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E-Cordial Labeling of Some Families of Graphs

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Abstract:
An E-cordial labeling σ: E →{0,1} induces σ*: V →{0,1} on graph G=(V,E), where $(\sigma(v) = (\sum_{u\in V} \sigma(uv))$ mod 2 is taken over all edges $uv \in E$, and the labelling satisfies the conditions $|v\sigma\ (0) - v\sigma\ (1)| \leq 1$, $|e\sigma\ (0) - e\sigma\ (1)| \leq 1$. Where $v\sigma$ (k) represent the count of vertices within the graph G that bear the label k based on the labeling function σ (here k can be either 0 or 1). Similarly, $e\sigma*\ (k)$ denotes the quantity of edges in the graph G that link vertices labeled with the value k according to the labeling function σ^* (here k can be 0 or 1). A graph along with E-cordial labeling is called an E-cordial graph. We prove that the graphs such as Herschel graph H, Durer graph, Frucht graph, Tietze graph, hypohamiltonian graph, truncated tetrahedron graph, cubic graph with 12 vertices, Wagner graph, Moser spindle graph, Goldner-Harary graph and diamondgraph are E-cordial graphs.

Keywords: Cordial labeling, E-cordial labeling, E-cordial graphs.

1. Introduction

In this paper, our focus is solely on studying graphs that have a limited number of elements, are uncomplicated with no loops or multiple edges between the same vertices, and have no specified direction for their edges. Graph labeling is a fundamental concept in graph theory, involving the assignment of numerical values, typically integers, to the V and/or edges of a graph. This area of research holds significant importance and finds practical applications in communication networks and coding theory. In coding theory, graph labeling is instrumental in designing error-correcting codes to ensure reliable data transmission and storage. In communication networks, it plays a vital role in optimizing data routing and resource management. Understanding the key terms of vertex labeling and edge labeling is essential in exploring the intricacies of graph labeling. Overall, graph labeling offers valuable insights into complex network structures and properties, facilitating the development of efficient systems for various real-world applications.

The investigation and analysis of E - cordial labeling are undertaken in this chapter, building upon the foundational concepts of graceful, harmonious, and cordial labeling. Yil - maz and Cahit's work in

1997 laid the groundwork for this intriguing labeling scheme, shedding light on its properties and implications within graph theory.

Cordial labeling, along with its various extensions and adaptations, continues to be an exciting area of research, and E-cordial labeling presents a captivating avenue for exploring the relationships between different labeling schemes.

2. Preliminaries

Definition 2.1: A labeling refers to the process of assigning integers to the vertices(nodes), edges(links), or both of a graph G. This assignment of integers helps uniquely identify the elements of the graph and allows for various analysis and studies of its properties.

Definition 2.2: A binary vertex labeling function $\sigma: V \rightarrow \{0,1\}$assigns labels 0 or 1 to each vertex in a graph. Specifically, σ(w) represents label of vertex win the graph. The induced edge label $\sigma^*(e)$ is equal to absolute difference between labels of two vertices u and v that form the edge e = uv. The resulting value of $\sigma^*(e)$ will be either 0 or 1, depending on the specific labels of vertices u and v under σ.

Let $v_\sigma(k)$ represent the count of vertices within the graph G that bear the label k based on the labeling function σ (wherek can be either 0 or 1). Similarly, $e_{\sigma^*}(k)$ denotes the quantity of edges in the graph G that link vertices labeled with the value k according to the labeling function σ^* (where again, k can be 0 or 1).

β -valuation of a graph is defined by Rosa. A [7].

Definition 2.3: The β-valuation function σ maps each vertex in graph G to a unique label in {0, 1, 2, . . . , q}, and label of an edge isabsolute difference between labels of its endpoints. To satisfy the condition of non-repeated edge labels, function σ must be an injection. The β -valuation of a graph with q edges is a way of assigning unique labels to vertices such that the absolute differences of labels on edges are distinct within graph.

The concept of "graceful labeling" was originally introduced by mathematician Solomon W. Golomb in his seminal work [3]. Even to this day, the logic of graceful labelingremains widely popular and continues to find applications in various fields.

Definition 2.4: A mapping $\sigma: V \rightarrow \{0,1,2,\ldots,q\}$ is said to be graceful labeling for each edge e = uw in graph, it assigns $|\sigma(u) - \sigma(w)|$as a unique label to edge.

Definition 2.5: A harmonious labeling of a graph G with q edges is an injective function $\sigma: V \rightarrow \{0,1,2,\ldots q-1\}$, where V is the set of vertices of the graph. This labeling ensures that each edge $e = uw$ has a distinct label based on the followingrelation $(\sigma(u) + \sigma(w)) \ (mod\ q)$.

The notion of edge-graceful and cordial labeling of graphs was first proposed by Lo in 1985 [6] and later by I. Cahit in 1987 [1]. These labeling schemes have become significant in graph theory as they offer elegant methods for assigning labelsto the edges of a graph while preserving certain graceful and cordial properties.

Definition 2.6: A cordial labeling of a graph G is a binary vertex labelingin which each edge $e=uw$ is assigned a label based on the absolute difference between the labels of its endpoints, $|v_\sigma(u) - v_\sigma(w)|$. Additionally, for every vertex v in the graph, $|v_\sigma(0) - v_\sigma(1)| \leq 1$. A graph that admits a cordial labeling is called a *cordial graph*.

E-cordial labeling, first introduced in 1997 by Yilmaz and Cahit [9], is a graph labeling concept.

Definition 2.7: An E-cordial labeling σ:E→{0, 1} induces σ^*:V→{0,1} on a graph G, where (σ(v) = ($\sum_{u\in V} \sigma(uv)$) mod 2 is taken over all edges uv E. The graph G is called an E-cordial graph if it admits such an E-cordial labeling, and the labeling satisfies the conditions $|v_\sigma(0) - v_\sigma(1)| \le 1$, $|e_\sigma(0) - e_\sigma(1)| \le 1$.

3. Main Results

Theorem 3.1: The Herschel graph H is an E-cordial graph.

Proof. Let H be the Herschel graph. Let $\{v_1, v_2, ..., v_{11}\}$ be the vertex set with central vertex v_{10} and E(H)=$\{v_iv_{i+1}:1\le i\le 7\}\cup\{v_{10}v_{2i+1} : \mathbf{0 \le i \le 3}\}\cup$ $\{v_1v_8, v_9v_2, v_9v_4, v_9v_8, v_1v_4, v_{11}v_6, v_{11}v_8\}$ be the edge set of the Herschel graph H.

The total count of vertices is 11 and edges is 18. We define a function σ that maps edges from the set E(H) to the binaryset {0, 1}. In other words, for each edge in the graph H, the function σ assigns either the label 0 or the label 1.

$\sigma(v_1v_8) = 1$

$\sigma(v_5v_{10}) = 1$

$\sigma(v_7v_{10}) = 1$

$\sigma(v_4v_{11}) = 1$

$\sigma(v_9v_4) = 1$

$\sigma(v_1v_{10}) = 0$

$\sigma(v_2v_9) = 0$

$\sigma(v_3v_{10}) = 0$

$\sigma(v_6v_{11}) = 0$

$\sigma(v_8v_{11}) = 0$

$\sigma(v_iv_{i+1}) = 1$; if $i = 2, 5, 7, 8$ and

$\sigma(v_iv_{i+1}) = 0$; if $i = 1, 3, 4, 6$.

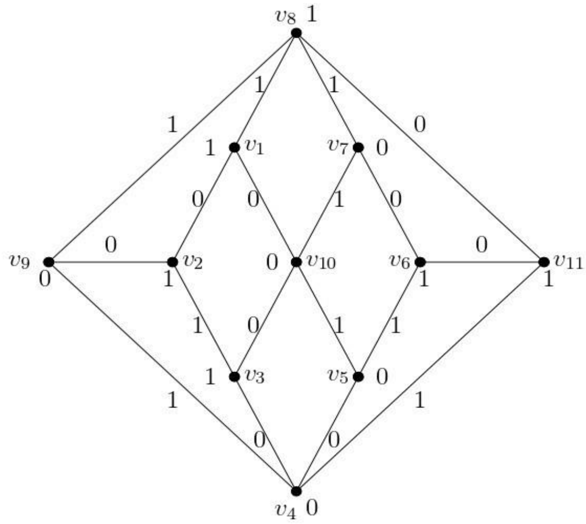

Figure 1: E-cordial labeling of the Herschel graph H

The labels associated with the nodes and links of the Herschel graph H are depicted in the provided Considering the labeling pattern σ defined above, the resultis as follows:

$v_\sigma(0) = 5$, $v_\sigma(1) = 6$ and $e_\sigma(0) = e_\sigma(1) = 9$.

Thus, we have $|v_\sigma(0)-v_\sigma(1)|= 1$ and $|e_\sigma(0)-e_\sigma(1)| = 0$.

Thus, the Herschel graph H can be characterized as an E-cordial graph.

Theorem 3.2: The Durer graph is an E-cordial graph.

Proof. Let G be the Durer graph. Let $V(G)=\{v_1, v_2, ..., v_{12}\}$ and $E(G)=\{v_iv_{i+1}:1\leq i\leq 11\}\cup\{v_1v_{12}, v_1v_9, v_2v_7, v_3v_5, v_4v_{12}, v_6v_{11}, v_8v_{10}\}$.

The total count of vertices is 12 and edges is 18. We definea function σ that maps edges from the set E to the binaryset 0, 1. In other words, for each edge in the graph G, the function σ assigns either the label 0 or the label 1.

$\sigma(v_1v_{12}) = 1$

$\sigma(v_4v_{12}) = 1$

$\sigma(v_6v_{11}) = 1$

$\sigma(v_8v_{10}) = 1$

$\sigma(v_1v_9) = 0$

$\sigma(v_2v_7) = 0$

$\sigma(v_3v_5) = 0$

$\sigma(v_iv_{i+1}) = 1$; if $i = 2, 4, 6, 8, 10$ and

$\sigma(v_iv_{i+1}) = 0$; if $i = 1, 3, 5, 7, 9, 11$.

The labels of the nodes and links of the Durer graph areshown following Figure 2.

Considering the labeling pattern σ defined above, the resultis as follows:

$v_\sigma(0) = 6, v_\sigma(1) = 6$ and $e_\sigma(0) = e_\sigma(1) = 9$.

Thus, we have $|v_\sigma(0)-v_\sigma(1)|= 0$ and $|e_\sigma(0)-e_\sigma(1)|= 0$.

Hence the Durer graph admits E-cordial labeling.

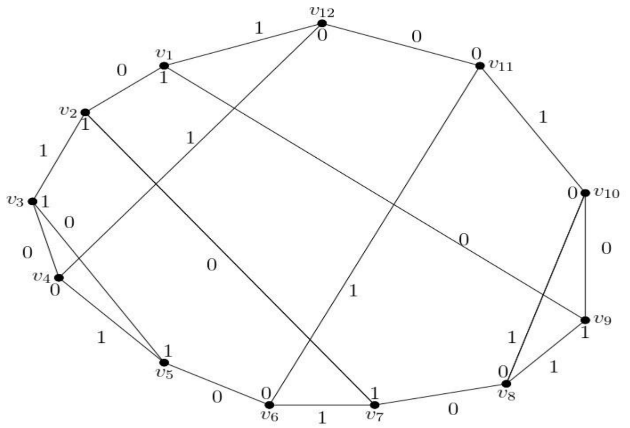

Figure 2: E-cordial labeling of the Durer graph

Theorem 3.3: The Frucht graph is an E-cordial graph.

Proof. Let G be the Frucht graph.

Let $V(G) = \{v_1, v_2, ..., v_{12}\}$ and $E(G) = \{v_iv_{i+1}:1 \leq i \leq 6\} \cup \{v_1v_7, v_1v_8, v_2v_8, v_8v_{11}, v_3v_9, v_4v_9, v_9v_{12}, v_5v_{10}, v_6v_{10}, v_{10}v_{12}, v_7v_{11}, v_{11}v_{12}\}$.

The total count of vertices is 12 and edges is 18. We definea function σ that maps edges from the set E to the binaryset 0, 1 . In other words, for each edge in the graph G, the function σ assigns either the label 0 or the label 1.

$\sigma(v_1v_8) = 1$

$\sigma(v_4v_9) = 1$

$\sigma(v_9v_{12}) = 1$

$\sigma(v_5v_{10}) = 1$

$\sigma(v_{10}v_{12}) = 1$

$\sigma(v_7v_{11}) = 1$

$\sigma(v_1v_7) = 0$

$\sigma(v_2v_8) = 0$

$\sigma(v_8v_{11}) = 0$

$\sigma(v_3v_9) = 0$

$\sigma(v_6v_{10}) = 0$

$\sigma(v_{11}v_{12}) = 0$

$\sigma(v_iv_{i+1}) = 1$; if i = 1, 3, 5 and

$\sigma(v_iv_{i+1}) = 0$; if i = 2, 4, 6.

The labels of the nodes and links of the Frucht graph are shown in following Figure 3.

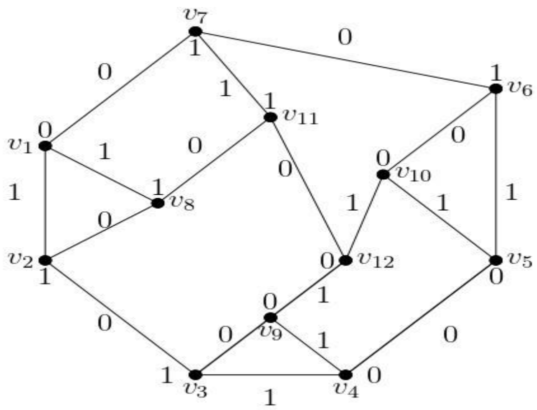

Figure 3: E-cordial labeling of the Frucht graph

Considering the labeling pattern σ defined above, the resultis as follows:

$v_\sigma(0) = v_\sigma(1) = 6$ and $e_\sigma(0) = e_\sigma(1) = 9$.

Thus, we have $|v_\sigma(0)-v_\sigma(1)|= 0$ and $|e_\sigma(0)-e_\sigma(1)|= 0$. Hence the Frucht graph G is an E-cordial graph.

Theorem 3.4: The Tietze graph is an E-cordial graph.

Proof. Let G be the Tietze graph.

Let $V(G) = \{v_1, v_2, ..., v_{12}\}$ and $E(G) = \{v_iv_{i+1} : 1 \leq i \leq 8\} \cup \{v_1v_9, v_{10}v_1, v_{10}v_{11}, v_{10}v_{12}, v_4v_{11}, v_{11}v_{12}, v_7v_{12}, v_2v_6, v_3v_8, v_5v_9\}$.

The total count of vertices is 12 and edges is 18. We definea function σ that maps edges from the set E to the binaryset 0, 1. In other words, for each edge in the graph G, thefunction σ assigns either the label 0 or the label 1.

$\sigma(v_{10}v_{11})=1$

$\sigma(v_{11}v_{12})=1$

$\sigma(v_7v_{12})=1$

$\sigma(v_2v_6)=1$

$\sigma(v_1v_9)=0$

$\sigma(v_{10}v_1)=0$

$\sigma(v_{10}v_{12})=0$

$\sigma(v_4v_{11})=0$

$\sigma(v_3v_8)=0$

$\sigma(v_5v_9)=0$

$\sigma(v_iv_{i+1})=1$; if i = 1, 3, 4, 6, 8 and

$\sigma(v_iv_{i+1})=0$; if i = 2, 5, 7.

The labels of nodes and links of the Tietze graph are shownin following Figure 4.

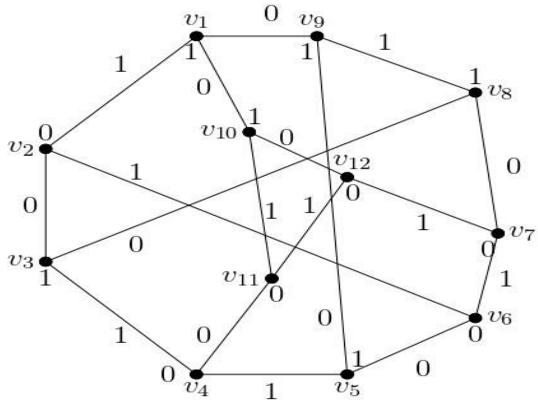

Figure 4: E-cordial labeling of the Tietze graph

Considering the labeling pattern σ defined above, the resultis as follows:

$v_\sigma(0) = v_\sigma(1) = 6$ and $e_\sigma(0) = e_\sigma(1) = 9$.

Thus, we have $|v_\sigma(0)-v_\sigma(1)|= 0$ and $|e_\sigma(0)-e_\sigma(1)|= 0$. Hence the Tietze graph G admits E-cordial labeling.

Theorem 3.5: The hypohamiltonian graph is an E-cordial graph.

Proof. Let G be the hypohamiltonian graph. Let $\{ v_1, v_2, ..., v_{16}\}$ be the vertex set with central vertex v_{16}

and $E(G) = \{ v_iv_{i+1}:1\leq i\leq 14\}U\{v_{16}v_{3i+1}:0\leq i\leq 4\}U\{v_{3i+2}v_{3i+6}:0\leq i\leq 3\}U\{v_1v_{15}, v_{14}v_3\}$ be the edgeset of the hypohamiltonian graph G.

The total count of vertices is 16 and edges is 25. We definea function σ that maps edges from the set E to the binaryset 0, 1. In other words, for each edge in the graph G, thefunction σ assigns either the label 0 or the label 1.

$\sigma(v_1v_{15})=1$

$\sigma(v_3v_{14})=1$

$\sigma(v_{11}v_{15})=1$

$\sigma(v_8v_{12})=1$

$\sigma(v_5v_9)=1$

$\sigma(v_1v_{16})=1$

$\sigma(v_2v_6)=0$

$\sigma(v_4v_{16})=0$

$\sigma(v_7v_{16})=0$

$\sigma(v_{10}v_{16})$=0

$\sigma(v_{13}v_{16})$=0

$\sigma(v_iv_{i+1})$=1 ; if i = 1, 3, 5, 7, 9, 11, 13 and

$\sigma(v_iv_{i+1})$=0 ; if i = 2, 4, 6, 8, 10, 12, 14.

The labels of nodes and links of the hypohamiltonian graphare shown in following Figure 5.

Considering the labeling pattern σ defined above, the resultis as follows:

$v_\sigma(0) = v_\sigma(1) = 8$ and $e_\sigma(0) = 12$, $e_\sigma(1) = 13$.

Thus, we have $|v_\sigma(0)\text{-}v_\sigma(1)|= 0$ and $|e_\sigma(0)\text{-}e_\sigma(1)|= 1$.

Hence the hypohamiltonian graph G is an E-cordial graph.

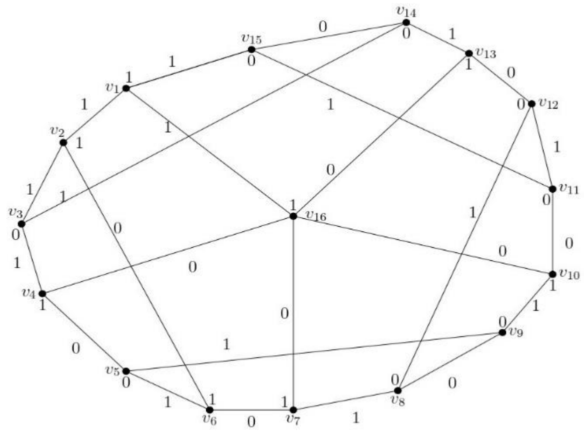

Figure 5: E-cordial labeling of the hypohamiltonian graph

Theorem 3.6: The truncated tetrahedron graph is an E-cordial graph.

Proof. Let G be the truncated tetrahedron graph. Let V(G)= $\{v_1, v_2, ..., v_{12}\}$ and

E(G) = $\{v_iv_{i+1}:1\leq i \leq 7\} \cup \{v_1v_8\ v_9v_1, v_9v_8\ v_9v_{11}, v_{10}v_2, v_{10}v_3, v_{10}v_{12}, v_{11}v_4, v_{11}v_5, v12v6, v12v7\}$.

The total count of vertices is 12 and edges is 18.

We define a function σ that maps edges from the set E to the binary set 0, 1. In other words, for each edge in the graphG, the function σ assigns either the label 0 or the label 1.

$\sigma(v_1v_8)$=1

$\sigma(v_8v_9)$=1

$\sigma(v_7v_{12})$=1

$\sigma(v_4v_{11})$=1

$\sigma(v_2v_{10})$=1

$\sigma(v_{10}v_{12})$=1

$\sigma(v_1v_9)$=0

$\sigma(v_9v_{11})$=0

$\sigma(v_3v_{10})$=0

$\sigma(v_5v_{11})$=0

$\sigma(v_6v_{12})$=0

$\sigma(v_iv_{i+1})$=1 ; if i = 2, 4, 6 and

$\sigma(v_iv_{i+1})$=0 ; if i = 1, 3, 5, 7.

The labels of nodes and links of the truncated tetrahedron graph are shown in following Figure 6.

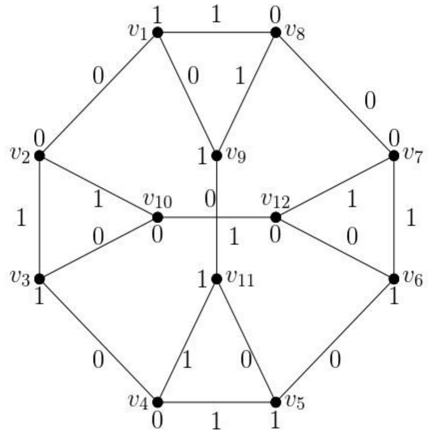

Figure 6: E-cordial labeling of the truncated tetrahedron graph

Considering the labeling pattern σ defined above, the result is as follows:

$v_\sigma(0) = v_\sigma(1) = 6$ and $e_\sigma(0) = e_\sigma(1) = 9$.

Thus, we have $|v_\sigma(0)-v_\sigma(1)|= 0$ and $|e_\sigma(0)-e_\sigma(1)|= 0$. Hence the truncated tetrahedron graph G admits E-cordial labeling.

Theorem 3.7: A cubic graph with 12 vertices is an E-cordial graph.

Proof. Let G be the cubic graph. Let V(G) = $\{v_1, v_2, ..., v_{12}\}$ and

E(G)=$\{v_iv_{i+1}: 1\leq i\leq 5\}U\{v_iv_{i+1}: 7\leq i\leq 11\}U\{v_1v_6, v_7v_{12}, v_1v_7, v_2v_8, v_3v_5, v_4v_{10}, v_5v_9, v_6v_{11}\}$.

The total count of vertices is 12 and edges is 18. We definea function σ that maps edges from the set E to the binaryset 0, 1 . In other words, for each edge in the graph G, the function σ assigns either the label 0 or the label 1.

$\sigma(v_1v_6)$ = 1

$\sigma(v_{12}v_7)$ = 1

$\sigma(v_1v_7)$ = 0

$\sigma(v_2v_8)$ = 0

$\sigma(v_3v_9)$ = 0

$\sigma(v_4v_{10})$ = 0

$\sigma(v_5v_{11})$ = 0

$\sigma(v_6v_{12}) = 0$

$\sigma(v_iv_{i+1}) = 1$; if $i = 2, 4, 7, 8, 9, 10, 11$ and

$\sigma(v_iv_{i+1}) = 0$; if $i = 1, 3, 5$.

The labels of nodes and links of the cubic graph are shownin following Figure 7.

Considering the labeling pattern σ defined above, the resultis as follows:

$v_\sigma(0) = v_\sigma(1) = 6$ and $e_\sigma(0) = e_\sigma(1) = 9$.

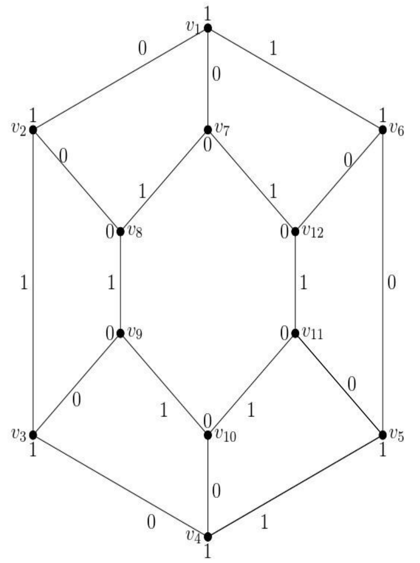

Figure 7: E-cordial labeling of the cubic graph with 12 vertices

Thus, we have $|v_\sigma(0)-v_\sigma(1)|= 0$ and $|e_\sigma(0)-e_\sigma(1)|= 0$. Hence the cubic graph with 12 vertices is an E-cordial graph.

Theorem 3.8: A Wagner graph with is an E-cordial graph.

Proof. Let G be the Wagner graph. Let V(G) $=\{ v_1, v_2, ..., v_8\}$ and
E(G)$=\{v_iv_{i+1}:1\leq i\leq 7\}\cup\{v_1v_8, v_1v_5, v_2v_6, v_3v_7, v_4v_8\}$.

The total count of vertices is 8 and edges is 12. We define afunction σ that maps edges from the set E to the binary setWe define a function σ that maps edges from the set E to thebinary set 0, 1. In other words, for each edge in the graphG, the function σ assigns either the label 0 or the label 1.

$\sigma(v_2v_6) = 1$

$\sigma(v_4v_8) = 1$

$\sigma(v_1v_8) = 0$

$\sigma(v_1v_5) = 0$

$\sigma(v_3v_7) = 0$

$\sigma(v_i v_{i+1}) = 1$; if $i = 1, 3, 5, 7$ and

$\sigma(v_i v_{i+1}) = 0$; if $i = 2, 4, 6$.

From the labeling pattern defined above, we obtain

$v_\sigma(0) = v_\sigma(1) = 4$ and $e_\sigma(0) = e_\sigma(1) = 6$.

Thus, we have $|v_\sigma(0) - v_\sigma(1)| = 0$ and $|e_\sigma(0) - e_\sigma(1)| = 0$.

The labels of nodes and links of the Wagner graph are shownin following Figure 8.

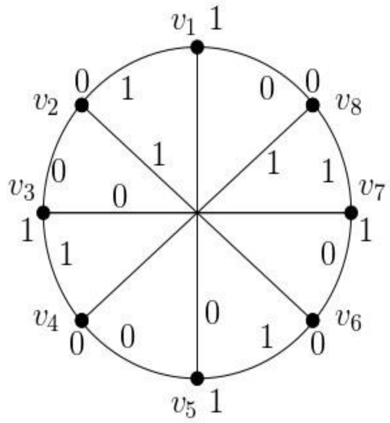

Figure 8: E-cordial labeling of the Wagner graph

Hence the Wagner graph admits E-cordial labeling.

Theorem 3.9: A Moser spindle graph is an E-cordial graph.

Proof. Let G be the Moser spindle graph. Let V(G) $=\{v_1, v_2, \ldots, v_7\}$ and E(G)$=\{v_i v_{i+1} : 1 \le i \le 4\} \cup \{v_1v_5, v_1v_6, v_2v_6, v_5v_6, v_3v_7, v_4v_7, v_5v_7\}$.

The total count of vertices is 7 and edges is 11. We definea function σ that maps edges from the set E to the binaryset 0, 1 . In other words, for each edge in the graph G, the function σ assigns either the label 0 or the label 1.

$\sigma(v_1v_5) = 1$

$\sigma(v_1v_6) = 1$

$\sigma(v_3v_7) = 1$

$\sigma(v_4v_7) = 1$

$\sigma(v_2v_6) = 0$

$\sigma(v_5v_6) = 0$

$\sigma(v_5v_7) = 0$

$\sigma(v_i v_{i+1}) = 1$; if $i = 1, 3$ and

$\sigma(v_i v_{i+1}) = 0$; if $i = 2, 4$.

The labels of nodes and links of the hypohamiltonian graphare shown in following Figure 9.

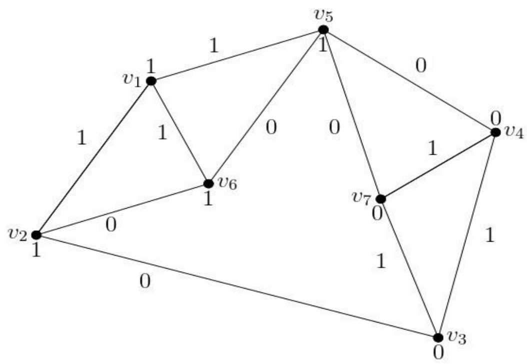

Figure 9: E-cordial labeling of the Moser spindle graph

Considering the labeling pattern σ defined above, the resultis as follows:

$v_\sigma(0) = 3$, $v_\sigma(1) = 4$ and $e_\sigma(0) = 5$, $e_\sigma(1) = 6$.

Thus, we have $|v_\sigma(0)-v_\sigma(1)|= 1$ and $|e_\sigma(0)-e_\sigma(1)|= 1$.

Hence the Moser spindle graph is an E-cordial graph.

Theorem 3.10: A Goldner-Harary graph is an E-cordial graph.

Proof. Let G be the Golden-Harary graph. Let $\{v_1, v_1, v_2, ..., v_{11}\}$ be the vertex set with central vertex v_{11} and $E(G)=\{v_iv_{i+1}:1\leq i\leq 2\} \cup \{v_1v_3, v_1v_4, v_2v_5, v_4v_8, v_1v_7, v_1v_8, v_7v_8, v_1v_5, v_1v_6, v_5v_6, v_3v_5, v_5v_{10}, v_3v_{10}, v_3v_9, v_3v_8, v_9v_{10}, v_1v_{11}, v_3v_{11}, v_6v_{11}, v_7v_{11}, v_9v_{11}, v_{10}v_{11}, v_5v_{11}, v_8v_{11}\}$ be the edge set of the Goldner- Harary graph G.

The total count of vertices is 11 and edges is 25. We definea function σ that maps edges from the set E to the binaryset {0, 1}. In other words, for each edge in the graph G, the function σ assigns either the label 0 or the label 1.

$\sigma(v_1v_3) = 1$

$\sigma(v_1v_5) = 1$

$\sigma(v_1v_6) = 1$

$\sigma(v_1v_7) = 1$

$\sigma(v_1v_{11}) = 1$

$\sigma(v_1v_4) = 1$

$\sigma(v_2v_5) = 1$

$\sigma(v_3v_8) = 1$

$\sigma(v_5v_{10}) = 1$

$\sigma(v_9v_{11}) = 1$

$\sigma(v_1v_8) = 0$

$\sigma(v_3v_5) = 0$

$\sigma(v_3v_9) = 0$

$\sigma(v_3v_{10}) = 0$

$\sigma(v_3v_{11}) = 0$

$\sigma(v_4v_8) = 0$

$\sigma(v_6v_{11}) = 0$

$\sigma(v_7v_{11}) = 0$

$\sigma(v_5v_{11}) = 0$

$\sigma(v_8v_{11}) = 0$

$\sigma(v_iv_{i+1}) = 1$; if i = 1, 8, 10 and

$\sigma(v_iv_{i+1}) = 0$; if i = 2, 3, 5, 7.

The labels of nodes and links of the Goldner-Harary graphare shown in following Figure 10.

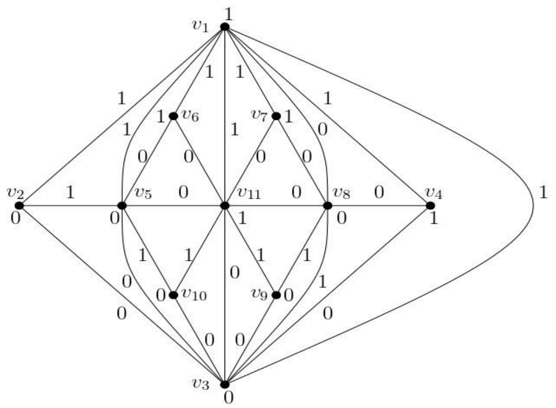

Figure 10: E-cordial labeling of the Goldner-Harary graph

Considering the labeling pattern σ defined above, the result is as follows:

$v_\sigma(0) = 6$, $v_\sigma(1) = 5$ and $e_\sigma(0) = 12$, $e_\sigma(1) = 13$.

Thus, we have $|v_\sigma(0)\ v_\sigma(1)|=1$ and $|e_\sigma(0)-e_\sigma(1)|=1$.

Hence the Goldner-Harary graph admits E-cordial labeling.

Theorem 3.11: A diamond graph is an E-cordial graph.

Proof. Let G be the diamond graph. Let $V(G)=\{v_1, v_2, v_3, v_4\}$ and $E(G)=\{v_iv_{i+1}:1\le i\le 3\}\cup\{v_1v_4, v_2v_4\}$.

The total count of vertices is 4 and edges is 5. We definea function σ that maps edges from the set E to the binaryset 0, 1. In other words, for each edge in the graph G, the function σ assigns either the label 0 or the label 1.

$\sigma(v_1v_4) = 0$

$\sigma(v_2v_4) = 0$

$\sigma(v_iv_{i+1}) = 1$; if i = 1, 2 and

$\sigma(v_iv_{i+1}) = 0$; if i = 3.

The labels of nodes and links of the edges of the diamond graph are shown in following Figure 11.

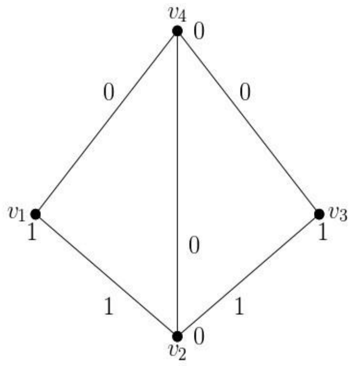

Figure 11: E-cordial labeling of the diamond graph

Considering the labeling pattern σ defined above, the result is as follows:

$v_\sigma(0) = v_\sigma(1) = 2$ and $e_\sigma(0) = 3$, $e_\sigma(1) = 2$.

Thus, we have $|v_\sigma(0)-v_\sigma(1)|=0$ and $|e_\sigma(0)-e_\sigma(1)|=1$.

Hence the diamond graph admits E-cordial labeling.

Considering the labeling pattern σ defined above, the result is as follows:

$v_\sigma(0) = v_\sigma(1) = 2$ and $e_\sigma(0) = 3$, $e_\sigma(1) = 2$.

Thus, we have $|v_\sigma(0)-v_\sigma(1)|=0$ and $|e_\sigma(0)-e_\sigma(1)|=1$.

Hence the diamond graph admits E-cordial labeling.

4. Conclusion

We proved the paper; different types of graphs are admitting E – Cordial labeling.

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