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Applying "AEM" Transform on Partial Differential Equations

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Abstract:

This work demonstrates the (Ahmed-Emad-Murat) "AEM" transform technique, where its important properties and its capability to evaluate a particular solution for partial differential equations have been introduced and shown via the solution of fundamental examples and physical partial differential equations.

Keywords: AEM Transform Technique, Inverse of AEM Transform, Partial Differential Equations.

1. Introduction

Partial differential equations (PDE) represented a special case of ordinary differential equations, with multiple partial derivatives of unknown variables. PDE degree is identified via the highest derivative that appears in operator equation. Using a mathematical approach that could solve PDE concludes a function converts to identity when substituted into the operator equation. PDEs have been applied in various scientific fields, which yield from their ability to express physical problems in a mathematical model that can be manipulated and solved by some mathematical approach [1-3,16-18].

The significance of PDEs necessitated applying the most effective mathematical approaches for their solution [4-7]. Integral technique's ability to transform problems from one domain to another to simplify their solution has positioned them as a priority in the domain of PDE solution. Authors have proposed numerous integral transforms to find the exact solution of PDEs; every proposed technique has particular cases where it shines [8-14]. The substantial field of partial operator differential equations, on the other hand, has not yet benefited from the revolutionary AEM- integral transform.

The AEM technique is applied in this paper to find the solution of first and second order partial operator differential equations, as well as practical applications of differential equations, which are regarded important in the mathematical physical field.

1. Fundamental Properties of AEM Transform [15]

AEM transform of $u(x,t)$ denoted by $E\big(x,H(\alpha),p(\alpha)\big)$ is given by:

$$AEM\big(u(x,t)\big) = H(\alpha)\int_{t=1}^{\infty} t^{-(p(\alpha)+1)}u(x,t)dt = E\big(x,H(\alpha),P(\alpha)\big).$$

Where x , t are variables and $H(\alpha), p(\alpha)$ are functions of parameter α.

1.1 AEM Transform Existence

AEM technique is considered to exist for sufficiently large parameter α, providing the integral:

$H(\alpha)\int_{t=1}^{\infty} t^{-(p(\alpha)+1)}u(x,t)dt = \lim_{m\to\infty}\int_{t=1}^{m} t^{-(p(\alpha)+1)}u(x,t)dt$.

Criteria for Convergence (I)

AEM transform for the function $u(x,t)$ exist, if it has exponential order and $\int_1^m |u(x,t)|dt$ exist for any $m > 0$.

Since the convergence is needed to be proven only for sufficient large parameter α, then it is going to be assumed that $p(\alpha) > c$ and $p(\alpha) > 0$.

$$H(\alpha)\int_{t=1}^{\infty}\left|t^{-(p(\alpha)+1)}u(x,t)\right|dt = H(\alpha)\left[\int_1^n \left|t^{-(p(\alpha)+1)}u(x,t)\right|dt + \int_n^{\infty}\left|t^{-(p(\alpha)+1)}u(x,t)\right|dt\right],$$

$$\leq H(\alpha)\left[\int_1^n |u(x,t)|dt + \int_n^{\infty} t^{-(p(\alpha)+1)}|u(x,t)|dt\right].$$

For: $\left[0 < H(\alpha)t^{-(p(\alpha)+1)} \leq 1\right]$

$$\leq H(\alpha)\left[\int_1^n |u(x,t)|dt + \int_n^{\infty} t^{-(p(\alpha)+1)}Mt^{(c(\alpha)+1)}dt\right],$$

$$= H(\alpha)\left[\int_1^n |u(x,t)|dt + M\left[\frac{t^{-(p(\alpha)-c(\alpha))+1}}{-(p(\alpha)-c(\alpha))+1}\Big|_n^{\infty}\right]\right].$$

For $p(\alpha) > c$

$$= H(\alpha)\left[\int_1^n |u(x,t)|dt + M\frac{n^{-(p(\alpha)-c(\alpha))+1}}{-(p(\alpha)-c(\alpha))+1}\right].$$

The first integral exists by assumption, and the second term is finite $p(\alpha) > c$.

The integral $H(\alpha)\int_{t=1}^{\infty} t^{-(p(\alpha)+1)}u(x,t)dt$, converges absolutely and $AEM\{u(x,t)\}$ exists.

Criteria for Convergence (II)

To satisfy criterion (I), AEM$\{u(x,t)\}$ exists if:

$u(x,t)$ is of exponential order and on the closed interval $[1,m]$.

$u(x,t)$ Is a bounded, piecewise, continuous and have a finite number of discontinuous requirements implying that $\int_b^0 |f(t)|dt$.

Where $F(p) \to 0$ as $p \to \infty$.

Assuming $u(x,t)$ satisfy criterion (I), which implies $F(p) = AEM\{u(x,t)\}$ will exist if $p \geq s$ for some s.

$F(p(\alpha)) = \left|H(\alpha)\int_{t=1}^{\infty} t^{-(p(\alpha)+1)}u(x,t)dt\right| \leq \int_{1}^{\infty}\left|H(\alpha)t^{-(p(\alpha)+1)}u(x,t)\right|dt = G(p)\ \ p \to \infty$, $H(\alpha)t^{-(p(\alpha)+1)} \to 0$. For $t \geq 1$.

1.2 AEM Transform Uniqueness

Assume that the functions f and g are exponential type b, piecewise and continuous on $[1,\infty)$. If $AEM\{f(x,t)\} = AEM\{g(x,t)\}$ when $s > b$, then $f(x,t) = g(x,t)$ for all t greater than or equal one.

1.3 The Inverse of AEM Transform [15]

The inverse of AEM technique of $E\big(x, H(\alpha), P(\alpha)\big)$ denoted by $(\text{AEM})^{-1}$ and defined as:

$$(\text{AEM})^{-1}\Big(AEM\big(u(x,t)\big)\Big) = u(x,t) = \frac{1}{2\pi i}\int_{\delta-i\varepsilon}^{\delta+i\varepsilon}\frac{t^{(p(\alpha)+1)}}{H(\alpha)}E\big(x, H(\alpha), P(\alpha)\big)d\alpha\ .$$

In general, $\mu = \delta + i\varepsilon$ with δ and ε being real numbers,$i \in \mathbb{C}$. The integral converges when

$Re(P(\alpha)) = \delta > 0$ .And $\delta < 0$, $E\big(x, H(\alpha), P(\alpha)\big) = 0$.

1.4 Properties of AEM transform [15]

Theorem (2.4.1). (Linearity): If $u(x,t) = Cu_1(x,t) \pm Du_2(x,t)$,where C and D are constants then

$$\text{AEM}\big(Cu_1(x,t) \pm Du_2(x,t)\big) = CH(\alpha)\int_{t=1}^{\infty} t^{-(p(\alpha)+1)}u_1(x,t)dt \pm \text{D}H(\alpha)\int_{t=1}^{\infty} t^{-(p(\alpha)+1)}u_2(x,t)dt,$$

$$= C\ AEM\big(u_1(x,t)\big) \pm D\ AEM\big(u_2(x,t)\big).$$

Table (2.4.1) [15]

$u(t)$	$E\big(H(\alpha), P(\alpha)\big)$
K , constant	$K\dfrac{H(\alpha)}{p(\alpha)}$
t^m , m=1, 2, 3, ...	$\dfrac{H(\alpha)}{p(\alpha)-m}$
$\ln(t)$	$\dfrac{H(\alpha)}{\big(p(\alpha)\big)^2}$
$t^m \ln(t)$	$\dfrac{H(\alpha)}{(p(\alpha)-m)^2}$
$\sin\big(aln(t)\big)$, $a\ constsnt$	$\dfrac{aH(\alpha)}{\big(p(\alpha)\big)^2 + a^2}$
$\cos\big(aln(t)\big)$	$\dfrac{p(\alpha)H(\alpha)}{\big(p(\alpha)\big)^2 + a^2}$
$\sinh\big(aln\ (t)\big)$	$\dfrac{aH(\alpha)}{\big(p(\alpha)\big)^2 - a^2}$

$\cosh\left(a ln(t)\right)$	$\dfrac{p(\alpha)H(\alpha)}{\left(p(\alpha)\right)^2 - a^2}$

Theorem (2.4.2): $AEM((\ln(t))^m) = \frac{m!H(\alpha)}{(p(\alpha))^{m+1}}$ $, where\ m = 1,2,3 \ldots$.

Proof: By the definition $AEM((\ln(t))^m) = H(\alpha)\int_1^\infty (\ln(t))^m t^{-(p(\alpha)+1)} dt$,

$$= H(\alpha)\left[(\ln(t))^m \frac{t^{-p(\alpha)}}{-p(\alpha)}\Bigg|_1^\infty - \int_1^\infty m(\ln(t))^{m-1}\frac{t^{-(p(\alpha)+1)}}{-p(\alpha)}\mathrm{dt}\right]$$

$$= \mathrm{m}\frac{H(\alpha)}{p(\alpha)}\int_1^\infty (\ln(t))^{m-1} t^{-(p(\alpha)+1)} dt \quad ,$$

$= m\frac{H(\alpha)}{p(\alpha)} AEM((\ln(t))^{m-1})$,

$$= m(m-1)\frac{H(\alpha)}{\left(p(\alpha)\right)^2}\int_1^\infty (\ln(t))^{m-2} t^{-(p(\alpha)+1)} dt \ ,$$

$$= m(m-1)(m-2)\ldots 4\frac{H(\alpha)}{\left(p(\alpha)\right)^{m-2}}\int_1^\infty (\ln(t))^3 t^{-(p(\alpha)+1)} dt \ ,$$

$$= m(m-1)(m-2)\ldots 4\frac{H(\alpha)}{\left(p(\alpha)\right)^{m-2}}\left[(\ln(t))^3 \frac{t^{-p(\alpha)}}{-p(\alpha)}\Bigg|_1^\infty - \int_1^\infty 3(\ln(t))^2 \frac{t^{-(p(\alpha)+1)}}{-p(\alpha)}\mathrm{dt}\right],$$

$$= m(m-1)(m-2)\ldots 4.3\frac{H(\alpha)}{\left(p(\alpha)\right)^{m-1}}\int_1^\infty (\ln(t))^2 t^{-(p(\alpha)+1)} dt = m(m-1)(m-2)\ldots 4.3\frac{E\left((\ln(t))^2\right)}{\left(p(\alpha)\right)^{m-1}}$$

,

$$= m(m-1)(m-2)\ldots 4.3\frac{H(\alpha)}{\left(p(\alpha)\right)^{m-1}}\left[(\ln(t))^2 \frac{t^{-p(\alpha)}}{-p(\alpha)}\Bigg|_1^\infty - \int_1^\infty 2\ln(t)\frac{t^{-(p(\alpha)+1)}}{-p(\alpha)}\mathrm{dt}\right],$$

$$= m(m-1)(m-2)\ldots 4.3.2\frac{H(\alpha)}{\left(p(\alpha)\right)^{m}}\int_1^\infty \ln(t)\, t^{-(p(\alpha)+1)} dt$$

$$= m(m-1)(m-2)\ldots 4.3.2\frac{H(\alpha)}{\left(p(\alpha)\right)^{m+1}},$$

$$= \frac{m!\,H(\alpha)}{\left(p(\alpha)\right)^{m+1}} \ .$$

Theorem (2.4.3) [15]: If $AEM\left(u(x,t)\right) = E\left(x, H(\alpha), p(\alpha)\right)$, then $AEM\left(t^m u(x,t)\right) = E(x, H(\alpha), p(\alpha) - m)$.

Proof: By the definition, we have:

$$AEM\big(t^m u(x,t)\big) = H(\alpha)\int_1^\infty t^{-(p(\alpha)+1)} t^m u(x,t)dt = H(\alpha)\int_1^\infty t^{-(p(\alpha)+1)+m} u(x,t)dt \ ,$$

$$= H(\alpha)\int_1^\infty t^{-((p(\alpha)-m)+1)} u(x,t)dt = E(x,H(\alpha),p(\alpha)-m).$$

2. AEM Transform of derivatives of $\boldsymbol{u(x,t)}$

(1) $AEM\big(tu_t(x,t)\big) = H(\alpha)\int_1^\infty t^{-(p(\alpha)+1)} t u_t(x,t)dt \ ,$

$$= H(\alpha)\left[t^{-p(\alpha)}u(x,t)\Big|_1^\infty - \int_1^\infty u(x,t).-p(\alpha)t^{-(p(\alpha)+1)}\,\mathrm{dt}\right] ,$$

$= H(\alpha)\big[-u(x,1) + p(\alpha)\int_1^\infty u(x,t).t^{-(p(\alpha)+1)}dt\big] = -H(\alpha)u(x,1) + p(\alpha)E\big(x,H(\alpha),p(\alpha)\big).$

(2) $AEM\big(t^2u_{tt}(x,t)\big) = H(\alpha)\int_1^\infty t^{-(p(\alpha)+1)}\, t^2 u_{tt}(x,t)dt = H(\alpha)\int_1^\infty t^{-(p(\alpha)-1)}\, u_{tt}(x,t)dt,$

$$= H(\alpha)\left[t^{-(p(\alpha)-1)}u_t(x,t)\Big|_1^\infty - \int_1^\infty u_t(x,t).-(p(\alpha)-1)t^{-p(\alpha)}\,\mathrm{dt}\right],$$

$$= -H(\alpha)u_t(x,1) - (p(\alpha)-1)H(\alpha)u(x,t) + p(\alpha)(p(\alpha)-1)E\big(x,H(\alpha),p(\alpha)\big)\,.$$

2.1 Theorem: Let $u(x,t)$ be continuous on $(1,\infty)$ and if $u_t, u_{tt}, \ldots, u_t^{(m)}$ exist, then

$AEM\Big[t^m u_t^{(m)}(x,t)\Big] = -H(\alpha)u_t^{(m-1)}(x,1) - H(\alpha)\big(p(\alpha)-(m-1)\big)u_t^{(m-2)}(x,1) - \cdots$

$-H(\alpha)\big(p(\alpha)-(m-1)\big)\big(p(\alpha)-(m-2)\big)\ldots(p(\alpha)-1)u(x,1) + p(\alpha)(p(\alpha)-1)\ldots$

$\big(p(\alpha)-(m-1)\big)E\big(x,H(\alpha),p(\alpha)\big).$

Proof:

By mathematical induction

- If $m = 1$

$$AEM\big(tu_t(x,t)\big) = H(\alpha)\int_1^\infty t^{-(p(\alpha)+1)} t u_t(x,t)dt\,,$$

$$= H(\alpha)\left[t^{-p(\alpha)}u(x,t)\Big|_1^\infty - \int_1^\infty u(x,t).-p(\alpha)t^{-(p(\alpha)+1)}\,\mathrm{dt}\right],$$

$$= H(\alpha)\left[-u(x,1) + p(\alpha)\int_1^\infty u(x,t).t^{-(p(\alpha)+1)}dt\right]$$

$$= -H(\alpha)u(x,1) + p(\alpha)E\big(x,H(\alpha),p(\alpha)\big).$$

- True for m

$AEM\left[t^m u_t^{(m)}(x,t)\right] = -H(\alpha)u_t^{(m-1)}(x,1) - H(\alpha)\big(p(\alpha)-(m-1)\big)u_t^{(m-2)}(x,1) - \cdots$
$-H(\alpha)\big(p(\alpha)-(m-1)\big)\big(p(\alpha)-(m-2)\big)\dots(p(\alpha)-1)u(x,1) + p(\alpha)(p(\alpha)-1)\dots$
$\big(p(\alpha)-(m-1)\big)E\big(x,H(\alpha),p(\alpha)\big).$

- Is it True for (m+1)?

$$AEM\left[t^{m+1}u_t^{(m+1)}(x,t)\right] = H(\alpha)\int_1^\infty t^{-(p(\alpha)+1)}t^{m+1}u_t^{(m+1)}(x,t)dt,$$

$$= H(\alpha)\int_1^\infty t^{-(p(\alpha)-m)}u_t^{(m+1)}(x,t)dt,$$

$$= H(\alpha)\left[t^{-(p(\alpha)-m)}u_t^{(m)}(x,t)\Big|_1^\infty + \int_1^\infty u_t^{(m)}(x,t).(p(\alpha)-m)t^{-((p(\alpha)-m)-1)}\,\mathrm{dt}\right]$$

$$= -H(\alpha)u_t^{(m)}(x,1) + (p(\alpha)-m)AEM\left(t^m u_t^{(m)}(x,t)\right),$$

$$\begin{aligned} = -H(\alpha)u_t^{(m)}(x,1) \\ &+ (p(\alpha)-m)\Big[-H(\alpha)u_t^{(m-1)}(x,1) - H(\alpha)\big(p(\alpha)-(m-1)\big)u_t^{(m-2)}(x,1) \\ &- H(\alpha)\big(p(\alpha)-(m-1)\big)\big(p(\alpha)-(m-2)\big)u_t^{(m-3)}(x,1) - \cdots \\ &+ p(\alpha)(p(\alpha)-1)(p(\alpha)-2)\dots\big(p(\alpha)-(m-1)\big)E\big(x,H(\alpha),p(\alpha)\big)\Big], \end{aligned}$$

$= -H(\alpha)u_t^{(m)}(x,1) - (p(\alpha)-m)H(\alpha)u_t^{(m-1)}(x,1) - H(\alpha)(p(\alpha)-m)\big(p(\alpha)-(m-$
$1)\big)u_t^{(m-2)}(x,1) - H(\alpha)(p(\alpha)-m)\big(p(\alpha)-(m-1)\big)\big(p(\alpha)-(m-2)\big)u_t^{(m-3)}(x,1) - \cdots +$
$p(\alpha)(p(\alpha)-1)(p(\alpha)-2)\dots\big(p(\alpha)-(m-1)\big)(p(\alpha)-m)E\big(x,H(\alpha),p(\alpha)\big).$

2.2 Examples of Applying AEM transform on partial differential equations with variable coefficients

Problem (3.2.1): Consider the following partially differential equation:

$$t^2u_{tt}(x,t) + tu_t(x,t) + 2u(x,t) = t^{-1}\ln(t).$$

$$With\ initial\ conditions\ (x,1) = 10, u_t(x,1) = -1. \qquad (3.2.1)$$

By Applying AEM transform on equation (3.2.1), we obtain

$$AEM\big(t^2u_{tt}(x,t)\big) + AEM\big(tu_t(x,t)\big) + 2AEM\big(u(x,t)\big) = AEM(t^{-1}\ln(t))$$

$$\begin{aligned} -H(\alpha)u_t(x,1) - H(\alpha)(p(\alpha)-1)u(x,1) + p(\alpha)(p(\alpha)-1)E\big(x,H(\alpha),p(\alpha)\big) - H(\alpha)u(x,1) \\ + p(\alpha)E\big(x,H(\alpha),p(\alpha)\big) + 2E\big(x,H(\alpha),p(\alpha)\big) \\ = \frac{H(\alpha)}{(p(\alpha)+1)^2}\,. \end{aligned} \qquad (3.2.2)$$

And by applying the initial condition and simplify equation (3.2.2)

$E\big(x,H(\alpha),p(\alpha)\big)=\frac{H(\alpha)}{(p(\alpha)+1)^2(p(\alpha))^2+2)}-\frac{H(\alpha)}{(p(\alpha))^2+2)}+\frac{10H(\alpha)p(\alpha)}{(p(\alpha))^2+2)}$.

After using partial fraction of the last equation, we get

$$E\big(x,H(\alpha),p(\alpha)\big)=H(\alpha)\left[\frac{2/9}{(p(\alpha)+1)}-\frac{3/9}{\big((p(\alpha)+1)\big)^2}-\frac{2/9\,p(\alpha)}{(p(\alpha))^2+2)}-(1/9)\frac{1}{(p(\alpha))^2+2)}-\frac{1}{(p(\alpha))^2+2)}+\frac{10p(\alpha)}{(p(\alpha))^2+2)}\right].\quad(3.2.3)$$

By applying the inverse of AEM transform to equation (3.2.3), we require

$u(x,t)=(2/9)t^{-1}+(3/9)t^{-1}\ln t+\frac{88}{9}\cos(\sqrt{2}\ln t)-\frac{10}{9\sqrt{2}}\sin(\sqrt{2}\ln t)$.

Problem (3.2.2): Consider the following partially differential equation:

$$2tu_t(x,t)-7u(x,t)=x^2\sin(\ln(t)).$$

$$With\ initial\ condition\ u(x,1)=-3.\quad(3.2.4)$$

By Applying AEM transform on equation (3.2.4), we obtain:

$AEM\big(2tu_t(x,t)\big)-7AEM\big(u(x,t)\big)=AEM(x^2\sin\ln(t))$.

$$-2H(\alpha)u(x,1)+2p(\alpha)E\big(x,H(\alpha),p(\alpha)\big)-7E\big(x,H(\alpha),p(\alpha)\big)=\frac{x^2H(\alpha)}{\Big(\big(p(\alpha)\big)^2+1\Big)}\ .\quad(3.2.5)$$

Via applying the initial condition and simplify equation (3.2.5):

$E\big(x,H(\alpha),p(\alpha)\big)=\frac{x^2H(\alpha)}{\big((p(\alpha))^2+1\big)(2p(\alpha)-7)}-\frac{6H(\alpha)}{(2p(\alpha)-7)}$.

After using partial fraction and simplify the last equation, we get:

$$E\big(x,H(\alpha),p(\alpha)\big)=x^2H(\alpha)\left(\frac{-2}{53}\frac{p(\alpha)}{\Big(\big(p(\alpha)\big)^2+1\Big)}-\frac{7}{53}\frac{1}{\Big(\big(p(\alpha)\big)^2+1\Big)}+\frac{4}{53}\frac{1}{(2p(\alpha)-7)}\right)-\frac{6H(\alpha)}{(2p(\alpha)-7)}\ .\quad(3.2.6)$$

By applying the inverse of AEM transform to equation (3.2.6), that is:

$$u(x,t)=(-2/53)x^2\cos(ln(t))-(7/53)x^2\sin(lnt)+\frac{2x^2}{53}t^{7/2}-3t^{7/2}.$$

3. Conclusion

The AEM integral technique has been used to evaluate the exact solution of partial operator differential equations. The proofs that accompanied using the AEM technique to partial differential operator equations and the solution of a practical example solidify the AEM transform's ability to efficiency handle and provide the solution to the PDEs, making it a strong competitor to other integral transforms in solving partial differential operator equations.

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