Advances in Nonlinear Variational Inequalities
ISSN: 1092-910X
Vol 27 No. 4 (2024)

Resolving Decision Making Issues with Nano Soft Logical Operators

R. Abinprakash [1], E. Lathanayagam [2], A. Saranya [3], R. Nishanthi [4]

[1] Associate Professor, [2]Assistant Professor, Department of Science and Humanities, Akshaya college of engineering and technology, Kinathukadavu, Coimbatore, Tamilnadu, India.

[3,4] Assistant Professor, Department of Mathematics, Dr. Mahalingam College of Engineering and Technology, Pollachi, Coimbatore, Tamilnadu, India.

[1] E-mail: abinprakash6343@gmail.com, [2] E-mail: shrilishanth@gmail.com,
[3] E-mail: saranyasaraswathy@gmail.com, [4] E-mail: raghunisha94@gmail.com

Article History:

Received: *29-03-2024*

Revised: *27-05-2024*

Accepted: *14-06-2024*

Abstract:

In this paper we have introduced the nano soft operators, such as $\widetilde{AND}$, $\widetilde{OR}$, $\widetilde{NOT}$and Max-Min product operators. We have solved a lot of real-world issues with the help of these operators. These operators supports correct decision-leading in various decision-making processes.

2020 Mathematics Subject classification: 54A05

Keywords: $\widetilde{AND}$- operator, $\widetilde{OR}$- operator, $\widetilde{NOT}$ - operator, Nano soft Max-Min operators.

1. Introduction

Soft sets, a concept introduced by Molodtsov [4] in 1999, extend traditional set theory by incorporating uncertainty and elaborately explained the definition of soft sets, soft membership function from P(U) to [0,1] in [4]. Soft decision-making problems arise when traditional binary (yes/no) decisions are insufficient due to uncertain or incomplete information. Here, soft sets provide a flexible approach to handling such uncertainties by assigning degrees of acceptance or rejection to each potential decision or outcome. Maji et. al. [5] extended his thoughts into the framework of decision-making and problem-solving. This allows decision-makers to account for varying levels of confidence or ambiguity in their decision processes, leading to more robust and adaptable solutions.

In 2013, Lellis thivagar et al.,[2] introduced the Nano topology, which is strong form of topology. Because it contains only atmost five elements. This can define by using Nano approximation spaces. Later Lellis Thivagar et al.,[3] used these approximation spaces in soft sets and he named as nano soft approximation spaces. Lellis thivagar et al., defined Nano soft topology by using Nano soft approximation spaces. He given a definition of Nano soft approximation is,

A non empty finite set of objects $\widetilde{U}$ is called an universe, $\tilde{\rho}_A$ be a $\widetilde{SS}$ over $\widetilde{U}$ and $\widetilde{\rho_s}$ be an equivalence relation on $\widetilde{\rho_A}$. Elements belonging to the equivalence class $\tilde{\rho}$(a) are said to be Soft indiscernible with one another and it is denoted by [$\tilde{\rho}$(a)]. The pair ($\widetilde{U}, \tilde{\rho}_A$) is called soft approximation space. Let $\tilde{\iota}_B \subseteq \tilde{\rho}_A$.

- If $L_{\widetilde{\rho_s}}(\widetilde{\iota_B}) = \bigcup_{a\in A}\{\tilde{\rho}(a): \tilde{\rho}(a) \subseteq \widetilde{\iota_B}\}$ is a soft lower approximation of $\tilde{\rho}_A$ with respect to $\widetilde{\iota_B}$
- If $U_{\widetilde{\rho_s}}(\widetilde{\iota_B}) = \bigcup_{a\in A}\{\tilde{\rho}(a): \tilde{\rho}(a) \cap \widetilde{\iota_B} \neq \emptyset\}$ is a soft upper approximation of $\tilde{\rho}_A$ with respect to $\widetilde{\iota_B}$
- If $B_{\widetilde{\rho_s}}(\widetilde{\iota_B}) = U_{\widetilde{\rho_s}}(\widetilde{\iota_B}) - L_{\widetilde{\rho_s}}(\widetilde{\iota_B})$ is a boundary region of $\tilde{\rho}_A$ with respect to $\widetilde{\iota_B}$.

In [1], Abinprakash has defined Nano soft Matrices by using Nano soft characterstic function and he consider oscillatory decisions while defining Nano soft characteristic function. Using these concept of Nano soft Matrices, every real-life problem can able to frame matrices and it can easily solvable or obtained solutions. Extension of the article [1], we have defined some logical operators like, $\widetilde{AND}$, $\widetilde{OR}$, $\widetilde{NOT}$ and Max-Min product operator. Using these nano soft logical operators, we can arrive better solutions of a given problems. This article aid in the process of making precise conclusions regarding the challenges. In addition, we have provided remedies for actual issues.

2. Applications of Nano Soft matrices

Example 2.1. A farmer, Mr. X has planned to plant a catch crop across the coconut grove.

Algorithm 1 in [1], helps to select the exact catch crop to Mr. X. Let $\widetilde{U}$ $=\{\widetilde{\iota_1}, \widetilde{\iota_2}, \widetilde{\iota_3}, \widetilde{\iota_4}, \widetilde{\iota_5}\}$ be the catch crops Areca, Papaya, Mango, Lemon and chocho respectively.

The parameter set $\tilde{E}$ = { ε_1= Not affecting Coconut trees yield, ε_2 = Ocupying less space,

ε_3= Exporting commodity and long-term yield, ε_4= Easy marketting facility, ε_5= High water

consumable}. Suppose that Mr. X chooses a parameter set A = $\{\varepsilon_1, \varepsilon_3, \varepsilon_4\}$.

Let $(\widetilde{\rho_E}, \tilde{E})$ be a Nano Soft Set ($\widetilde{NS}$− set) over $\widetilde{U}$ and $(\widetilde{\rho_A}, \tilde{E})$ be a $\widetilde{NS}$ -subsets of $(\widetilde{\rho_E}, \tilde{E})$.

$(\widetilde{\rho_{\tilde{E}}}, \tilde{E})$= $\{\tilde{\rho}(\varepsilon_1), \tilde{\rho}(\varepsilon_2), \tilde{\rho}(\varepsilon_3), \tilde{\rho}(\varepsilon_4), \tilde{\rho}(\varepsilon_5)\}$, where

$\tilde{\rho}(\varepsilon_1)$ = $\{\widetilde{\iota_1}, \widetilde{\iota_2}, \widetilde{\iota_3}, \widetilde{\iota_4}, \widetilde{\iota_5}\}$

$\tilde{\rho}(\varepsilon_2)$= $\{\widetilde{\iota_1}, \widetilde{\iota_2}, \widetilde{\iota_5}\}$

$\tilde{\rho}(\varepsilon_3)$ = $\{\widetilde{\iota_1}, \widetilde{\iota_3},\}$

$\tilde{\rho}(\varepsilon_4)$ = $\{\widetilde{\iota_2}, \widetilde{\iota_3}, \widetilde{\iota_4}\}$

$\tilde{\rho}(\varepsilon_5)$ = $\{\widetilde{\iota_1}, \widetilde{\iota_2}, \widetilde{\iota_4}, \widetilde{\iota_5}\}$

$(\widetilde{\rho_A}, \tilde{E})$= $\{\tilde{\rho}(\varepsilon_1), \tilde{\rho}(\varepsilon_3), \tilde{\rho}(\varepsilon_4)\}$

Let $\tilde{\rho}_s$ be the Nano Soft equivalence relation on $\tilde{\rho}_{\tilde{E}}$,

$\tilde{\rho}_s$=$\{\tilde{\rho}(\varepsilon_1) \times \tilde{\rho}(\varepsilon_1), \tilde{\rho}(\varepsilon_2) \times \tilde{\rho}(\varepsilon_2), \tilde{\rho}(\varepsilon_3) \times \tilde{\rho}(\varepsilon_3), \tilde{\rho}(\varepsilon_4) \times \tilde{\rho}(\varepsilon_4), \tilde{\rho}(\varepsilon_5) \times \tilde{\rho}(\varepsilon_5), \tilde{\rho}(\varepsilon_2) \times \tilde{\rho}(\varepsilon_4), \tilde{\rho}(\varepsilon_4) \times \tilde{\rho}(\varepsilon_2)\}$

Then the Nano Soft equivalence class es ($\widetilde{NS}$- equivalence classes) of $\tilde{\rho}_s$ are,

$[\tilde{\rho}(\varepsilon_1)]$= $\{\tilde{\rho}(\varepsilon_1)\}$

$[\tilde{\rho}(\varepsilon_2)]$= $\{\tilde{\rho}(\varepsilon_2), \tilde{\rho}(\varepsilon_4)\}$

$[\tilde{\rho}(\varepsilon_3)]$= $\{\tilde{\rho}(\varepsilon_3)\}$

$[\tilde{\rho}(\varepsilon_4)]$= $\{\tilde{\rho}(\varepsilon_2), \tilde{\rho}(\varepsilon_4)\}$

$[\tilde{\rho}(\varepsilon_5)]= \{\tilde{\rho}(\varepsilon_5)\}$

Now, $\widetilde{\rho_{\tilde{E}}} / \tilde{\rho}_s = \{\tilde{\rho}(\varepsilon_1), \{\tilde{\rho}(\varepsilon_2), \tilde{\rho}(\varepsilon_4)\}, \tilde{\rho}(\varepsilon_3), \tilde{\rho}(\varepsilon_5)\}$

If $L_{\widetilde{\rho_s}}(\widetilde{\rho_A}, \tilde{E})= L_{\widetilde{\rho_s}}(\widetilde{\rho_A})=\{ \tilde{\rho}(\varepsilon_1), \tilde{\rho}(\varepsilon_3)\}$

$U_{\widetilde{\rho_s}}(\widetilde{\rho_A}, \tilde{E})= U_{\widetilde{\rho_s}}(\widetilde{\rho_A})=\{ \tilde{\rho}(\varepsilon_1),\tilde{\rho}(\varepsilon_2), \tilde{\rho}(\varepsilon_3), \tilde{\rho}(\varepsilon_4)\}$

$B_{\widetilde{\rho_s}}(\widetilde{\rho_A}, \tilde{E})= B_{\widetilde{\rho_s}}(\widetilde{\rho_A})=\{ \tilde{\rho}(\varepsilon_2), \tilde{\rho}(\varepsilon_4)\}$

The tabular representation of a Nano Soft Matrix ($\widetilde{NSM}$) as follows

Ψ_N	ε_1	ε_2	ε_3	ε_4	ε_5
$\tilde{\iota}_1$	1	0.5	1	0	0
$\tilde{\iota}_2$	1	0.5	0	0.5	0
$\tilde{\iota}_3$	1	0	1	0.5	0
$\tilde{\iota}_4$	1	0	0	0.5	0
$\tilde{\iota}_5$	1	0.5	0	0	0

Table -1

The sets $\{\varepsilon_1, \varepsilon_2, \varepsilon_3, \varepsilon_4\}$,$\{\varepsilon_2, \varepsilon_3, \varepsilon_4, \varepsilon_5\}$ and $\{\varepsilon_2, \varepsilon_3, \varepsilon_4\}$ are the reduct of $(\widetilde{\rho_A}, \tilde{E})$. Now let us choose the reduct $\widetilde{NS}$ −set $(\widetilde{\rho_{\tilde{\rho}_s}}, \tilde{E})= \{\tilde{\rho}(\varepsilon_2), \tilde{\rho}(\varepsilon_3), \tilde{\rho}(\varepsilon_4)\}$ of of $(\widetilde{\rho_A}, \tilde{E})$. Then Table – 1 becomes,

Ψ_N	ε_2	ε_3	ε_4
$\tilde{\iota}_1$	0.5	1	0
$\tilde{\iota}_2$	0.5	0	0.5
$\tilde{\iota}_3$	0	1	0.5
$\tilde{\iota}_4$	0	0	0.5
$\tilde{\iota}_5$	0.5	0	0

Table -2

The choice value of Table -2 is

Ψ_N	ε_2	ε_3	ε_4	Choice value
$\tilde{\iota}_1$	0.5	1	0	1.5
$\tilde{\iota}_2$	0.5	0	0.5	1
$\tilde{\iota}_3$	0	1	0.5	1.5
$\tilde{\iota}_4$	0	0	0.5	0.5
$\tilde{\iota}_5$	0.5	0	0	0.5

Table-3

From Table-3, The $\widetilde{OC_K}$= {max$\{\xi_i\} = \tilde{\iota}_1$ or $\tilde{\iota}_3$. From this we conclude that the best catch crop for Mr. X is Areca or Mango ($\tilde{\iota}_1$ or $\tilde{\iota}_3$).

Definition 2.1. The entries v_{ij} in the weighted table of $\widetilde{NS} - set$, $v_{ij} = u_{ij} \times w_j$, Where w_j be the weightage given by user.

Definition 2.2. Let $\widetilde{U} = \{u_1, u_2, u_3, \dots \dots \dots, u_n\}$ be the set of objects and E= $\{\varepsilon_1, \varepsilon_2, \dots \dots, \varepsilon_n\}$ be a set of attributes of $\widetilde{U}$. The weighted choice value of u_i is ξ_i,

$\xi_i = \sum_j v_{ij}$.

Example 2.2. Now let us solve the Example 3.1 using the following Algorithm.

Suppose that Mr. X has given am weight for the parameter of reduct $\widetilde{NS} - set$, $\widetilde{\rho_\rho}$ is

$\varepsilon_2 \rightrightarrows w_2$=0.5, $\varepsilon_3 \rightrightarrows w_3$=0.6 and $\varepsilon_4 \rightrightarrows w_4$=0.7.

Algorithm – 1:

Step 1: Choose parameter subsets A of $\tilde{E}$.

Step 2: Construct nano soft subsets $\widetilde{\rho_A}$ for a given parameter set A.

Step 3: Find reduct Nano soft subset, Say $\widetilde{\rho_r}$ of $\widetilde{\rho_A}$.

Step 4: Use Definition 2.1 for finding weighted table of Nano soft set.

Step 5: Find weighted choice value by using Definition 2.2.

Step 6: Find the weighted choice value $\widetilde{OC_k}$= *max* $\{\xi_i\}$.

Then u_i is the optimum selection of object of $\widetilde{U}$.

Now Table-3 becomes,

Ψ_N	$\varepsilon_2 \,\tilde{*}\, w_2 = 0.5$	$\varepsilon_3 \,\tilde{*}\, w_3$= 0.6	$\varepsilon_4 \,\tilde{*}\, w_4 = 0.7$	Weighted Choice value
$\tilde{\iota}_1$	0.25	0.6	0	0.85
$\tilde{\iota}_2$	0.25	0	0.35	0.6
$\tilde{\iota}_3$	0	0.6	0.35	0.95
$\tilde{\iota}_4$	0	0	0.35	0.35
$\tilde{\iota}_5$	0.25	0	0	0.25

Table- 4

From Table- 4, The best catch crop for Mr. X is Mango ($\tilde{\iota}_3$)

Result 3.1. From Example 2.1 and Example 2.2, we conclude that, if the chooser gives some reasonable weightage to the parameters, then it will help to choose appropriate decision to them.

3. Logical Operators on Nano Soft matrices

Definition 3.1. Let $[\upsilon_{ij}] \in [\widetilde{NSM}]_{m\times n}$and $[\zeta_{ij}] \in [\widetilde{NSM}]_{m\times n}$ then $\widetilde{AND}$ -Product is defined by $\widetilde{\Lambda}^N: [\widetilde{NSM}]_{m\times n} \widetilde{\times} [\widetilde{NSM}]_{m\times n} \to [\widetilde{NSM}]_{m\times n^2}$. $[\upsilon_{ij}]\widetilde{\Lambda}^N[\zeta_{ik}] = [\xi_{iq}]$, where $[\xi_{iq}] = Min\{\upsilon_{ij}, \zeta_{ik}\}$, for all $i, q = n(j-1) + k$.

Definition 3.2. Let $[\upsilon_{ij}] \in [\widetilde{NSM}]_{m\times n}$and $[\zeta_{ij}] \in [\widetilde{NSM}]_{m\times n}$ then $\widetilde{OR}$ -Product is defined by $\widetilde{\vee}^N: [\widetilde{NSM}]_{m\times n} \widetilde{\times} [\widetilde{NSM}]_{m\times n} \to [\widetilde{NSM}]_{m\times n^2}$. $[\upsilon_{ij}]\ \widetilde{\vee}^N\ [\zeta_{ik}] = [\xi_{iq}]$, where $[\xi_{iq}] = Max\{\upsilon_{ij}, \zeta_{ik}\}$, for all $i, q = n(j-1) + k$.

Definition 3.3. Let $[\upsilon_{ij}] \in [\widetilde{NSM}]_{m\times n}$and $[\zeta_{ij}] \in [\widetilde{NSM}]_{m\times n}$ then $\widetilde{NOT}$ -Product is defined by $\widetilde{\neg}^N: [\widetilde{NSM}]_{m\times n} \widetilde{\times} [\widetilde{NSM}]_{m\times n} \to [\widetilde{NSM}]_{m\times n^2}$. $[\upsilon_{ij}]\widetilde{\neg}^N[\zeta_{ik}] = [\xi_{iq}]$, where $[\xi_{iq}] = Min\{\widetilde{\neg}^N\upsilon_{ij}, \widetilde{\neg}^N\zeta_{ik}\}$, for all $i, q = n(j-1) + k$.

Definition 3.4.

Let $[\zeta_{ij}] \in [\widetilde{NSM}]_{m\times n^2}$, $I_k = \{q: \exists i,\ \xi_{iq} \neq 0,\ (k-1)n < q \leq kn\}$ for all $k \in \{1,2,3,\ldots\ldots n\}$ then the Max-Min product $\oplus$ is defined by $\oplus: [\widetilde{NSM}]_{m\times n^2} \to [\widetilde{NSM}]_{m\times 1}$,

$\oplus\ [\xi_{iq}] = [Max\{\widetilde{s_k}\}]$, where

$$\widetilde{s_k} = \begin{cases} Min\{\xi_{iq}\}, q \in I_k\ , if\ I_k \neq 0 \\ 0 \qquad\qquad\qquad if\ I_k\ = 0 \end{cases}$$

The one column $\widetilde{NSM}(\oplus\ [\xi_{iq}])$ is called Nano Soft Max-Min Decision matrix.

Definition 3.4.

Let us define an optimum solution for the initial universal set $\widetilde{U}$=$\{u_1, u_2, \ldots\ldots\ldots u_n\}$ If $\oplus\ \{\xi_{iq}\} = [\zeta_{i1}]$, then $\widetilde{OS}_{[\zeta_{i1}]}(\widetilde{U}) = \{u_i: u_i \in \widetilde{U}\ , 0 < \xi_{i1} \leq 1\}$ is optimum solution of $\widetilde{U}$.

Example 3.1.

A couple, Mr. & Mrs. Ayoop are willing to buy a new bike from a Show room $\widetilde{U}$ which have five different company bikes with five different facility options. Let us consider, $\widetilde{U}$ = {ς_1 = Royal Enfield, ς_2 = Hero splendor, ς_3 = Honda Unicorn, ς_4= Bajaj Platena, ς_5 =TV S Star City} be the five different company bikes.

Let E = {$\in_1$ = Costly and Comfortable drive, $\in_2$= Family bikes, $\in_3$ = Royal look with high fuel capacity, $\in_4$ =Good mileage, $\in_5$ = Cheap and Maintanenence free} be the set of parameters.

Now using the following algorithm to solve this problem.

Algorithm 2:

Step 1: Choose parameter subsets A and B of $\widetilde{E}$.

Step 2: Construct Nano soft subsets $\widetilde{\wp}_A$ and $\widetilde{\wp}_B$ for given parameter set A and B respectively.

Step 3: Construct the $\widetilde{NSM}$ for the $\widetilde{NS}$-subsets $\widetilde{\wp}_A$ and $\widetilde{\wp}_B$

Step 4: Find the Max-Min product of $\widetilde{NSM}$.

Step 5: Find the suitable object of $\widetilde{U}$.

Step 1: Favourite attributes of Mr &Mrs. Ayoop are $A = \{\in_1, \in_3, \in_4\}$ and $B = \{\in_1, \in_2\}$ respectively.

Step 2: Let $(\widetilde{\wp}_{\tilde{E}}, \tilde{E})$ be a $\widetilde{NS}$-Set over $\widetilde{U}$ and $(\widetilde{\wp}_A, \tilde{E})$ and $(\widetilde{\wp}_B, \tilde{E})$ are the $\widetilde{NS}$-Subsets of $(\widetilde{\wp}_{\tilde{E}}, \tilde{E})$.

$(\widetilde{\wp}_{\tilde{E}}, \tilde{E}) = \{\widetilde{\wp}(\in_1), \widetilde{\wp}(\in_2), \widetilde{\wp}(\in_3), \widetilde{\wp}(\in_4), \widetilde{\wp}(\in_5)\}$, where

$\widetilde{\wp}(\in_1)=\{\varsigma_1, \varsigma_3\}$

$\widetilde{\wp}(\in_2)= \{\varsigma_1, \varsigma_2, \varsigma_3\}$

$\widetilde{\wp}(\in_3)=\{\varsigma_1, \varsigma_3\}$

$\widetilde{\wp}(\in_4)=\{\varsigma_2, \varsigma_3, \varsigma_4\}$

$\widetilde{\wp}(\in_5)=\{\varsigma_2, \varsigma_4, \varsigma_5\}$

$(\widetilde{\wp_A}, \tilde{E}) = \{\widetilde{\wp}(\in_1), \widetilde{\wp}(\in_3), \widetilde{\wp}(\in_4)\}$

$(\widetilde{\wp_B}, \tilde{E}) = \{\widetilde{\wp}(\in_1), \widetilde{\wp}(\in_2)\}$

Let $\widetilde{\wp_s}$ be the $\widetilde{NS}$ $-$equivalence relation on $\widetilde{\wp}_{\tilde{E}}$,

$\widetilde{\wp_s} = \{\widetilde{\wp}(\in_1) \tilde{\times} \widetilde{\wp}(\in_1), \widetilde{\wp}(\in_2) \tilde{\times} \widetilde{\wp}(\in_2), \widetilde{\wp}(\in_3) \tilde{\times} \widetilde{\wp}(\in_3), \widetilde{\wp}(\in_4) \tilde{\times} \widetilde{\wp}(\in_4),$
$\widetilde{\wp}(\in_5) \tilde{\times} \widetilde{\wp}(\in_5), \widetilde{\wp}(\in_2) \tilde{\times} \widetilde{\wp}(\in_4), \widetilde{\wp}(\in_4) \tilde{\times} \widetilde{\wp}(\in_2)\}$

Then the $\widetilde{NS}$ $-$equivalence classes of $\widetilde{\wp_s}$ are,

$[\widetilde{\wp}(\in_1)]=\{ \widetilde{\wp}(\in_1)\}$

$[\widetilde{\wp}(\in_2)]=\{\widetilde{\wp}(\in_2), \widetilde{\wp}(\in_4)\}$

$[\widetilde{\wp}(\in_3)]=\{ \widetilde{\wp}(\in_3)\}$

$[\widetilde{\wp}(\in_4)]=\{\widetilde{\wp}(\in_4), \widetilde{\wp}(\in_2)\}$

$[\widetilde{\wp}(\in_5)] = \{\widetilde{\wp}(\in_5)\}$

Now, $\widetilde{\wp}_{\tilde{E}}/\widetilde{\wp_s}= \{\widetilde{\wp}(\in_1), \widetilde{\{\wp}(\in_2), \widetilde{\wp}(\in_4)\}, \widetilde{\wp}(\in_3), \widetilde{\wp}(\in_5)\}$

$L_{\widetilde{\rho_s}}(\widetilde{\rho_A})= \{\widetilde{\wp}(\in_1), \widetilde{\wp}(\in_3)\}$,

$U_{\widetilde{\rho_s}}(\widetilde{\rho_A})= \{\widetilde{\wp}(\in_1), \widetilde{\wp}(\in_2), \widetilde{\wp}(\in_3), \widetilde{\wp}(\in_4)\}$,

$B_{\widetilde{\rho_s}}(\widetilde{\rho_A})= \{\widetilde{\wp}(\in_2), \widetilde{\wp}(\in_4)\}$.

Similarly, we can obtain,

$L_{\widetilde{\rho_s}}(\widetilde{\rho_B})= \{\widetilde{\wp}(\in_1)\}$,

$U_{\widetilde{\rho_s}}(\widetilde{\rho_B})= \{\widetilde{\wp}(\in_1), \widetilde{\wp}(\in_2), \widetilde{\wp}(\in_4)\}$,

$B_{\widetilde{\rho_s}}(\widetilde{\rho_B})= \{\widetilde{\wp}(\in_2), \widetilde{\wp}(\in_4)\}$.

Step 3: The $\widetilde{NSM}$ of $\widetilde{\rho_A}$ and $\widetilde{\rho_B}$ are,

Advances in Nonlinear Variational Inequalities
ISSN: 1092-910X
Vol 27 No. 4 (2024)

$$\widetilde{NSM}(\widetilde{\rho_{\mathrm{A}}}) = \begin{bmatrix} 1 & 0.5 & 1 & 0 & 0 \\ 0 & 0.5 & 0 & 0.5 & 0 \\ 1 & 0.5 & 1 & 0.5 & 0 \\ 0 & 0 & 0 & 0.5 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\widetilde{NSM}(\widetilde{\rho_{\mathrm{B}}}) = \begin{bmatrix} 1 & 0.5 & 0 & 0 & 0 \\ 0 & 0.5 & 0 & 0.5 & 0 \\ 1 & 0.5 & 0 & 0.5 & 0 \\ 0 & 0 & 0 & 0.5 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Step 4: Now Let us use Cagman's [6] $(\widetilde{\Lambda}^N) - product\ for\ the\ \widetilde{NSM}\ is,$

$$\widetilde{NSM}(\widetilde{\rho_{\mathrm{A}}})\ \widetilde{\Lambda}^N \widetilde{NSM}(\widetilde{\rho_{\mathrm{B}}}) = \begin{bmatrix} 1 & 0.5 & 1 & 0 & 0 \\ 0 & 0.5 & 0 & 0.5 & 0 \\ 1 & 0.5 & 1 & 0.5 & 0 \\ 0 & 0 & 0 & 0.5 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \widetilde{\Lambda}^N \begin{bmatrix} 1 & 0.5 & 0 & 0 & 0 \\ 0 & 0.5 & 0 & 0.5 & 0 \\ 1 & 0.5 & 0 & 0.5 & 0 \\ 0 & 0 & 0 & 0.5 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0.5 & 0 & 0 & 0 & 0.5 & 0.5 & 0 & 0 & 0 & 1 & 0.5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.5 & 0 & 0.5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.5 & 0 & 0.5 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0.5 & 0 & 0.5 & 0 & 0.5 & 0.5 & 0 & 0.5 & 0 & 1 & 0.5 & 0 & 0.5 & 0 & 0.5 & 0.5 & 0 & 0.5 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.5 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Step 5: Let us use Max-Min Product $(\oplus - \text{Product})$for the $\widetilde{NSM}$

$$\oplus\ [\widetilde{NSM}(\widetilde{\rho_{\mathrm{A}}})\ \widetilde{\Lambda}^N \widetilde{NSM}(\widetilde{\rho_{\mathrm{B}}})] = \begin{bmatrix} 0 \\ 0 \\ 0.5 \\ 0 \\ 0 \end{bmatrix}$$

Step 6: An optimum solution of $\widetilde{U}$ is

$\widetilde{OS}_{\oplus[\widetilde{NSM}(\widetilde{\rho_{\mathrm{A}}})\widetilde{\Lambda}^N \widetilde{NSM}(\widetilde{\rho_{\mathrm{B}}})]}$ = ς_3.

From this selection process the bike ς_3 = Honda unicorn is the most suitable bike for Mr. & Mrs. Ayoop.

Example 3.2. Suppose that Mr.X and Mr. Y are analysing the best school to admit their childrens. Let $\widetilde{U}$ = {s_1 ,s_2, s_3, s_4} be their near by schools, which can have parameters $\tilde{E}$= {$\in_1$ = Good infrastructure, $\in_2$=Good quality of education, $\in_3$ =Remarkable experienced staff members, $\in_4$ =Low fees structure, $\in_5$ = High fees structure}.

Step 1: The parameter sets of Mr. X &Mr. Y are $A = \{\in_{1,}\in_2\}$ and $B = \{\in_{2,}\in_4\}$ respectively.

Step 2: Let $(\widetilde{\wp}_{\tilde{E}}, \tilde{E})$ be a $\widetilde{NS}$-Set over $\widetilde{U}$ and $(\widetilde{\wp}_A, \tilde{E})$ and $(\widetilde{\wp}_B, \tilde{E})$ are the $\widetilde{NS}$-Subsets of $(\widetilde{\wp}_{\tilde{E}}, \tilde{E})$.

$(\widetilde{\wp}_{\tilde{E}}, \tilde{E}) = \{\widetilde{\wp}(\in_1), \widetilde{\wp}(\in_2), \widetilde{\wp}(\in_3), \widetilde{\wp}(\in_4), \widetilde{\wp}(\in_5)\}$, where

$\widetilde{\wp}(\in_1) =\{s_1, s_2, s_4\}$

$\widetilde{\wp}(\in_2)=\{s_1, s_2, s_3\}$

$\widetilde{\wp}(\in_3)$ ={s_2, s_3, s_4}

$\widetilde{\wp}(\in_4)$={s_2, s_3}

$\widetilde{\wp}(\in_5)$={s_1,s_4}

$(\widetilde{\wp_A}, \tilde{E}) = \{\widetilde{\wp}(\in_1), \widetilde{\wp}(\in_2)\}$

$(\widetilde{\wp_B}, \tilde{E}) = \{\widetilde{\wp}(\in_1), \widetilde{\wp}(\in_2), \widetilde{\wp}(\in_4)\}$

Let $\widetilde{\wp_s}$ be the $\widetilde{NS}$ −equivalence relation on $\widetilde{\wp}_{\tilde{E}}$,

$\widetilde{\wp_s} = \{\widetilde{\wp}(\in_1) \tilde{\times} \widetilde{\wp}(\in_1), \widetilde{\wp}(\in_2) \tilde{\times} \widetilde{\wp}(\in_2), \widetilde{\wp}(\in_3) \tilde{\times} \widetilde{\wp}(\in_3), \widetilde{\wp}(\in_4) \tilde{\times} \widetilde{\wp}(\in_4),$
$\widetilde{\wp}(\in_5) \tilde{\times} \widetilde{\wp}(\in_5), \widetilde{\wp}(\in_2) \tilde{\times} \widetilde{\wp}(\in_3), \widetilde{\wp}(\in_3) \tilde{\times} \widetilde{\wp}(\in_2)\}$

Then the $\widetilde{NS}$ −equivalence classes of $\widetilde{\wp_s}$ are,

[$\widetilde{\wp}(\in_1)$]={ $\widetilde{\wp}(\in_1)$}

[$\widetilde{\wp}(\in_2)$]={$\widetilde{\wp}(\in_2), \widetilde{\wp}(\in_3)$}

[$\widetilde{\wp}(\in_3)$]={ $\widetilde{\wp}(\in_3), \widetilde{\wp}(\in_2)$}

[$\widetilde{\wp}(\in_4)$]={$\widetilde{\wp}(\in_4)$}

[$\widetilde{\wp}(\in_5)$] = {$\widetilde{\wp}(\in_5)$}

Now, $\widetilde{\wp}_{\tilde{E}}/\widetilde{\wp_s}$= {$\widetilde{\wp}(\in_1)$, $\widetilde{\{\wp}(\in_2), \widetilde{\wp}(\in_3)\}$,$\widetilde{\wp}(\in_4), \widetilde{\wp}(\in_5)$}

$L_{\widetilde{\rho_s}}(\widetilde{\rho_A})$= {$\widetilde{\wp}(\in_1)$},

$U_{\widetilde{\rho_s}}(\widetilde{\rho_A})$= {$\widetilde{\wp}(\in_1), \widetilde{\wp}(\in_2), \widetilde{\wp}(\in_3)$},

$B_{\widetilde{\rho_s}}(\widetilde{\rho_A})$= {$\widetilde{\wp}(\in_2), \widetilde{\wp}(\in_3)$}.

Similarly, we can obtain,

$L_{\widetilde{\rho_s}}(\widetilde{\rho_B})$= {$\widetilde{\wp}(\in_4)$},

$U_{\widetilde{\rho_s}}(\widetilde{\rho_B})$= {$\widetilde{\wp}(\in_2), \widetilde{\wp}(\in_3), \widetilde{\wp}(\in_4)$},

$B_{\widetilde{\rho_s}}(\widetilde{\rho_B})$= {$\widetilde{\wp}(\in_2), \widetilde{\wp}(\in_3)$}.

Step 3: The $\widetilde{NSM}$ of $\widetilde{\rho_A}$ *and* $\widetilde{\rho_B}$ *are* ,

$$\widetilde{NSM}(\widetilde{\rho_A}) = \begin{bmatrix} 1 & 0.5 & 0 & 0 & 0 \\ 1 & 0.5 & 0.5 & 0 & 0 \\ 0 & 0.5 & 0.5 & 0 & 0 \\ 1 & 0 & 0.5 & 0 & 0 \end{bmatrix}$$

$$\widetilde{NSM}(\widetilde{\rho_B}) = \begin{bmatrix} 0 & 0.5 & 0 & 0 & 0 \\ 0 & 0.5 & 0.5 & 1 & 0 \\ 0 & 0.5 & 0.5 & 1 & 0 \\ 0 & 0 & 0.5 & 0 & 0 \end{bmatrix}$$

Step 4: Now Let us use Cagman's [6] $(\tilde{\Lambda}^N) - product\ for\ the\ \widetilde{NSM}\ is,$

$$\widetilde{NSM}(\widetilde{\rho_{\mathrm{A}}})\ \widetilde{\Lambda}^{N}\widetilde{NSM}(\widetilde{\rho_{\mathrm{B}}}) = \begin{bmatrix} 1 & 0.5 & 0 & 0 & 0 \\ 1 & 0.5 & 0.5 & 0 & 0 \\ 0 & 0.5 & 0.5 & 0 & 0 \\ 1 & 0 & 0.5 & 0 & 0 \end{bmatrix} \widetilde{\Lambda}^{N} \begin{bmatrix} 0 & 0.5 & 0 & 0 & 0 \\ 0 & 0.5 & 0.5 & 1 & 0 \\ 0 & 0.5 & 0.5 & 1 & 0 \\ 0 & 0 & 0.5 & 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0.5 & 0 & 0 & 0 & 0 & 0.5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.5 & 0.5 & 1 & 0 & 0 & 0.5 & 0.5 & 0.5 & 0 & 0 & 0.5 & 0.5 & 0.5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.5 & 0.5 & 0.5 & 0 & 0 & 0.5 & 0 & 0.5 & 0.5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Step 4: Let us use Max-Min Product ($\oplus - \text{Product}$)for the $\widetilde{NSM}$

$$\oplus\ [\widetilde{NSM}(\widetilde{\rho_{\mathrm{A}}})\ \widetilde{\Lambda}^{N}\widetilde{NSM}(\widetilde{\rho_{\mathrm{B}}})] = \begin{bmatrix} 0 \\ 0.5 \\ 0 \\ 0 \end{bmatrix}$$

Step 5: An optimum solution of $\widetilde{U}$ is

$$\widetilde{OS}_{\oplus[\widetilde{NSM}(\widetilde{\rho_{\mathrm{A}}})\widetilde{\Lambda}^{N}\widetilde{NSM}(\widetilde{\rho_{\mathrm{B}}})]} = \mathrm{s}_2.$$

The school s_2 is the best school for Parents Mr. X and Mr. Y to admit their childrens.

4. Conclusion

This study found that compared to soft Matrices, Nano Soft Matrices produce out comes that are more accurate. While just yes or No possibilities were explored in soft Matrice, Nano Soft Matrices concerned on intermediate Possibilities. We can apply these ideas to computer programes in the future.

References

[1] R. Abinprakash, Nano Soft Matrix approach to Decision making problems, (Communicated)

[2] M. LellisThivagar and CarmelRichard, On Nano Forms of Weakly Open Sets, International Journal of Mathematics and Statistics Invention,1(1) (2013),31-37.

[3] M. LellisThivagar, S.P.R. Priyalatha, An innovative Approach on Nano Soft Topological Spaces, South East Asian J. of Math. And Math. Sci. 13(2), (2017),47-62.

[4] D. A. Molodtsov Soft Set Theory- First Result, Computers and Mathematics with Applications, Vol. 37 (1999), 19-31.

[5] P.K. Maji, R. Biswas and A.R. Roy, An Application of soft sets in a decision-making problem, Computers and mathematics with applications,44(2002), 1077-1083.

[6] Naim Cagman, Serdar Enginoglu, Soft set theory and its decision making, Computers and mathematics with applications,59(2010),3308-3314.