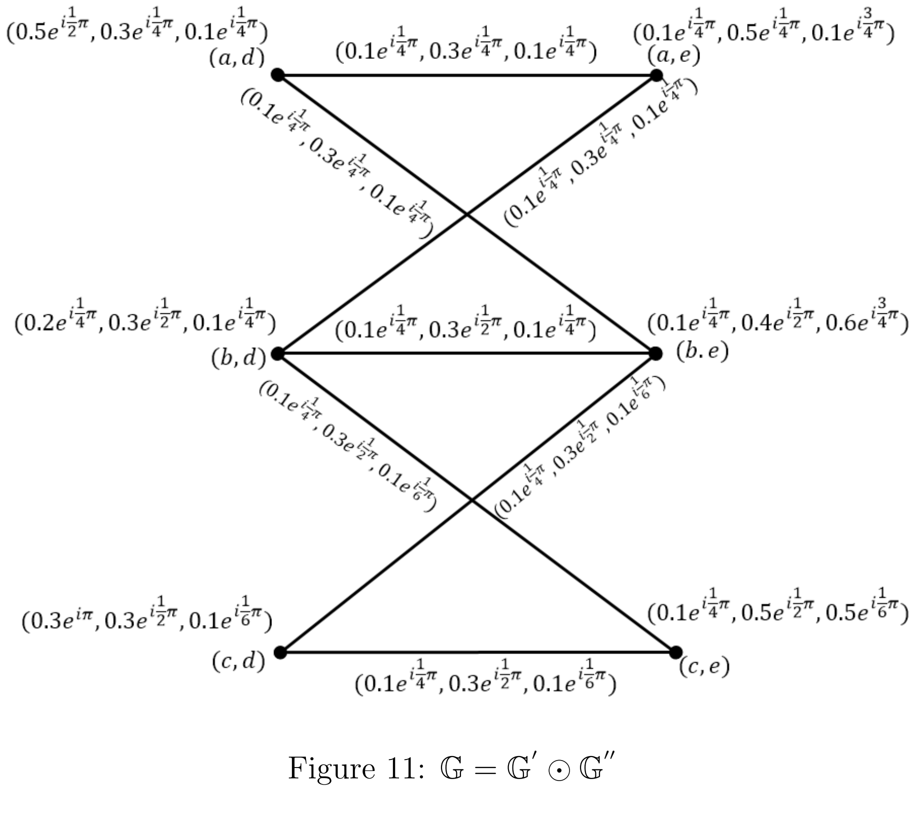

Figure 11: $\mathbb{G} = \mathbb{G}' \odot \mathbb{G}''$

Proposition 3.4.2:

Let $\mathbb{G}' = (\mathbb{P}_1, \mathbb{Q}_1)$ and $\mathbb{G}'' = (\mathbb{P}_2, \mathbb{Q}_2)$ be two cohesive fuzzy graphs, where $\mathbb{P}$ is a vertex set and $\mathbb{Q}$ is a edge set respectively. Then there is a semi-strong product between them $\mathbb{G}' \odot \mathbb{G}''$ of $\mathbb{G}' = (\mathbb{P}_1, \mathbb{Q}_1)$ and $\mathbb{G}'' = (\mathbb{P}_2, \mathbb{Q}_2)$ is a cohesive fuzzy graph.

Proof:

Suppose that $\mathbb{G}' = (\mathbb{P}_1, \mathbb{Q}_1)$ and $\mathbb{G}'' = (\mathbb{P}_2, \mathbb{Q}_2)$ are two Cohesive fuzzy graphs, where $\mathbb{P}$ is a vertex set and $\mathbb{Q}$ is a edge set respectively. Then semi-strong product $\mathbb{G}' \odot \mathbb{G}''$ of $G' = (\mathbb{P}_1, \mathbb{Q}_1)$ and $\mathbb{G}'' = (\mathbb{P}_2, \mathbb{Q}_2)$ can be proved. Let $(a_1, a_2)(b_1, b_2) \in \mathbb{Q}_1 \times \mathbb{Q}_2$. For proving this we have three cases.

case 1: It is trivial.

case 2: If $a_1 = b_1 = c$ and Let $a \in \mathbb{P}_1$ and $a_2 b_2 \in \mathbb{Q}_2$

$$\mu_{\mathbb{Q}_1 \odot \mathbb{Q}_2}((c, a_2)(c, b_2)) e^{i\alpha_{\mathbb{Q}_1 \odot \mathbb{Q}_2}((c,a_2)(c,b_2))}$$
$$= \min\{\mu_{\mathbb{P}_1}(c), \mu_{\mathbb{Q}_2}(a_2 b_2)\} e^{i \min\{\alpha_{\mathbb{P}_1}(c), \alpha_{\mathbb{Q}_2}(a_2 b_2)\}},$$
$$\leq \min\{\mu_{\mathbb{P}_1}(c), \min\{\mu_{\mathbb{P}_2}(a_2), \mu_{\mathbb{P}_2}(b_2)\}\} e^{i \min\{\alpha_{\mathbb{P}_1}(c), \min\{\alpha_{\mathbb{P}_2}(a_2), \alpha_{\mathbb{P}_2}(b_2)\}\}},$$
$$= \min\{\min\{\mu_{\mathbb{P}_1}(c), \mu_{\mathbb{P}_2}(a_2)\}, \min\{\mu_{\mathbb{P}_1}(c), \mu_{\mathbb{P}_2}(b_2)\}\} e^{i \min\{\min\{\alpha_{\mathbb{P}_1}(c), \alpha_{\mathbb{P}_2}(a_2)\}, \min\{\alpha_{\mathbb{P}_1}(c), \alpha_{\mathbb{P}_2}(b_2)\}\}}$$
$$= \min\{\mu_{\mathbb{P}_1 \odot \mathbb{P}_2}(c, a_2), \mu_{\mathbb{P}_1 \odot \mathbb{P}_2}(c, b_2)\} e^{i \min\{\alpha_{\mathbb{P}_1 \odot \mathbb{P}_2}(c, a_2), \alpha_{\mathbb{P}_1 \odot \mathbb{P}_2}(c, b_2)\}},$$

$$\gamma_{\mathbb{Q}_1 \odot \mathbb{Q}_2}((c, a_2)(c, b_2)) e^{i\beta_{\mathbb{Q}_1 \odot \mathbb{Q}_2}((c,a_2)(c,b_2))}$$
$$= \min\{\gamma_{\mathbb{P}_1}(c), \gamma_{\mathbb{Q}_2}(a_2 b_2)\} e^{i \min\{\beta_{\mathbb{P}_1}(c), \beta_{\mathbb{Q}_2}(a_2 b_2)\}},$$
$$\leq \min\{\gamma_{\mathbb{P}_1}(c), \min\{\gamma_{\mathbb{P}_2}(a_2), \gamma_{\mathbb{P}_2}(b_2)\}\} e^{i \min\{\beta_{\mathbb{P}_1}(c), \min\{\beta_{\mathbb{P}_2}(a_2), \beta_{\mathbb{P}_2}(b_2)\}\}},$$
$$= \min\{\min\{\gamma_{\mathbb{P}_1}(c), \gamma_{\mathbb{P}_2}(a_2)\}, \min\{\gamma_{\mathbb{P}_1}(c), \gamma_{\mathbb{P}_2}(b_2)\}\} e^{i \min\{\min\{\beta_{\mathbb{P}_1}(c), \beta_{\mathbb{P}_2}(a_2)\}, \min\{\beta_{\mathbb{P}_1}(c), \beta_{\mathbb{P}_2}(b_2)\}\}}$$
$$= \min\{\gamma_{\mathbb{P}_1 \odot \mathbb{P}_2}(c, a_2), \gamma_{\mathbb{P}_1 \odot \mathbb{P}_2}(c, b_2)\} e^{i \min\{\beta_{\mathbb{P}_1 \odot \mathbb{P}_2}(c, a_2), \beta_{\mathbb{P}_1 \odot \mathbb{P}_2}(c, b_2)\}},$$

$$\delta_{\mathbb{Q}_1 \odot \mathbb{Q}_2}((c, a_2)(c, b_2)) e^{i\omega_{\mathbb{Q}_1 \odot \mathbb{Q}_2}((c,a_2)(c,b_2))}$$
$$= \min\{\delta_{\mathbb{P}_1}(c), \delta_{\mathbb{Q}_2}(a_2 b_2)\} e^{i \min\{\omega_{\mathbb{P}_1}(c), \omega_{\mathbb{Q}_2}(a_2 b_2)\}},$$
$$\leq \min\{\delta_{\mathbb{P}_1}(c), \min\{\delta_{\mathbb{P}_2}(a_2), \delta_{\mathbb{P}_2}(b_2)\}\} e^{i \min\{\omega_{\mathbb{P}_1}(c), \min\{\omega_{\mathbb{P}_2}(a_2), \omega_{\mathbb{P}_2}(b_2)\}\}},$$
$$= \min\{\min\{\delta_{\mathbb{P}_1}(c), \delta_{\mathbb{P}_2}(a_2)\}, \min\{\delta_{\mathbb{P}_1}(c), \delta_{\mathbb{P}_2}(b_2)\}\} e^{i \min\{\min\{\omega_{\mathbb{P}_1}(c), \omega_{\mathbb{P}_2}(a_2)\}, \min\{\omega_{\mathbb{P}_1}(c), \omega_{\mathbb{P}_2}(b_2)\}\}}$$
$$= \min\{\delta_{\mathbb{P}_1 \odot \mathbb{P}_2}(c, a_2), \delta_{\mathbb{P}_1 \odot \mathbb{P}_2}(c, b_2)\} e^{i \min\{\omega_{\mathbb{P}_1 \odot \mathbb{P}_2}(c, a_2), \omega_{\mathbb{P}_1 \odot \mathbb{P}_2}(c, b_2)\}},$$

case 3: If $a_1 \neq b_1, a_2 \neq b_2$ and $a_1 b_1 \in \mathbb{Q}_1, a_2 b_2 \in \mathbb{Q}_2$

$$\mu_{\mathbb{Q}_1 \odot \mathbb{Q}_2}((a_1, a_2)(b_1, b_2)) e^{i\alpha_{\mathbb{Q}_1 \odot \mathbb{Q}_2}((a_1,a_2)(b_1,b_2))}$$
$$= \min\{\mu_{\mathbb{Q}_1}(a_1 b_1), \mu_{\mathbb{Q}_2}(a_2 b_2) e^{i \min\{\alpha_{\mathbb{Q}_1}(a_1 b_1), \alpha_{\mathbb{Q}_2}(a_2 b_2)\}}$$
$$\leq \min\{\min\{\mu_{\mathbb{P}_1}(a_1), \mu_{\mathbb{P}_1}(b_1)\}, \{\mu_{\mathbb{P}_2}(a_2), \mu_{\mathbb{P}_2}(b_2)\}\} e^{i \min\{\min\{\alpha_{\mathbb{P}_1}(a_1), \alpha_{\mathbb{P}_1}(b_1)\}, \{\alpha_{\mathbb{P}_2}(a_2), \alpha_{\mathbb{P}_2}(b_2)\}\}}$$
$$= \min\{\min\{\mu_{\mathbb{P}_1}(a_1), \mu_{\mathbb{P}_2}(a_2)\}, \min\{\mu_{\mathbb{P}_1}(b_1), \mu_{\mathbb{P}_2}(b_2)\}\} e^{i \min\{\min\{\alpha_{\mathbb{P}_1}(a_1), \alpha_{\mathbb{P}_2}(a_2)\}, \min\{\alpha_{\mathbb{P}_1}(b_1), \alpha_{\mathbb{P}_2}(b_2)\}\}}$$
$$= \min\{\mu_{\mathbb{P}_1 \odot \mathbb{P}_2}(a_1, a_2), \mu_{\mathbb{P}_1 \odot \mathbb{P}_2}(b_1, b_2)\} e^{i \min\{\alpha_{\mathbb{P}_1 \odot \mathbb{P}_2}(a_1, a_2), \alpha_{\mathbb{P}_1 \odot \mathbb{P}_2}(b_1, b_2)\}}$$

Similar way we can prove for γ and δ.

Hence the Semi-strong product $\mathbb{G}' \odot \mathbb{G}''$ of two cohesive fuzzy graphs is a cohesive fuzzy graph .

Conclusion:

In this paper we studied some cohesive fuzzy graph products like cartesian product, direct product, strong product, semi-strong product with some suitable graphical illustrations for better understanding and also few of the theoretical results are proved using above products of cohesive fuzzy graph.

References:

[1]. L. A. Zadeh, Fuzzy sets, Inf. Control, 8 (1965), 338-353. https://doi.org/10.1016/S0019-9958(65)90241-X

[2]. K. T. Atanassov, Intutionistic fuzzy sets, Fuzzy Sets Syst., 20(1986), 87-96. https://doi.org/10.1016/S0165-0114(86)80034-3

[3]. V. Torra, Hesitant fuzzy sets, Int. J. Intell. Syst., 25(2010), 529-539. https://doi.org/10.1002/int.20418

[4]. D. Ramot, R. Milo, M. Friedman, A. Kandel, Complex fuzzy sets, IEEE Trans. Fuzzy Syst., 10 (2002), 171-186. https://doi.org/10.1109/91.995119

[5]. A. S. Alkouri, A. R. Salleh, Complex intuitionistic fuzzy sets, AIP Conf. Proc.,1482(2012), 464-470.

[6]. N. Chen, Z. Xu, M. Xia, Interval-valued hesitant preference relations and their applications to group decision making, Knowl Based Syst., 37(2013), 528-540. https://doi.org/10.1016/j.knosys.2012.09.009

[7]. A.Rosenfeld, Fuzzy graphs, Academic Press, 1975, 77-95. https://doi.org/10.1016/B978-0-12-775260-0.50008-6

[8]. T.Pathinathan, J. J. Arockiaraj, J.J. Roseline, Hesitancy fuzzy graphs, Indian J. Sci. Technol., 8(2015), 1-5. https://doi.org/10.17485/ijst/2015/v8i35/86672

[9]. D. E. Tamir, N. D. Rishe, M. Last, A. Kandel, Soft computing based epidemical crisis prediction, Intell. Meth. Cyber Warf., vol. 563, pp. 43-67,2015.

[10]. F. Karaaslan, Hesitant fuzzy graphs and their applications in decision making, J. Intell. Fuzzy Syst., 36(2019), 2729-2741.https://doi.org/10.3233/JIFS-18865

[11]. M. Talafha, A. Alkouri, S. Alqaraleh, H. Zureigat, A.Aljarrah. Complex hesitant fuzzy sets and its application in multiple attributes decision-making problems, J. Intell. Fuzzy Syst.,41(2021),7299-7327.
https://doi.org/10.3233/JIFS-211156

[12]. N. Yaqoob, M. Gulistan, S. Kadry, H. A. Wahab, Complex intuitionistic fuzzy graphs with appication in Cellular network provider companies, Mathematics, 7(2019), 35. https://doi.org/10.3390/math70100035

[13]. M. Javaid, A. Kashif, and T. Rashid, Hesitant fuzzy graphs and their products, Fuzzy inf. Eng., vol. 12, no. 2, pp. 238-252, 2020.

[14]. E. AbuHijleh, Complex hesitant fuzzy graph, Fuzzy Inf. Eng., 15(2023), 149-161.

https://doi.org/10.26599/FIE.2023.9270010
[15]. Xue, X; Poonia, M.; Abdulsahib, G.M.; Shukla, V. On Cohesive Fuzzy Sets, Operations and Electromagnetic Signals and Solar Activities. Symmetry 2023, 15,595. https://doi.org/10.3390/Sym1530595
[16]. J. Jon Arockiaraj, S. Sheela Rani, Cohesive fuzzy graph and it's Properties (communicated)