Advances in Nonlinear Variational Inequalities
ISSN: 1092-910X
Vol 28 No. 2 (2025)

Relatively Prime Domination Number in Triangular Snake Graphs

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Article History:

Received: *27-07-2024*

Revised: *10-09-2024*

Accepted: *18-09-2024*

Abstract:

A set S⊆V is said to be relatively prime dominating set if it is a dominating set with at least two elements and for every pair of vertices u and v in S, (deg(u),deg 〖(v))〗 =1 and the minimum cardinality of a relatively prime dominating set is called relatively prime domination number and it is denoted by γ_rpd (G). If there is no such pair exist, then γ_rpd (G)=0. For a finite undirected graph G(V,E) and a subset V, the switching of G by is defined as the graph (V,) which is obtained from G by removing all edges between and its complement V- and adding as edges all non-edges between and V- . This article delves into the discussion of the relatively prime domination number on triangular snake graphs and their complements. The findings reveal that for triangular snake graphs, the relatively prime domination number γ_rpd(G^v) equals either 2 or 3. Similarly, for alternate triangular snake graphs, the γ_rpd(G^v) is determined to be 2 or 3. In the case of double triangular snake graphs, the relatively prime domination number γ_rpd(G^v) is established as 2, 3, 4, or 6, while for double alternate triangular snake graphs, it is 2, 3, or 4. Notably, the complements of alternate triangular, double triangular, and double alternate triangular snake graphs exhibit a relatively prime domination number of 2.

Keywords: Dominating Set, Domination Number, Relatively Prime Dominating Set, Relatively Prime Dominating Number

1. Introduction

By a graph G = (V, E) we mean a finite undirected graph without loops and multiple edges. The order and size of G are denoted by p and q respectively. For graph theoretical terms, we refer to Harary [2] and for terms related to domination we refer to Haynes [7]. A subset S of V is said to be a dominating set in G if every vertex in V– S is adjacent to at least one vertex in S. The domination number γ(G) is the minimum cardinality of a dominating set in G. Berge [1] and Ore [6] formulated the concept of domination in graphs. It was further extended to define many other dominations related parameters in graphs. In 2017, C. Jayasekaran and A. Jancy Vini [3] have introduced the concept of relatively prime domination number in graph theory. Let G be a non – trivial graph. A set $S \subseteq V$ is said to be a relatively prime dominating set if it is a dominating set and for every pair of vertices u and v in S such that (d(u), d(v)) = 1. The minimum cardinality of a relatively prime dominating set is called the relatively prime domination number and it is denoted by $\gamma_{rpd}(G)$. Further they have introduced the concept of relatively prime dominating polynomial in [4]. Switching in graphs was introduced by Lint and Seidel [5]. For a finite undirected graph G(V, E) and a subset $\sigma \subseteq V$, the switching of G by σ is defined as the graph $G^{\sigma}(V, E')$ which is obtained from G by removing all edges between σ and its complement $V-\sigma$ and adding as edges all non-edges between σ and $V-\sigma$. For σ = {v}, we write G^v instead of $G^{\{v\}}$ and the corresponding switching is called as vertex switching. In this paper we determine the relatively prime domination number $\gamma_{rpd}(G^v)$ and $\gamma_{rpd}(\overline{G})$, where G is a triangular snake graph.

2. Preliminaries

Definition 2.1. The triangular snake is obtained from the path P_n by replacing each edge of the path by a triangle C_3. It is denoted by T_n.

Definition 2.2. An alternate triangular snake is obtained from a path $v_1, v_2, \ldots, v_n$ by joining v_i and v_{i+1}(alternately) to a new vertex a_i. It is denoted by $A(T_n)$.

Definition 2.3. A double triangular snake consists of two triangular snakes that have a common path. It is denoted by $D(T_n)$.

Definition 2.4. A double alternate triangular snake consists of two alternate triangular snakes that have a common path. It is denoted by $A(D(T_n))$.

3. Relatively Prime Domination Number of Triangular Snake Graphs

In this section we have discussed the relatively prime domination number for snake graphs.

Theorem 3.1. Let G be the triangular snake graph with p vertices, where p = 2n+1, n ≥ 2. Then for p ≠ 5, $\gamma_{rpd}(G^v)$ = 2 or 3.

Proof: Let G be a triangular snake graph with p vertices, where p = 2n+1, n ≥ 2. Let the vertices in the path be v_i, 1≤ i≤ m and the vertices in the triangle be u_i, 1≤ i≤ m-1. Then $d(v_i)$ = 4, 2≤ i≤ m-1, $d(v_1) = d(v_m)$ = 2, $d(u_i)$ = 2, 1≤ i≤ m-1. Let v be any vertex in G. We have the following cases.

Case 1: v = u_i , 1≤ i≤ m-1

Without loss of generality, let v = u_i, $i = 1,2, \ldots, m-1$. Then $d(v)$ = p-3. Since, v covers all the vertices of G^v except the two vertices, namely v_i and v_{i+1}. Then $d(v_i)$ = 1 if v_i is an initial vertex, otherwise 3. Similarly, $d(v_{i+1})$ = 1 if v_{i+1} is an end vertex, otherwise 3. Note that, v_i and v_{i+1} are adjacent in G. If v_i is an initial vertex, then $\{v, v_i\}$ is a relatively prime dominating set and hence $\gamma_{rpd}(G^v)$ = 2. If v_{i+1} is an end vertex, then $\{v, v_{i+1}\}$ is a relatively prime dominating set and hence $\gamma_{rpd}(G^v)$ = 2. Now, we consider that neither v_i is an initial vertex nor v_{i+1} is an end vertex. Then $d(v_i)$ = $d(v_{i+1})$ = 3. Since |V| = 2n+1, d(v) is always even. If d(v) is not a multiple of 3, then $\{v, v_i\}$ is a relatively prime dominating set and hence $\gamma_{rpd}(G^v)$ = 2. Suppose d(v) is a multiple of 3. Then we cannot take the vertices v_i and v_{i+1} together. So, we consider the vertices which are adjacent to v_i and v_{i+1}. Since, $d(u_i)$ = 3 and thus we cannot take these vertices and hence we consider the vertices in the path. Let them be v_{i-1} and v_{i+2}. Then $d(v_{i-1})$ = 3, if it is an initial vertex, otherwise 5. Similarly, $d(v_{i+2})$ = 3, if it is an end vertex, otherwise 5. If v_{i-1} and v_{i+2} is an initial and end vertex respectively, then relatively prime dominating set does not exist. Otherwise degree is 5. To cover the vertices v_i and v_{i+1}, we must take the vertices v_{i-1} and v_{i+2}. It is not possible to take these vertices. Thus relatively prime dominating set does not exist.

Case 2: v is an initial or end vertex of the path.

Without loss of generality, let it be v_1. Then $d(v_1)$ = p-3 in G^v. This vertex covers all the vertices of G^v except two vertices, namely v_2 and u_1. Here $d(u_1)$ = 1 and $d(v_2)$ =3. Since u_1 and v_2 are adjacent in G and (d(u_1), d(v_2)) = (p-3, 1) = 1. Therefore, $\{v, u_1\}$ is a relatively prime dominating set and hence $\gamma_{rpd}(G^v)$ = 2.

Case 3: v is any internal path vertex.

Then d(v_i) = p-5. This vertex covers all the vertices of G^v except four vertices, namely v_{i-1}, v_{i+1}, u_i, and u_{i-1}. Then d(u_i) = d(u_{i-1}) =1 and d(v_{i-1}) =1, if v_{i-1} is an initial vertex, otherwise 3. Similarly, d(v_{i+1}) = 1, if v_{i+1} is an end vertex, otherwise 3. Since u_i and v_{i-1}, u_{i+1} and v_{i+1} are adjacent in G, then $\{v, u_i, u_{i+1}\}$ is a relatively prime dominating set and hence $\gamma_{rpd}(G^v)$ = 3.

Theorem 3.2. Let G be the alternate triangular snake graph with p vertices, where p = 3n, n ≥ 2. Then for p ≠ 6, $\gamma_{rpd}(G^v) = 2$ or 3.

Proof: Let G be an alternate triangular snake graph with p vertices, where p = 3n, n ≥ 2. Let the vertices in the path be v_i, 1≤ i≤ m, where v_1 and v_m denotes the initial and end vertex of the path and the remaining vertices in the triangle be $u_1, u_3, \dots, u_{m-1}$. Clearly, d(v_i) = 3, 2≤ i≤ m-1 and d(v_1) = d(v_m) = 2. Also d(u_1) = d(u_3) = … = d(u_{m-1}) = 2. Let v be any vertex in an alternate triangular snake graph. We consider the following cases.

Case 1: v is any vertex from $\{u_1, u_3, \dots, u_{m-1}\}$.

Without loss of generality, let v = u_i, $i = 1,3, \dots, m-1$. Then in G^v, d(u_i) = p-3. This vertex u_i covers all the vertices in G^v other than the two vertices which are adjacent to u_i in G and the two vertices are v_k and v_{k+1}.

Case1.1: v_k is an initial vertex.

Then d(v_k) =1 and d(v_{k+1}) = 2. Note that v_k and v_{k+1} are adjacent in G^v. Hence the two vertices u_i and v_k covers all the vertices of G^v and (d(u_i), d(v_k)) = (p-3, 1) = 1. Thus, $\{u_i, v_k\}$ is a relatively prime dominating set and hence $\gamma_{rpd}(G^v)$ = 2 in this case.

Case 1.2: v_{k+1} is a end vertex.

Proof same as Case 1.1.

Case 1.3: Neither v_k is an initial vertex nor v_{k+1} is a end vertex.

Then d(v_k) = d(v_{k+1}) = 2. To cover the vertex v_k, either we have to take v_k or a vertex which is adjacent to v_k. Let the vertex which is adjacent to v_k be v_{k-1} and d(v_{k-1}) = 3. Since p-3 is multiple of 3, we cannot take the vertex v_{k-1}. Hence the only possibility vertex is v_k. Note that, if n is odd, then p-3 is even and hence (p-3, 2) = 2. Therefore, relatively prime dominating set does not exist in this case. If n is even, then p-3 is odd and so (p-3, 2) = 1. Therefore, relatively prime dominating set is $\{u_i, v_k\}$ and $\gamma_{rpd}(G^v)$ = 2 in this case too.

Case 2: v is an initial vertex or an end vertex of a path.

Without loss of generality, let v = v_1. Then in G^v, d(v_1) = p-3. This vertex v_1 covers all the vertices in G^v except the two vertices which are adjacent to v_1 in G, i.e., v_1 is not adjacent to u_1 and v_2. Note that in G^v, d(u_1) =1 and d(v_2) = 2 and in G^v, the vertices u_1 and v_2 are adjacent. Hence, the two vertices v_1 and u_1 covers all the vertices of G^v and (d(v_1), d(u_1)) = (p-3, 1) = 1. Therefore, $\{v_1, u_1\}$ is the relatively prime domination set and hence $\gamma_{rpd}(G^v)$ = 2.

Case 3: v is any internal path vertex.

Without loss of generality, let v = v_i, where i ≠ 1, m. Then d(v_i) = p-4. The vertex v_i covers all the vertices in G^v other than the three vertices which are adjacent to v_i in G, say u_i, v_{i-1}, v_{i+1}. Then either

v_{i-1} and u_i are adjacent or v_{i+1} and u_i are adjacent. Without loss of generality, let us take v_{i-1} and u_i are adjacent. Since d(u_i)=1, we have left with only one vertex to cover, i.e., the vertex v_{i+1}. To cover the vertex v_{i+1}, either we have to choose this vertex or a vertex which is adjacent to v_{k+1}. In G^v, the vertex adjacent to v_{i+1} are v_{i+2} and u_{i+2}. Note that if p is odd, then d(v_i) is odd and if p is even, then d(v_i) is even. In G^v, d(v_{k+1}) = 2 and d(v_{i+1})= d(u_{i+2}) = 3. Since v_{i+1} is adjacent to v_i in G and v_{i+2} and u_i are not adjacent to v_i in G. Suppose that p is odd, then we take the vertex v_{i+1}, since (d(v_i), d(v_{i+1})) = (p-4, 2) = 1. Therefore, the relatively prime dominating set is $\{v_i, u_i, v_{k+1}\}$ and thus $\gamma_{rpd}(G^v)$ = 3. Otherwise, that is, if p is even, then we cannot choose the vertex v_{i+1}. The remaining possibilities are either v_{i+1} or u_{i+2}. But degree of both the vertices is three. Since d(v_i) = p-4 is even and |V| = 3n, it cannot be a multiple of 3. Hence the set $\{v_i, u_i, v_{k+2}\}$ is a relatively prime dominating set and thus relatively prime domination number is 3.

Observation 3.3. Let G be an alternate triangular snake graph with 6 vertices. Then γ_{rpd}(G) = 2.

Theorem 3.4. Let G be a double triangular snake graph with p vertices, where p = 3n+1, n $\geq$ 2. Then $\gamma_{rpd}(G^v)$ = 2, 3, 4 or 6.

Proof: Let G be a double triangular snake graph with p vertices. Let the vertices in the path be v_i, $1\leq i\leq m$ and the vertices in the upper triangle be u_i, $1\leq i\leq m-1$ and the vertices in the lower triangle be w_i, $1\leq i\leq m-1$ such that 3m -2 = p. Then d(v_1) = d(v_m) = 3 and d(v_i) = 6, $2\leq i\leq m-1$, d(u_i) = d(w_i) = 2, $1\leq i\leq m-1$. Let v be any vertex in the double triangular snake graph. The following cases arise.

Case 1: v is either u_i or w_i, $1\leq i\leq m-1$

Without loss of generality, let v = u_i, where i = 1, 2, …, m-1. In G^v, d(v) = p-3. This vertex v covers all the vertices in G^v, except the two vertices, say v_{i-1} and v_{i+1}. Note that v_{i-1} and v_{i+1} are adjacent in G^v. Hence to cover the vertices v_{i-1} and v_{i+1}, either we can take anyone of v_{i-1} and v_{i+1} or choose a vertex which is adjacent to both v_{i-1} and v_{i+1}. Since G is a double triangular snake graph, the vertex w_i is adjacent to v_{i-1} and v_{i+1} . Then d(w_i) = 3. Now, since $|V|$ = 3n+1, n ≥ 2, the degree of the vertex v, cannot be a multiple of 3 and so (p-3, 3) = 1. Hence the relatively prime dominating set is $\{v, w_i\}$ and thus $\gamma_{rpd}(G^v)$ = 2 in this case .

Case 2: v is an initial or an end vertex of a path.

Without loss of generality, Let v = v_1. Then d(v_1)= p-4 in G^v. This vertex v_1 covers all the vertices except three vertices, namely u_1, w_1 and v_2. Then d(u_1) = 1 = d(w_1) and d(v_2) = 5. Note that the vertex v_2 has adjaceny with the vertices u_1 and w_1. If d(v_1) is not multiple of 5, then we can take the set { v_1, v_2} as relatively prime dominating set and hence $\gamma_{rpd}(G^v)$ = 2 in this case; if d(v_1) is multiple of 5, then take the set { v_1, u_1 , w_1 } as relatively prime dominating set and hence $\gamma_{rpd}(G^v)$ = 3 in this case.

Case 3: v is any internal path vertex.

Then d(v_i) = 6 in G, d(v_i) = p-7 in G^v. Since $|V|$ = 3n+1, d(v_i) is a multiple of 3. Note that this vertex v_i is a midpoint for 4 triangle in G. This vertex v_i covers all the vertices of G^v, other than the six vertices, namely $v_{i-1}, v_{i+1}, u_{i-1}, u_{i+1}, w_{i-1}$, and w_{i+1}. Then d(u_{i-1}) = d(u_{i+1}) = d(w_{i-1}) = d(w_{i+1}) =1. And d(v_{i-1}) = 2 if v_{i-1} is an initial vertex; otherwise d(v_{i-1}) = 5. Similarly d(v_{i+1}) = 2 if v_{i+1} is an end vertex; otherwise d(v_{i+1}) = 5.

Case 3.1: v_{i-1} is an initial vertex.

Note that the vertex v_{i-1} is adjacent to both the vertices u_{i-1} and w_{i-1}. Similarly, the vertex v_{i+1} is adjacent to both the vertices u_{i+1} and w_{i+1}. If d(v) is even, then we cannot take the vertex v_{i-1}. Hence the set $\{v_i, u_{i-1}, w_{i-1}, v_{i+1}\}$ is a relatively prime dominating set and hence $\gamma_{rpd}(G^v)$ = 4 in this case. If d(v) is odd and a multiple of 5, then the set $\{v_i, v_{i-1}, u_{i+1}, w_{i+1}\}$ is a relatively prime dominating set and hence $\gamma_{rpd}(G^v)$ = 4 in this case also.

Case 3.2: v_{i+1} is an end vertex.

Similar to Case 3.1.

Case 3.3: Neither v_{i-1} is an initial vertex nor v_{i+1} is an end vertex.

Then d(v_{i-1}) = d(v_{i+1}) = 5. If d(v) is even and not a multiple of 5, then the set $\{v_i, v_{i-1}, u_{i+1}, w_{i+1}\}$ is a relatively prime dominating set and hence $\gamma_{rpd}(G^v)$ = 4 in this case. If d(v) is odd and a multiple of 5, then the set $\{v_i, v_{i-1}, u_{i-1}, w_{i-1}, u_{i+1}, w_{i+1}\}$ is a relatively prime dominating set and hence $\gamma_{rpd}(G^v)$ = 6 in this case.

Theorem 3.5. Let G be a double alternate triangular snake graph with p vertices, where p = 4n, n ≥ 2. Then $\gamma_{rpd}(G^v)$ = 2, 3 or 4.

Proof: Let G be a double alternate triangular snake graph with p vertices. Let the vertices in the path be v_i, 1≤ i≤ m where v_1 and v_m denote the initial vertex and end vertex respectively and the vertices in the upper triangle be u_i, 1≤ i≤ m-1 and the vertices in the lower triangle be w_i, 1≤ i≤ m-1. Then d(v_i) = 4, 2≤ i≤ m-1, d(u_i) = d(w_i) = 2, 1≤ i≤ m-1. Let v be any vertex in G. We have the following cases.

Case 1: v = u_i or w_i, 1≤ i≤ m-1

Without loss of generality, let v = $u_i, i = 1,2, \dots, m-1$. Then in G^v, d(v) = p-3. This vertex covers all the vertices of G except two vertices, namely v_i and v_{i+1}. in the path. Note that v_i and v_{i+1} are adjacent in G^v. Since |V| = 4n, d(v) = p–3 is always odd. We have the following subcases.

Case 1.1: v_i is an initial vertex.

Then d(v_i) = 2. Therefore (p–3, 2) = 1 and these two vertices covers all the vertices of G^v. Hence the vertices v and v_i satisfies the condition for being a relatively prime dominating set. Therefore $\gamma_{rpd}(G^v)$ = 2.

Case 1.2: v_{i+1} is an end vertex.

Same as Case 1.1.

Case 1.3: Neither v_i is an initial vertex nor v_{i+1} is an end vertex.

Then d(v_i) = d(v_{i+1}) = 3. If d(v) is not a multiple of 3, then the set {v,v_i} is a relatively prime dominating set and so $\gamma_{rpd}(G^v)$ = 2 in this case. Suppose that d(v) is a multiple of 3. Note that degree of each vertex in the path except the four vertices v_1,v_m,v_i,v_{i+1} is 5; d(v_i) = d(v_{i+1}) = 3; d(v_1) = d(v_m) = 4. Since d(v) is a multiple of 3, we cannot take the vertices of v_i and v_{i+1}. Also, we cannot take the vertex in the lower triangle which is adjacent to v_i and v_{i+1}, since degree of that vertex is 3. So, we consider the vertices of degree 5 which are adjacent to v_i and v_{i+1}. Since they are in the path, we have to take the two vertices, one from the left side of v_i and the other from the right side of v_{i+1}. Since both the vertex has degree 5, we cannot cover the two vertices v_i and v_{i+1}. Hence relatively prime dominating set does not exist in this case.

Case 2: v is an initial vertex or an end vertex of a path.

Without loss of generality, let v = v_1. Then d(v) = p–4. This vertex covers all the vertices of G^v, except three vertices, namely u_1, w_1 and v_2. Since G is a double alternate triangular snake graph, u_1 and v_2,

w_1 and v_2 are adjacent in G^v. Also d(u_1) = d(w_1) = 1 and d(v_2) = 3. If d(v) is not multiple of 3, then the set $\{v_1, v_2\}$ is a relatively prime dominating set and hence $\gamma_{rpd}(G^v)$ = 2. Otherwise, the set {v, u_1, w_1} satisfies all the conditions for being a relatively prime dominating set. Therefore, $\gamma_{rpd}(G^v)$ = 3 in this case.

Case 3: v is any internal path vertex of a path.

Without loss of generality, let v = v_i, $i = 2,3,\ldots,m-1$. Then d(v) = p–5. This vertex cover all the vertices of G^v, namely v_{i-1}, v_{i+1}, u_i and w_i. Then d(v_{i-1}) = 2 if v_{i-1} is an initial vertex, otherwise 3. Similarly, d(v_{i+1}) = 2 if v_{i+1} is an end vertex, otherwise 3. Also d(u_i) = d(w_i) = 1. Since |V| = 4n, d(v) = p–5 is always odd. We have the following subcases.

Case 3.1: v_{i-1} is an initial vertex.

Then d(v_{i-1}) = 2. If d(v) is not multiple of 3, then the set {v, v_{i-1}, v_{i+1}} is a relatively prime dominating set and hence $\gamma_{rpd}(G^v)$ = 3. Suppose d(v) is multiple of 3, then we cannot take the vertex v_{i+1}. To cover the vertex v_{i+1}, we have to consider a vertex which is adjacent to v_{i+1}. Since degree of each vertex in the set $\{u_1, u_2, \ldots u_{m-1}, w_1, w_2, \ldots, w_{m-1}\}$ other than u_i and w_i is three, we cannot take any vertex from this set. Consider the vertex in the path which is adjacent to v_{i+1}, say v_{i+2}. Note that d(v_{i+2}) = 5. Therefore, if d(v) is not multiple of 5, then the set {v, v_{i-1}, v_{i+2}} is a relatively prime dominating set and so $\gamma_{rpd}(G^v)$ = 3. If d(v) is multiple of 3 and 5, then relatively prime dominating set does not exist.

Case 3.2: v_{i+1} is an end vertex.

Same as Case 3.1.

Case 3.3: Neither v_{i-1} is an initial vertex nor v_{i+1} is an end vertex.

Then d(v_{i-1}) = d(v_{i+1}) = 3. Hence we cannot take these vertices together to obtain a relatively prime dominating set. Consider the vertex in the path which is adjacent to v_{i+1}, say v_{i+2}. If v_{i+2} is an end vertex, then d(v_{i+2}) = 4, otherwise 5. Therefore, if v_{i+2} is an end vertex and d(v) is not a multiple of 3, then $\{v_i, v_{i-1}, v_{i+2}\}$ is a relatively prime dominating set and hence $\gamma_{rpd}(G^v)$ = 3. Otherwise, the set $\{v_i, v_{i+2}, u_i. w_i\}$ is a relatively prime dominating set and thus $\gamma_{rpd}(G^v)$ = 4. Therefore, assume that the vertex adjacent to v_{i-1} is not an initial vertex and the vertex adjacent to v_{i+1} is an end vertex. If d(v) is not a multiple of 3 and 5, then the set $\{v_i, v_{i-1}, v_{i+2}\}$ is a relatively prime dominating set and hence $\gamma_{rpd}(G^v)$ = 3. If d(v) is a multiple of 3 not a multiple of 5, then the set $\{v_i, u_i, w_i, v_{i+2}\}$ is a relatively prime dominating set and hence $\gamma_{rpd}(G^v)$ = 4. If d(v) is a multiple of 3 and 5, then relatively prime dominating set does not exist.

4. Relatively Prime Domination Number on Complement of Triangular Snake Graphs

In this section we have shown that the relatively prime domination number for complement of snake graphs is 2.

Theorem 4.1. Let G be an alternate triangular snake graph. Then for p≠6, $\gamma_{rpd}(\bar{G})$ = 2.

Proof: Let G be an alternate triangular snake graph with p vertices, p > 6. Let the vertices in G be $v_1, v_2, \ldots, v_p$. In an alternate triangular snake graph, the degree of each vertex is either 2 or 3. Hence in the complement of alternate triangular snake graph $\bar{G}$, degree of each vertex is either p-3 or p-4. Consider a vertex which has degree p-3, say v_i in $\bar{G}$. Then it does not have any adjacency with two vertices say v_k and v_l. Now, we choose a vertex in $\bar{G}$ which has degree p-4 and have adjacency with the vertices v_k and v_l, say v_j. Such a vertex is always exist, since $|V| \geq 6$. Clearly, the two vertices v_i

and v_j covers all the vertices of $\bar{G}$, and (d(v_i), d(v_j)) = (p-3, p-4)=1. Hence, the set {v_i, v_j }satisfies all conditions for being the relatively prime dominating set. Thus $\gamma_{rpd}(\bar{G})$ = 2.

Theorem 4.2. Let G be a double triangular snake graph. Then for p$\neq$ *7*, $\gamma_{rpd}(\bar{G})$ = 2 .

Proof: Let G be a double triangular snake graph with p vertices. Let the vertices in G be v_1,v_2, … ,v_p. In G, degree of each vertex is either 2, 3 or 6 and thus degree of each vertex in $\bar{G}$ is p-3, p-4 or p-7. First we choose a vertex of degree p-3 , say v_i. This vertex covers all vertices of $\bar{G}$, except two vertices, say v_k and v_l. Next, we choose a vertex of degree p-4 such that it has adjacent with the two vertices v_k and v_l, say v_j. Such a vertex always exists, since $|V| \geq 6$. Clearly these two vertices v_i and v_j cover all vertices of $\bar{G}$ and (d(v_i), d(v_j)) = (p-3 , p-4) = 1. Hence {v_i,v_j} is a relatively prime dominating set . Thus, $\gamma_{rpd}(\bar{G})$ = 2.

Theorem 4.3. For any double alternate triangular snake graph G, γ_{rpd}**(**$\overline{\text{G}}$**) =** 2.

Proof: Let G be a double alternate triangular snake graph with p vertices. Let them be v_1,v_2, … v_p. In double alternate triangular snake graph G, the degree of each vertex is either 2,3 or 4 and hence in the complement of double alternate triangular snake graph $\bar{G}$, degree of each vertex is either p–3, p–4 or p–5. Choose a vertex of degree p–3, say v_i. This vertex cover all vertices of $\overline{\text{G}}$, except the two vertices, say v_k and v_l. Now, choose a vertex of degree p–4 such that it has adjacent with the vertices v_k and v_l. Such a vertex always exists, as |V| $\geq$ 6 and let it be v_j. Note that, these two vertices v_i and v_j cover all the vertices of $\bar{G}$ and (d(v_i),d(v_j)) = (p-3, p-4) = 1. Therefore, $\{v_i, v_j\}$ is a relatively prime dominating set. Hence $\gamma_{rpd}(\bar{G})$ = 2.

5. Conclusion

Dominations in graph theory is a wide area with more applications to real life which helps the researchers to get more ideas to manage the problems in real life situation. The standard purpose of the paper is to explain the significance of dominating sets and relatively prime domination number. We have examined the idea of relatively prime dominations in various types of triangular snake graphs and also their complements.

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