Advances in Nonlinear Variational Inequalities
ISSN: 1092-910X
Vol 28 No. 2 (2025)

An Inflation-Affected Supply-Chain Dual Warehouse Inventory Model for Decaying Things by Investment in Preservation Technology

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Article History:

Received: *31-07-2024*

Revised: *18-09-2024*

Accepted: *27-09-2024*

Abstract:

It is common to observe that when demand increases, an industry with limited storage capacity (OW) must rent a warehouse (RW). We must purchase more product in larger quantities due to the COVID-19 epidemic, sales discounts, or other factors. When OW's capacity is reached, extra stock is held in RW at a high holding charge compared to OW. The dual warehouse inventory problem of a supply chain constrained by limited storage capacity in OW for non-instantaneously decaying commodities under inflationary conditions has been investigated in the current study. There is a partial backlog in shortages, and the demand rate is depending on inventory. Because of the quick changes in the environment, preservation technology has become increasingly crucial in today's global society. In order to slow down the deterioration of goods with a high rate of deterioration, preservation technology is specifically used in this article. Our primary goal is to reduce the suggested model's overall cost function. At the conclusion of this work, A sensitivity analysis and numerical illustration have been performed to showcase the model.

Keywords: Dual warehouse, demand dependent on stock, non-instantaneous deterioration, partial backorder, preservation technology.

1.Introduction

Another essential component of inventory analysis is the two-warehouse inventory system. Owing to the competitive marketing environment, a warehouse's location is crucial to corporate strategy. Retailers naturally need to locate a stock in a well-known selling area. They therefore want more storage space because there isn't enough room in a busy marketplace. Items have little storage space in crowded markets (like supermarkets). If there's a bulk discount available, or if certain foods are seasonal, or if acquiring goods costs more compared to other inventory expenses, or if there's a surge in product demand or frequent procurement hurdles, management may choose to buy significant quantities of products in a single transaction. These products are frequently unable to be accommodated in a busy marketplace's on-hand warehouse (the company's own warehouse, OW). In this situation, a second warehouse (a rented warehouse, RW) is necessary for keeping excess items, and it may be located some distance from the OW. As a result, a RW is used to store items that exceed the OW's fixed limit. The RW typically incurs greater unit holding charges compared to the

OW, yet its superior preservation facilities lead to a lesser rate of product deterioration compared to the OW. It is more cost effective to utilize RW items as soon as possible to reduce inventory costs. As a result, the vendor saves things in the OW first, but consumes stock from the RW first.

Hartley (1976) pioneered the two-warehouse concept, and since then, numerous authors have concentrated on two-warehouse inventory problems. Sarma (1987) was the initial step in the direction of OW's restricted storage capability. He has also created a two-warehouse model to accommodate decaying items and limited storage capacity. Gothi and colleagues (2016) explored a dual warehouse model by quadratic demand also variable holding cost for decaying products. The backorder rate is believed to be directly proportional to the time interval between replenishments. Tiwari and his team (2017) proposed a dual-warehouse model for deteriorating items, incorporating partial backlog and demand influenced by stock levels, amidst inflationary conditions, utilizing particle swarm optimization.

Deterioration is described as the process of decay, spoiling, off-trend damage, also evaporation. Some items, such as milk, have an ending time. They are considered to be in pristine condition until the expiration date. Some preservation technologies can help items keep their freshness longer (Iqbal and Sarkar, 2020). A suitable approach is utilized to decrease degradation by investing in various preservation technology, such as refrigerators, air conditioners, and drying machines, which are used for various products. Contemporary preservation methods can be employed to reduce the pace of degradation, proving to be an efficient conservation strategy. Perishable goods such as fruits, vegetables, volatile liquids, blood, fashion items, among others, are subject to deterioration.

Ghare and Scharder (1963) were pioneers in presenting inventory models involving degradation. They looked at the constant rate of decay. In actual life, numerous goods begin to decay after their maximum life period, which is known as non-instantaneous degeneration. Sekar and Uthayakumar (2018) suggested a perishable model sensitive to both time and price variations. Feng (2019) included a pricing decision on degrading things as well, but for the consequence of a quality investment. It was considered that demand was price and quality dependent. Kaliraman and colleagues (2017) recommended a dual-warehouse inventory structure tailored for perishable goods, featuring an exponential demand rate and allowable payment postponement. Mashud and collaborators (2021) introduced an inventory methodology to explore carbon secretions and deterioration within the realm of efficient green investment and conservation technology.

In the current environment, inflation is a critical concern for all sectors. In practice, Inflation denotes the gradual rise in the overall price level of goods and services within an economy as time progresses. Accordingly, annual inflation also entails a reduction in the value of holding costs. Holding costs are influenced by the inflation rate used to assess the ending inventory's worth, as the value of inventory items fluctuates rather than remaining constant. In the global economy, it's crucial not to underestimate the significance of inflation and the concept of the time worth of money. Buzacott (1975) was the pioneer in incorporating the inflationary impact on costs into an EOQ model. Bierman and Thomas (1977) subsequently enhanced Buzacott's (1975) model by incorporating discount rates to accommodate inflation. Misra (1979) further advanced the EOQ model by introducing diverse inflation rates to account for various associated costs. Pervin and colleagues (2016) constructed an inventory model for degrading goods within a market experiencing decreasing demand, incorporating a trade credit strategy. Shaikh (2017) devised a dual warehouse inventory model tailored for degrading products, which considers fluctuating demand within another trade credit framework. Mishra and colleagues (2019) present a demand model that is dependent on both stock levels and prices. Xu and Song (2020) developed an integrated optimization approach aimed at optimizing production capacity, raw material procurement, and manufacture forecasting in the face of quantity uncertainty. Hasan and colleagues (2020) introduced a model for agricultural products incorporating product separation and diverse discount policies to minimize product deterioration.

Table-1: Key contributions of the planned inventory model for dual warehouses.

Literature	Dual Warehouse/ Solo Warehouse	Demand rate	Deterioration	Preservation Technology	Shortage
Liang and Zhou (2011)	Dual	Stable	Stable	-	-
Hsu and co-authors (2010)	Solo	Stable	Stable	Stable	Partial backordered
Maiti and co-authors (2009)	Solo	Price varying (Non-linear function)	-	-	Wholly backordered
Taleizadeh and co-authors (2014)	Solo	Stable	Stable	-	Partial backordered
Lashgari and co-authors (2016)	Solo	Stable	-	-	No shortages, Wholly and Partial backordered
Teng and co-authors (2016)	Solo	Stable	Time-Varying	-	Partial backordered
Tiwari and co-authors (2017)	Dual	Dependent on stocks	Stable	-	Wholly backordered
Jaggi and co-authors (2017)	Dual	Price varying	Stable	-	Wholly backordered
Tiwari and co-authors (2018a)	Solo	Price varying	Termination	-	Partial backordered
Tiwari and co-authors (2018b)	Dual	Price varying	Stable	-	Wholly backordered
This paper	Dual	Stock dependent	Stable	Stable	Partial backordered

The primary contributions of the suggested inventory model are outlined below:

- ❖ Preservation technology in a dual warehouse system
- ❖ Inflationary inventory model
- ❖ Product demand is contingent on inventory levels.
- ❖ Partially backlogged shortages (backlog rate inversely proportionate to waiting time)

2. Assumptions and Notations

2.1. Notations

$D(t)$	Rate of demand
Q	Order quantity
θ	Actual rate of deterioration
ϵ	Cost of preservation technology to mitigate deterioration and ensure product conservation, where ϵ is a positive value.
λ	Resultant rate of decay, where $\lambda = \theta - m(\epsilon)$.
r	Rate of inflation
h_r	The cost of holding per unit over a given period in a rented warehouse (RW).
h_o	The cost of holding per unit over a given period in an owned warehouse (OW).
A	Cost of ordering
p_c	The cost per unit purchased
C_s	The cost of shortages
l_c	The cost of lost sales
C_T	Total cost
W_2	Confined spatial area of OW
W_1	The upper limit of inventory in RW
I_b	The highest level of backorders
T_d	The maximum lifespan of an item
T_1	The timeframe when the stock level in RW reaches zero
T_2	The timeframe when the stock level in OW reaches zero
T	The duration of the cycle
$I_1(t)$	The level of stock in RW within the time interval $[0, T_d]$
$I_2(t)$	The level of stock in RW within the time interval $[T_d, T_1]$

$I_3(t)$ The level of stock in OW within the time interval $[0, T_d]$

$I_4(t)$ The level of stock in OW within the time interval $[T_d, T_1]$

$I_5(t)$ The level of stock in OW within the time interval $[T_1, T_2]$

$I_6(t)$ The level of stock in OW within the time interval $[T_2, T]$

2. 2. Assumptions

1. The demand rate varies by stock levels also expressed in the following form

$$D(t) = \begin{cases} a + bI(t) & I(t) > 0 \\ a & I(t) \leq 0 \end{cases}$$

2. Shortages are permitted and partial backordered, with a backorder rate given by

$I_b(t) = \frac{1}{1+\delta(T-t)}$ where t represents the duration of waiting, and $0 < \delta < 1$ is the backorder constraint.

3. The model pertains to a sole non-instantaneous deteriorating item. There's no degradation within the timeframe $[0, T_d]$ but it occurs during the interval $[T_d, T_2]$ at a deterioration rate of $\theta(t) = \theta t$, where $0 < \theta << 1$ represents the deterioration parameter.

4. The time horizon extends indefinitely, and the replenishment rate has no limit, and there is zero lead time.

5. The Owned Warehouse (OW) has a finite space of W_o units, whereas the capacity of the Rented Warehouse is infinite.

6. The holding cost (h_r) of RW is higher than the holding cost (h_o) of OW. As a result, the items from OW are used only once the inventory in RW is depleted.

7. Transportation charges and the duration between RW and OW are insignificant.

3. Formulating the inventory model mathematically

As illustrated in Fig.1, we segregate the following time intervals separately, $[0, T_d]$, $[T_d, T_1]$, $[T_1, T_2]$ and $[T_2, T]$. At time $t = 0$, the stock level is S, with W_2 units are stored in OW while W_1 units are stored in RW. Throughout the intermission $[0, T_d]$, stock levels remain positive at both RW and OW. Throughout this interval, demand leads to a decrease in the inventory level of RW, whereas the stock level of OW remains steady. At $t = T_d$, deterioration sets in. Throughout the interval $[T_d, T_1]$, inventory level of RW declines because of both demand and deterioration, reaching zero at $t = T_1$ whereas the stock level of OW decreases due to deterioration. Throughout the intermission $[T_1, T_2]$, demand is fulfilled by using OW inventory. In the interval $[T_2, T]$, shortages occur and are somewhat backlogged.

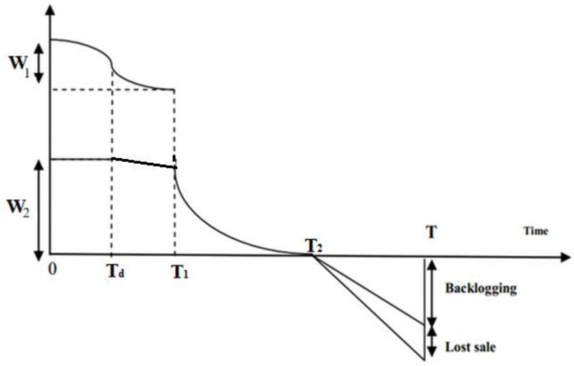

Fig. 1: Visual depiction of the inventory model involving dual warehouses with shortages

The inventory level at RW is described by the subsequent set of differential equations

$$\frac{dI_1(t)}{dt} = -(a + bI_1(t)) \qquad 0 \le t \le T_d \tag{1}$$

$$\frac{dI_2(t)}{dt} + \lambda t I_2(t) = -(a + bI_2(t)) \qquad T_d \le t \le T_1 \tag{2}$$

and the inventory level at OW is described by the subsequent set of differential equations:

$$I_3(t) = W_2 \qquad 0 \le t \le T_d \tag{3}$$

$$\frac{dI_4(t)}{dt} + \lambda t I_4(t) = 0 \qquad T_d \le t \le T_1 \tag{4}$$

$$\frac{dI_5(t)}{dt} + \lambda t I_5(t) = -(a + bI_5(t)) \qquad T_1 \le t \le T_2 \tag{5}$$

$$\frac{dI_6(t)}{dt} = -\left(\frac{a}{1+\delta(T-t)}\right) \qquad T_2 \le t \le T \tag{6}$$

The boundary conditions are given below:

$I_1(0) = W_1$, $I_1(T_d) = I_2(T_d)$, $I_2(T_1) = 0$, $I_3(T_d) = I_4(T_d) = W_2$, $I_5(T_2) = I_6(T_2) = 0$,
$I_6(T) = -I_b$.

The solutions to the aforementioned equations are derived using boundary conditions

$$I_1(t) = \frac{a}{b}\left(e^{b(T_d - t)} - 1\right) + ae^{-\left(tb + \frac{\lambda T_d^2}{2}\right)}\left\{(T_1 - T_d) + b\left(\frac{T_1^2 - T_d^2}{2}\right) + \lambda\left(\frac{T_1^3 - T_d^3}{6}\right)\right\} \tag{7}$$

$$I_2(t) = e^{-\left(tb + \frac{\lambda t^2}{2}\right)}\left\{(T_1 - t) + b\left(\frac{T_1^2 - t^2}{2}\right) + \lambda\left(\frac{T_1^3 - t^3}{6}\right)\right\} \tag{8}$$

$$I_4(t) = W_2 e^{-\lambda\left(\frac{T_d^3 - t}{2}\right)} \tag{9}$$

$$I_5(t) = e^{-\left(tb + \frac{\lambda t^2}{2}\right)}\left\{(T_2 - t) + b\left(\frac{T_2^2 - t^2}{2}\right) + \lambda\left(\frac{T_2^3 - t^3}{6}\right)\right\} \tag{10}$$

$$I_6(t) = -a(t - T_2) \tag{11}$$

Since $I_1(0) = W_1$ and from equation (7) we get

$$W_1 = \frac{a}{b}\left(e^{b(T_d)} - 1\right) + ae^{-\left(\frac{\lambda T_d^2}{2}\right)}\left\{(T_1 - T_d) + b\left(\frac{T_1^2 - T_d^2}{2}\right) + \lambda\left(\frac{T_1^3 - T_d^3}{6}\right)\right\} \quad (12)$$

Since $I_4(T_1) = I_5(T_1)$ and using equations (9) and (10) we get

$$T_1 = \left(-\frac{1}{2\lambda W_2}\right)(-6a - 3bW_2 + \lambda T_2 W_2) + \{(6a + 3bW_2 - \lambda T_2 W_2)^2 + 4\lambda W_2 - 6aT_2 + 6W_2 - 3bT_2W_2 - T_2^2 W_2 \lambda + 3T_d^2 W_2 \lambda\}^{1/2} \quad (13)$$

Maximum backlogged amount $I_b = -I_6(T)$

$$I_b = a(T - T_2) \quad (14)$$

Order quantity $Q = W_1 + W_2 + I_b$

$$Q = \frac{a}{b}\left(e^{b(T_d)} - 1\right) + ae^{-\left(\frac{\lambda T_d^2}{2}\right)}\left\{(T_1 - T_d) + b\left(\frac{T_1^2 - T_d^2}{2}\right) + \lambda\left(\frac{T_1^3 - T_d^3}{6}\right)\right\} + W_2 + a(T - T_2) \quad (15)$$

The overall relevant cost encompasses the following cost factors:

1. Cost of ordering (OC) = A
2. The cost per unit purchased (PC) = $p_c Q$
3. The cost of holding (HC) is

$$HC = h_r \int_0^{T_d} I_1(t)e^{-rt}\,dt + h_r \int_{T_d}^{T_1} I_2(t)e^{-rt}\,dt + h_o \int_o^{T_d} I_3(t)e^{-rt}\,dt + h_o \int_{T_d}^{T_1} I_4(t)e^{-rt}\,dt + h_o \int_{T_1}^{T_2} I_5(t)e^{-rt}\,dt$$

$$= h_r\left[a\left(\frac{T_d^2}{2} - \frac{rT_d^3}{6}\right) + a\left(T_d - \frac{(b+r)T_d^2}{2} + \frac{(2rb-3\lambda)T_d^3}{6} + \frac{\lambda r T_d^4}{4}\right)\left((T_1 - T_d) + b\left(\frac{T_1^2 - T_d^2}{2}\right) + \lambda\left(\frac{T_1^3 - T_d^3}{6}\right)\right) + a\left\{T_1 + b\frac{T_1^2}{2} + \lambda\frac{T_1^3}{6}\right\}\left\{(T_1 - T_d) - b\left(\frac{T_1^2 - T_d^2}{2}\right) + \lambda\left(\frac{T_1^3 - T_d^3}{6}\right)\right\} - a\left\{\frac{T_1^2 - T_d^2}{2} - (b+r)\frac{T_1^3 - T_d^3}{3} - \lambda\left(\frac{T_1^4 - T_d^4}{8}\right)\right\} - b\left\{\left(\frac{T_1^3 - T_d^3}{6}\right) - (b+r)\left(\frac{T_1^4 - T_d^4}{8}\right) - \lambda\left(\frac{T_1^5 - T_d^5}{20}\right)\right\} - \lambda\frac{T_1^4 - T_d^4}{24} - (b+r)\left(\frac{T_1^5 - T_d^5}{30}\right) - \lambda\left(\frac{T_1^6 - T_d^6}{72}\right)\right]$$

$$+h_o\left\{W_2\left(T_d - \frac{rT_d^2}{2}\right) + W_2\left((T_1 - T_d)\left(1 + \lambda\frac{T_d^2}{2}\right) - r\left(\frac{T_1^2 - T_d^2}{2}\right) - \lambda\left(\frac{T_1^3 - T_d^3}{6}\right)\right) + a\left(T_2 + b\frac{T_2^2}{2} + \lambda\frac{T_2^3}{6}\right)\left((T_2 - T_1) - b\left(\frac{T_2^2 - T_1^2}{2}\right) - \lambda\left(\frac{T_2^3 - T_1^3}{6}\right)\right) - a\left(\left(\frac{T_2^2 - T_1^2}{2}\right) - (b+r)\left(\frac{T_2^3 - T_1^3}{3}\right) - \lambda\left(\frac{T_2^4 - T_1^4}{8}\right)\right) - b\left(\left(\frac{T_2^3 - T_1^3}{6}\right) - (b+r)\left(\frac{T_2^4 - T_1^4}{8}\right) - \lambda\left(\frac{T_2^5 - T_1^5}{20}\right)\right) - (b+r)\left(\frac{T_2^5 - T_1^5}{30}\right) - \lambda\left(\frac{T_2^6 - T_1^6}{72}\right)\right\} \quad (16)$$

4. The cost of shortage (SC) is

$$SC = c_s \int_{T_2}^{T} I_6(t)e^{-rt}dt$$

$$= ac_s T_2\left[(r+1)\left(\frac{T^2 - T_2^2}{2}\right) - T + T_2 - r\left(\frac{T^3 - T_2^3}{3}\right)\right] \quad (17)$$

5. The cost of lost sale (LC) is

$$LC = l_c a \int_{T_2}^{T}\left(1 - \frac{1}{(1 + \delta(T-t))}\right)e^{-rt}dt$$

$$= l_c\left[a(T-T_2) - r\left(\frac{T^2-T_2^2}{2}\right) - \frac{1}{\delta}\log\big(1+\delta(T-T_2)\big) + \frac{r}{\delta}(-(\delta T+1)\log\big(1+\delta(T-T_2) - (T-T_2)\big)\right] \quad (18)$$

6. Present value of deterioration cost (DC) during $[0, T_1]$ is

$$DC = \lambda D_c\left\{\int_{T_d}^{T_1} I_2(t)\, e^{-rt}dt + \int_{T_d}^{T_1} I_4(t)\, e^{-rt}dt + \int_{T_1}^{T_2} I_5(t)\, e^{-rt}dt\right\}$$

$$= \lambda D_c\Bigg\{(b+r)\left(\left(\frac{T_1 {T_d}^2}{2} - \frac{{T_d}^3}{3}\right) + \frac{b}{2}\left(\frac{{T_1}^2 {T_d}^2}{2} - \frac{{T_d}^4}{4}\right) + \frac{\lambda}{6}\left(\left(\frac{{T_1}^3 {T_d}^2}{2} - \frac{{T_d}^5}{5}\right)\right)\right)$$

$$-\left(T_1 T_d - \frac{{T_d}^2}{2}\right) + \frac{b}{2}\left({T_1}^2 T_d - \frac{{T_d}^3}{3}\right) + \frac{\lambda}{6}\left(\left({T_1}^3 T_d - \frac{{T_d}^4}{4}\right)\right)$$

$$-(br - \frac{\lambda}{2})\left(\left(\frac{T_1 {T_d}^3}{3} - \frac{{T_d}^4}{4}\right) + \frac{b}{2}\left(\frac{{T_1}^2 {T_d}^3}{3} - \frac{{T_d}^5}{5}\right) + \frac{\lambda}{6}\left(\left(\frac{{T_1}^3 {T_d}^3}{3} - \frac{{T_d}^6}{6}\right)\right)\right)$$

$$-\frac{\theta r}{2}\left(\left(\frac{T_1 {T_d}^4}{4} - \frac{{T_d}^5}{5}\right) + \frac{b}{2}\left(\frac{{T_1}^2 {T_d}^4}{4} - \frac{{T_d}^6}{6}\right) + \frac{\lambda}{6}\left(\left(\frac{{T_1}^3 {T_d}^4}{4} - \frac{{T_d}^7}{7}\right)\right)\right)$$

$$+W_2\left((T_1 - T_d)\left(1 + \lambda\frac{T_d^2}{2}\right) - r\left(\frac{T_1^2 - T_d^2}{2}\right) - \lambda\left(\frac{T_1^3 - T_d^3}{6}\right)\right) + a\left(T_2 + b\frac{T_2^2}{2} + \lambda\frac{T_2^3}{6}\right)$$

$$\left((T_2 - T_1) - b\left(\frac{T_2^2 - T_1^2}{2}\right) - \lambda\left(\frac{T_2^3 - T_1^3}{6}\right)\right) - a\left(\left(\frac{T_2^2 - T_1^2}{2}\right) - (b+r)\left(\frac{T_2^3 - T_1^3}{3}\right) - \lambda\left(\frac{T_2^4 - T_1^4}{8}\right)\right)$$

$$-b\left(\left(\frac{T_2^3 - T_1^3}{6}\right) - (b+r)\left(\frac{T_2^4 - T_1^4}{8}\right) - \lambda\left(\frac{T_2^5 - T_1^5}{20}\right)\right) - (b+r)\left(\frac{T_2^5 - T_1^5}{30}\right) - \lambda\left(\frac{T_2^6 - T_1^6}{72}\right)\Bigg\} \quad (19)$$

Total cost, $C_T = OC + PC + HC + DC + SC + LC$

$$C_T =$$

$$\left[\begin{array}{c} A + P_c Q \\ +h_r\left[a\left(\frac{T_d^2}{2}-\frac{rT_d^3}{6}\right)+a\left(T_d-\frac{(b+r)T_d^2}{2}+\frac{(2rb-3\lambda)T_d^3}{6}+\frac{\lambda r T_d^4}{4}\right)\left((T_1-T_d)+b\left(\frac{T_1^2-T_d^2}{2}\right)+\lambda\left(\frac{T_1^3-T_d^3}{6}\right)\right)\right. \\ +a\left\{T_1+b\frac{T_1^2}{2}+\lambda\frac{T_1^3}{6}\right\}\left\{(T_1-T_d)-b\left(\frac{T_1^2-T_d^2}{2}\right)+\lambda\left(\frac{T_1^3-T_d^3}{6}\right)\right\}-a\left\{\frac{T_1^2-T_d^2}{2}-(b+r)\frac{T_1^3-T_d^3}{3}-\lambda\left(\frac{T_1^4-T_d^4}{8}\right)\right\} \\ \left.-b\left\{\left(\frac{T_1^3-T_d^3}{6}\right)-(b+r)\left(\frac{T_1^4-T_d^4}{8}\right)-\lambda\left(\frac{T_1^5-T_d^5}{20}\right)\right\}-\lambda\frac{T_1^4-T_d^4}{24}-(b+r)\left(\frac{T_1^5-T_d^5}{30}\right)-\lambda\left(\frac{T_1^6-T_d^6}{72}\right)\right] \\ +h_o\left\{W_2\left(T_d-\frac{rT_d^2}{2}\right)+W_2\left((T_1-T_d)\left(1+\lambda\frac{T_d^2}{2}\right)-r\left(\frac{T_1^2-T_d^2}{2}\right)-\lambda\left(\frac{T_1^3-T_d^3}{6}\right)\right)+a\left(T_2+b\frac{T_2^2}{2}+\lambda\frac{T_2^3}{6}\right)\right. \\ \left((T_2-T_1)-b\left(\frac{T_2^2-T_1^2}{2}\right)-\lambda\left(\frac{T_2^3-T_1^3}{6}\right)\right)-a\left(\left(\frac{T_2^2-T_1^2}{2}\right)-(b+r)\left(\frac{T_2^3-T_1^3}{3}\right)-\lambda\left(\frac{T_2^4-T_1^4}{8}\right)\right) \\ \left.-b\left(\left(\frac{T_2^3-T_1^3}{6}\right)-(b+r)\left(\frac{T_2^4-T_1^4}{8}\right)-\lambda\left(\frac{T_2^5-T_1^5}{20}\right)\right)-(b+r)\left(\frac{T_2^5-T_1^5}{30}\right)-\lambda\left(\frac{T_2^6-T_1^6}{72}\right)\right\} \\ +\lambda D_c\left\{(b+r)\left(\left(\frac{T_1{T_d}^2}{2}-\frac{{T_d}^3}{3}\right)+\frac{b}{2}\left(\frac{{T_1}^2{T_d}^2}{2}-\frac{{T_d}^4}{4}\right)+\frac{\lambda}{6}\left(\left(\frac{{T_1}^3{T_d}^2}{2}-\frac{{T_d}^5}{5}\right)\right)\right)\right. \\ -\left(T_1T_d-\frac{{T_d}^2}{2}\right)+\frac{b}{2}\left({T_1}^2T_d-\frac{{T_d}^3}{3}\right)+\frac{\lambda}{6}\left(\left({T_1}^3T_d-\frac{{T_d}^4}{4}\right)\right) \\ -\left(br-\frac{\lambda}{2}\right)\left(\left(\frac{T_1{T_d}^3}{3}-\frac{{T_d}^4}{4}\right)+\frac{b}{2}\left(\frac{{T_1}^2{T_d}^3}{3}-\frac{{T_d}^5}{5}\right)+\frac{\lambda}{6}\left(\left(\frac{{T_1}^3{T_d}^3}{3}-\frac{{T_d}^6}{6}\right)\right)\right) \\ -\frac{\theta r}{2}\left(\left(\frac{T_1{T_d}^4}{4}-\frac{{T_d}^5}{5}\right)+\frac{b}{2}\left(\frac{{T_1}^2{T_d}^4}{4}-\frac{{T_d}^6}{6}\right)+\frac{\lambda}{6}\left(\left(\frac{{T_1}^3{T_d}^4}{4}-\frac{{T_d}^7}{7}\right)\right)\right) \\ +W_2\left((T_1-T_d)\left(1+\lambda\frac{T_d^2}{2}\right)-r\left(\frac{T_1^2-T_d^2}{2}\right)-\lambda\left(\frac{T_1^3-T_d^3}{6}\right)\right)+a\left(T_2+b\frac{T_2^2}{2}+\lambda\frac{T_2^3}{6}\right) \\ \left((T_2-T_1)-b\left(\frac{T_2^2-T_1^2}{2}\right)-\lambda\left(\frac{T_2^3-T_1^3}{6}\right)\right)-a\left(\left(\frac{T_2^2-T_1^2}{2}\right)-(b+r)\left(\frac{T_2^3-T_1^3}{3}\right)-\lambda\left(\frac{T_2^4-T_1^4}{8}\right)\right) \\ \left.-b\left(\left(\frac{T_2^3-T_1^3}{6}\right)-(b+r)\left(\frac{T_2^4-T_1^4}{8}\right)-\lambda\left(\frac{T_2^5-T_1^5}{20}\right)\right)-(b+r)\left(\frac{T_2^5-T_1^5}{30}\right)-\lambda\left(\frac{T_2^6-T_1^6}{72}\right)\right\} \\ +ac_sT_2\left[(r+1)\left(\frac{T^2-T_2^2}{2}\right)-T+T_2-r\left(\frac{T^3-T_2^3}{3}\right)\right] \\ +l_c\left[a(T-T_2)-r\left(\frac{T^2-T_2^2}{2}\right)-\frac{1}{\delta}\log\left(1+\delta(T-T_2)\right)+\frac{r}{\delta}\left(-(\delta T+1)\log\left(1+\delta(T-T_2)-(T-T_2)\right)\right)\right] \end{array}\right] \quad (20)$$

To minimize the total average cost per unit time, the optimal values of T_2 can be found by solving the following equation:

$$\frac{\partial C_T}{\partial T_2} = 0 \quad (21)$$

As long as they meet the sufficient conditions:

$$\frac{\partial^2 C_T}{\partial {T_2}^2} > 0 \quad (22)$$

We utilize computer software, specifically MATLAB, to calculate the total cost and optimal values of T_2 in the subsequent section.

4. Numerical Illustration

The concept just discussed through the following numerical example, with parameters provided as follows: $\theta = 0.03$, $\epsilon = 10$, $m(\epsilon) = \theta(1 - e^{-0.5\epsilon})$, $r = 0.03$, $h_r = 0.4$,

$h_o = 0.3$, $A = 100$, $p_c = 50$, $C_s = 1.5$, $l_c = 0.5$, $W_2 = 40$, $a = 20$, $b = 6$, $T_d = 0.3$,

$\delta = 0.0001, d_c = 0.5, T_1 = 0.7, T = 2$

Then we get $T_2 = 1.6283, Q = 96.2670\ and\ C_T = 5150.6$

5. Sensitivity analysis

To analyze the sensitivity of this model, we conducted a sensitivity analysis by changing key parameters such as demand parameters 'a' and 'b', deterioration rate θ, etc. The consequence of change in parameters is specified in Table 1. Enthusiastic observation of the table 1 tells the following information:

1. Increases in a results in decrement in T_2 but increment in C_T and Q.
2. Increase in b results in decrement in T_2 but increment in C_T and Q.
3. Increase in θ results in decrement in T_2 but increment in C_T and Q.
4. Increases in r results in increment in T_2 but decrement in C_T and Q.

Table 1: Sensitivity analysis examines how optimal values of various parameters change in response to alterations in other parameters.

Parameter	Change in parameter	T_2	Q	C_T
a	18	1.6949	89.4401	4788.8
	22	1.5711	103.1511	5514.8
	26	1.4777	117.0614	6249.4
b	5	1.8667	84.5933	4522.8
	7	1.4539	107.3961	5753.0
	9	1.2168	130.5068	7020.1
θ	0.02	1.6283	96.2656	5150.5
	0.04	1.6282	96.2685	5150.7
	0.05	1.6281	96.2700	5150.8
r	0.02	1.4082	100.6686	5360.5
	0.04	1.8147	92.5375	4966.1
	0.05	1.9795	89.2412	4797.6

6. Conclusion

This investigation introduces a dual warehouse inventory model designed for non-instantaneous degrading products, which extends the traditional EOQ model. Our model accommodates a configurable deterioration rate, Demand influenced by stock levels, and allows for partial backlogging of shortages. We incorporate preservation techniques to mitigate deterioration during the inventory's deterioration period and suggest an approach to find the best replenishment cycles, order quantities, and preservation technology costs, all aimed at minimizing the total inventory cost per unit time. Through numerical illustrations, we demonstrate the efficacy of our model, particularly for suppliers employing preservation technology in both ordinary and reserve warehouses to control deterioration rates. Our numerical analysis confirms the stability of the proposed solution. Furthermore, we suggest avenues for expanding the model to include features like non-instantaneous degradation, variable backlogged shortages, and perpetual inventory policies. Future research could explore extensions incorporating nonlinear demand and holding costs, as well as introducing additional elements such as trade credit terms, Demand influenced by price, with price being a variable in decision-making, thereby enhancing the practical applicability and robustness of the inventory management approach proposed herein.

References

[1] Bierman Jr H & Thomas J. (1977). Inventory decisions under inflationary conditions. Decision Sciences, 8(1), 151-155.

[2] Buzacott J. A. (1975). Economic order quantities with inflation. Journal of the Operational Research Society, 26(3), 553-558.

[3] Feng L. (2019). Dynamic pricing, quality investment, and replenishment model for perishable items. International Transactions in Operational Research, 26(4), 1558–1575.

[4] Ghare P.M. & Schrader G. P. (1963). A model for an exponentially decaying inventory. Journal of Industrial Engineering, 14(5), 238-243.

[5] Ghiami Y, Williams T & Wu Y. (2013). A two-echelon inventory model for a deteriorating item with stock-dependent demand, partial backlogging and capacity constraints. European Journal of Operational Research, 231(3), 587 – 597.

[6] Gothi U.B., Mehta D & Parmar K. (2016). A two-warehouse inventory model with Weibull deterioration rate and time dependent demand rate and holding cost. IOSR Journal of Mathematics, 12(3), 95 – 102.

[7] Hartley R.V. (1976). Operations Research–A Managerial Emphasis, California: Good Year Publishing Company.

[8] Hasan, Rakibul, Abu Hashan Md Mashud, Yosef Daryanto, and Hui Ming Wee., (2020). A non- instantaneous inventory model of agricultural products considering deteriorating impacts and pricing policies. Kybernetes. ahead-of-print. [CrossRef]

[9] Hsu P. H., Wee H. M. & Teng H. M. (2010). Preservation technology investment for deteriorating inventory. International Journal of Production Economics, 124(2), 388-394.

[10] Iqbal M. W. & Sarkar B. (2020). Application of preservation technology for lifetime dependent products in an integrated production system. Journal of Industrial and Management Optimization, 16, 141–167.

[11] Jaggi C. K., Tiwari S. & Goel S. K. (2017). Credit financing in economic ordering policies for non-instantaneous deteriorating items with price dependent demand and two storage facilities. Annals of Operations Research, 248(1-2), 253-280.

[12] Kaliraman N.K., Raj R., Chandra S. & Chaudhary H. (2017). Two warehouse inventory model for deteriorating item with exponential demand rate and permissible delay in payment. Yugoslav Journal of Operations Research, 27(1), 109 – 124.

[13] Lashgari M., Taleizadeh A. A. & Ahmadi A. (2016). Partial up-stream advanced payment and partial down-stream delayed payment in a three-level supply chain. Annals of Operations Research, 238(1-2), 329-354.

[14] Liang Y. & Zhou F. (2011). A two-warehouse inventory model for deteriorating items under conditionally permissible delay in payment. Applied Mathematical Modelling, 35(5), 2221-2231.

[15] Maiti A. K., Maiti M. K. & Maiti M. (2009). Inventory model with stochastic lead-time and price dependent demand incorporating advance payment. Applied Mathematical Modelling, 33(5), 2433- 2443.

[16] Mashud A. H. M., Hasan M. R., Daryanto Y. & Wee H. M. (2021). A resilient hybrid payment supply chain inventory model for post Covid-19 recovery. Computers & Industrial Engineering, 157, 107249.

[17] Misra R. B. (1979). A note on optimal inventory management under inflation. Naval Research Logistics Quarterly, 26(1), 161-165.

[18] Mishra U., Wu J. Z. & Tseng M. L. (2019). Effects of a hybrid-price-stock dependent demand on the optimal solutions of a deteriorating inventory system and trade credit policy on re-manufactured product. Journal of Cleaner Production, 241, 118282.

[19] Nobil, Amir Hossein, Amir Hossein Afsar Sedigh and Leopoldo Eduardo Cardenas Barron., (2020). Reorder point for the EOQ inventory model with imperfect quality items. Ain Shams Engineering Journal, 11, 1339–1343.

[20] Pervin M., Mahata G. C. & Kumar Roy S. (2016). An inventory model with declining demand market for deteriorating items under a trade credit policy. International Journal of Management Science and Engineering Management, 11(4), 243-251.

[21] Ramya S and Sivakumar D (2024).“Optimizing inventory management for perishable goods: managing Exponential demand variations, stable holding charges, and partial backlog handling”, Communications on Applied Nonlinear Analysis, Vol. 31 No.3s, 303-311.

[22] Sarma K.V.S. (1987). A deterministic order level inventory model for deteriorating items with two storage facilities. European Journal of Operational Research, 29, 70 – 73.

[23] Shaikh A. A. (2017). A two-warehouse inventory model for deteriorating items with variable demand under alternative trade credit policy. International Journal of logistics Systems and Management, 27(1), 40-61.

[24] Sekar T. & Uthayakumar R. (2018). Optimization of an imperfect manufacturing system for deteriorating items with rework and shortage under inflation. Process Integration and Optimization for Sustainability, 2(4), 303-320.

[25] Taleizadeh A. A. (2014). An EOQ model with partial backordering and advance payments for an evaporating item. International Journal of Production Economics, 155, 185-193.

[26] Teng J. T., Cárdenas-Barrón L. E., Chang H. J., Wu J. & Hu Y. (2016). Inventory lot-size policies for deteriorating items with expiration dates and advance payments. Applied Mathematical Modelling, 40(19-20), 8605-8616.

[27] Tiwari S., Jaggi C. K., Bhunia A. K., Shaikh A. A. & Goh M. (2017). Two-warehouse inventory model for non-instantaneous deteriorating items with stock-dependent demand and inflation using particle swarm optimization. Annals of Operations Research, 254(1), 401-423.

[28] Tiwari S., Cárdenas-Barrón L. E., Goh M. & Shaikh A. A. (2018a). Joint pricing and inventory model for deteriorating items with expiration dates and partial backlogging under two-level partial trade credits in supply chain. International Journal of Production Economics, 200, 16-36.

[29] Tiwari S., Jaggi C. K., Gupta M. & Cárdenas-Barrón L. E. (2018b). Optimal pricing and lot-sizing policy for supply chain system with deteriorating items under limited storage capacity. International Journal of Production Economics, 200, 278-290.

[30] Xu W. & Song D. P. (2020). Integrated optimization for production capacity, raw material ordering and production planning under time and quantity uncertainties based on two case studies. Operational Research, 1-29.