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# Graphical and Matrix Coding Methods for Enhanced Message Security Using Star Graphs

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**Abstract:**
This article presents a new method for encoding secret messages using super-intermediate symbols in star charts. The system includes two super average images and two formats, as well as graphics and matrix encoding methods for added security. The authors provide a method to adjust the average of the three star maps to ensure the stability of the encoded message. This research provides a non-programming method for developing matrix graphics coding techniques and has implications for fields such as telecommunications and cryptography. The use of super-average symbols and star charts provides a secure method of communication with applications in data encryption, network security, and confidential communication. The results of this study show that the effectiveness of the proposed system in improving the security and privacy of encrypted messages makes it useful in business.

**Keywords**: SPFN, EFOF, super mean labeling, GMJ and star graphs.

## 1. Introduction

Matrix theory, numerical analysis, probability theory, topology, and combinatorics are the origin of most image signatures can be traced back to the method introduced by Rosa [6] in 1967 and the method given by Graham and Sloane in 1980. It works as a good model for many applications, for example: coding theory, x-ray crystallography etc. [3]. It would be better to take advantage of this and use the tag for sub-fields of coding theory and cryptography, so in this research we also propose research to expand the application of the secret process of the registration form.

In this study, we propose to continue the use of image signatures in the field of cryptography by introducing a new encryption method. The process will convert plain text to ciphertext in matrix and image format, and the key to decrypting the message is the ability to recognize the image. This method provides very secure data communication because the encrypted message cannot be decrypted without key information and the ability to interpret the image.

The idea of encrypting messages using image signatures has many advantages over traditional encryption methods. First, it's very secure because using images makes it very difficult for someone who doesn't know the key to decipher the message. Also, using image signatures allows for more creativity in the encryption process, as different images can be used to encode different messages.

This provides a level of customization not found in traditional encryption methods. It can also be applied to many fields such as research, data encryption, network security, and private communications. The use of image signatures in cryptography has not been widely explored, and this research represents an important step in that direction. Using the power of image technology and digital signatures, we can create new and innovative ways to protect our communications and information.

In conclusion, this research offers a new method to encrypt messages using image signatures. This approach provides high security and allows for many levels of creativity and customization in the encryption process. The plan can be used in many areas, including data encryption, network security, and communication privacy. By expanding the use of digital signatures in cryptography, we can create new and innovative ways to protect our communications and information.

## 2. Literature Review

Communication between sender and receiver will be limited and unintelligible to others. The idea of writing definitions was introduced by S. Somasundaram and R. Ponraj [7]. Some new translation families are discussed at [13]. The binary star with corresponding sides is an example for $m - n \leq 4$ Proven by Maheshwari and Ramesh.

The concept of over-defined flags was introduced and studied by D. Ramya et al. , [4]. R. Vasuki and A. Nagarajan [14] the upper average properties of the H-diagram of subdivisions and $K_{1,3}$ $K_{1,3}$, some pips and subdivided graphs R. Vasuki. A study on inverse graphs. Nagarajan [15]. [8], [9] in star map coding with top-average labels by Uma Maheswari et al. was developed by Uma Maheswari et al., [10], [9] using Fibonacci mesh encoding. 6[12].

Inspired by these studies, we are working on three star maps $K_{1,4} \cup K_{1,\ ,4} \cup K_{1,4}$ and $K_{1,5} \cup K_{1,,6} \cup K_{1,7}$ Using super intermediate symbols and GMJ (Graphic Message Scrambling) encoding methods, hence this article.

## 3. Definitions

**Definition 2.1** *Let G be a (a, b) graph and* $\lambda: V(G) \to \{one,\ two,\ three,\ \cdots,\ a+b\}$ *be an injection. For each edge* $e = xy$*, let*

$$\lambda^*(e) = \begin{cases} \frac{\lambda(x)+\lambda(y)}{2} & \text{if} \quad \lambda(x)+\lambda(y) \quad \text{is} \quad \text{even;} \\ \frac{\lambda(x)+\lambda(y)+one}{two} & \text{if} \quad \lambda(x)+\lambda(y) \quad \text{is} \quad \text{odd} \end{cases}$$

then $\lambda$ is called Super Mean Labeling if $\lambda(Y) \cup \{\lambda^*(e): e \in E(G)\} = \{one, two,\ three,\ \cdots,\ a+b\}$. A graph that admits a SML is called a Super Mean Graph.

## 4. A Rule for Super Mean Labeling on Three Star Graphs

1. Take 1, $\frac{p+q+1}{2}$ for the first and the last values of the vertices and edges of the star gaph and it is reffred by [9].

2. If $g(u_1) = three$, then $g(v_1) = four$ and if $g(u_1) = five$, then $g(v_1) = two$.

Start by placing the numbers on the first star, then complete the second and third stars next to each other. Place the least digit missing from the begining part of the graph in the first overhanging corner of the next part. Continuously assign the smallest number to the starting overhanging corner of the third star. That is, $f(v_1)$ and $f(w_1)$ must be distributed in order. This procedure is maintained when assigning digits to the angles of the remaining parts of the graph.

## 5. Coding Method

Cryptography is an essential part of securing information during communication. Cryptography restricts access to confidential information using various techniques such as encryption and decryption. A good example of cryptography is the Caesar cipher, also known as a scroll cipher, which involves changing the letters of certain numbers to create an encrypted message. However, many encryption methods have been developed, such as the GMJ encryption method, to increase the security of encrypted messages. This method involves assigning numbers or non-alphabetic symbols to the alphabet, finding the frequency of each letter in a word, and entering it in a special way. The encoded message is then encrypted for privacy and displayed as a digital image. In this research, we focus on developing algorithms to convert plain text to ciphertext using matrices and images. By encrypting messages, we can ensure that they are sent as unsaved messages that only authorized persons can decipher. Data encryption is an important process for ensuring the security of data transmission on the Internet. Our proposed algorithm aims to increase the security of these encrypted messages. We can create secure and confidential communications using various techniques such as coding and encoding. This can have broad applications in many fields such as online business, secure communications, and even military and government communications.

## 6. Illustrations

### 6.1 Illustration 1:

1. **Message:** Documents found hundred meters east of the flower garden in toy city.
2. **clue:** Twice the twins twinkling but thrice identical.

(Twinkling represents star, twins represents two implies two star, twice the twins represents four but thrice identical represents three times four, the three star graph is understood) and the graph shown in Figure 1.

3. **Graph:** The three star graph $G = K_{1,4} \cup K_{1,4} \cup K_{1,4}$.

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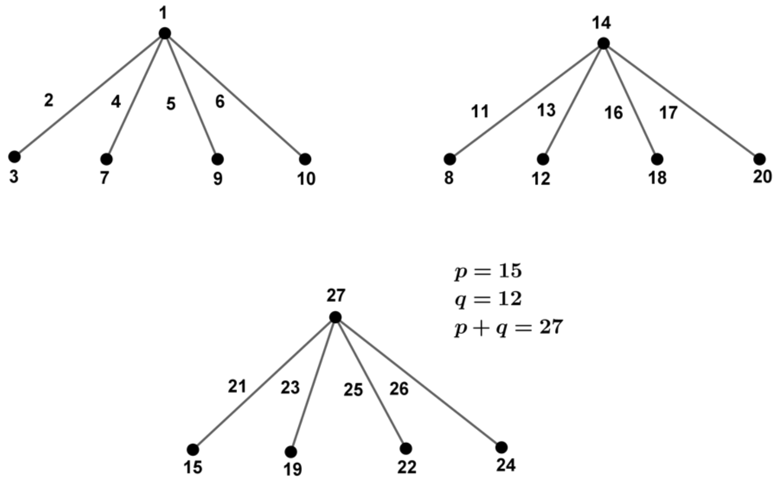


Figure 1: $G = K_{1,4} \cup K_{1,4} \cup K_{1,4}$

(p vertices and q edges)

4. **Splitting of alphabets: EFOF**

| | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 14 | 1 | 15 | 2 | 16 | 3 | 17 | 4 | 18 | 5 | 19 | 6 | 20 |
| Aa | Bb | Cc | Dd | Ee | Ff | Gg | Hh | Ii | Jj | Kk | Ll | Mm |
| 7 | 21 | 8 | 22 | 9 | 23 | 10 | 24 | 11 | 25 | 12 | 26 | 13 |
| Nn | Oo | Pp | Qq | Rr | Ss | Tt | Uu | Vv | Ww | Xx | Yy | Zz |

Numbers one through thirteen are placed in place of letters B through D and F. And numbers from 14 to 26 are assigned different functions of the alphabet, from A to C and E. This process is called EFOF (Even forward, Odds forward) as shown in Figure 2.

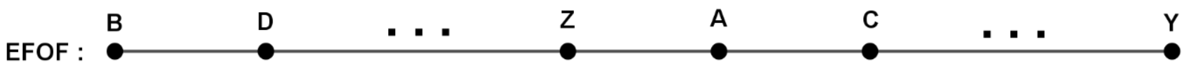


Figure 2: scale function

We represent the number of letters as an encoding function. Even places of alphabets the function $\phi$ is given by,

$$\phi(2r + two) = (r + one) \;\; for \;\; r = 0, one, two, \cdots, 12.$$

1,3,5,… places of alphabets the mathematical representation is,

$$\phi(2r - one) = (thirteen + r) \;\; for\, r = 1,3,5, \cdots, 13.$$

For decoding we reverse the process.

By using EFOF and the SML on $K_{1,4} \cup K_{1,4} \cup K_{1,4}$ the message is encoded.

5. **Coding:**(word wise)

| | |
|---|---|
| Documents | (1,0,1)(3,0,1)(3,1,0)(3,4,0)(2,4,0)(2,0,3)(1,2,0)(1,4,0) |
| found | (1,1,0)(3,0,1)(3,4,0)(1,2,0)(1,0,1) |
| two | (1,4,0)(1,0,3)(3,0,1) |
| hundred | (1,0,2)(3,4,0)(1,2,0)(1,0,1)(1,3,0)(2,0,3)(1,0,1) |
| meters | (2,4,0)(2,0,3)(1,4,0)(2,0,3)(1,3,0)(3,0,2) |
| east | (2,0,3)(1,1,1)(3,0,2)(1,4,0) |
| of | (3,0,1)(1,1,0) |
| the | (1,4,0)(1,0,2)(2,0,3) |
| flower | (1,1,0)(1,0,4)(3,0,1)(1,0,3)(2,0,3)(1,3,0) |
| garden | (2,0,4)(1,1,1)(1,3,0)(1,0,1)(2,0,3)(1,2,0) |
| in | (2,3,0)(1,2,0) |
| toy | (1,4,0)(3,0,1)(3,0,4) |
| city | (3,1,0)(2,3,0)(1,4,0)(3,0,4) |

6. **Presenting the letter codes:**

The triple variable form by adding the three elements of the number and is applied to defined the letters that are more difficult to find. They are correctly marked on the ends of the squares. If the denotion is (1,5,0), use the number 6 to represent it. It can be shown that there are 3 ways to check if it is the fifth dangling vertex or the value of five if 6=1+5,2+4 or 3+3 to get a letter. edges of the first. Star dangling vertex, next dangling vertex or fourth dangling value of the second star, or third dangling vertex or value edge of the three star graphs. A number representing a letter is simply written in a horizontal line, Figure 3 shows the encoding image.

7. **Horizontal string:**

244765352473254437324526555455355425352544546342535354745570

8. **Picture coding :**

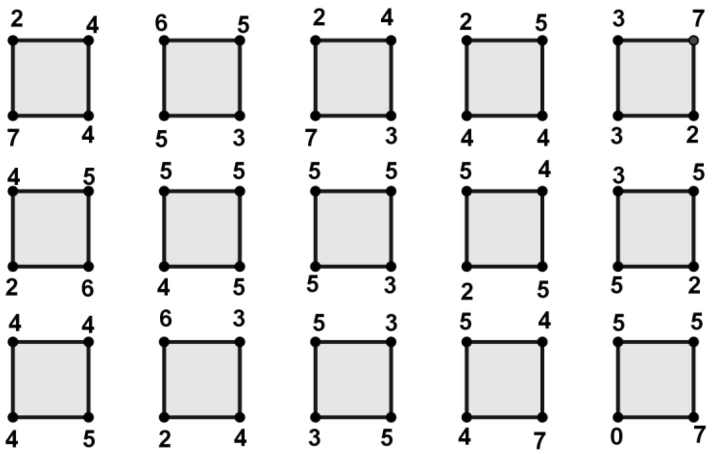


Figure 3: coded message EFOF

**6.2 Illustration 2:**

1. **Message:** Between first monday and third Thursday contact soldiers conduct coaching and assemble behind the headquarters.

2. **clue:** Numerical value of 4 brothers run together, a person to fall on times of other. (4 brothers are 2, 3, 5 and 7, they are single digit prime numbers, which implies three star graph.

3. **Graphical Representation:** The graph is shown in Figure 3.

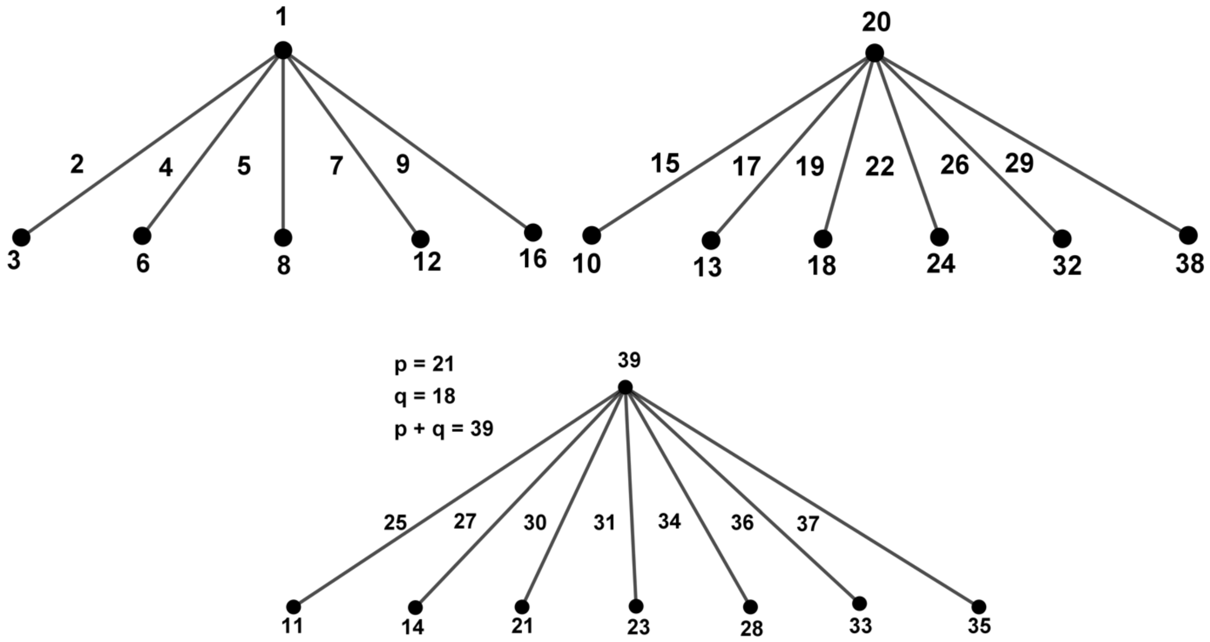


Figure 4: $K_{1,5} \cup K_{1,6} \cup K_{1,7}$

4. **Splitting of alphabets: SPFN**

| | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 6 | 7 | 2 | 8 | 9 | 10 | 11 | 3 | 12 | 13 | 14 | 15 |
| A | B | C | D | E | F | G | H | I | J | K | L | M |
| 16 | 17 | 4 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 25 | 5 | 26 |
| N | O | P | Q | R | S | T | U | V | W | X | Y | Z |

Location $1^2, 2^2, 3^2, 4^2$ and $5^2$ returns 1,2,3,4,5. Then the letter B takes the number 6, the letter C takes the number 7, the letter E takes the number 8, and so on. This method is called SPFN (Square Integer Initial Number) as shown in Figure 5.

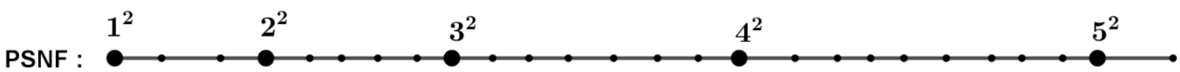


Figure 5: scale function

The function for encoding is given below:

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$\chi(m^2) = m$ for $m = 1,2,3,4,5.$
$\chi(m^2 + s) = 5 + s, m = 1, s = 1,2$
$\chi(m^2 + s) = 7 + s, m = 2, s = 1,2, \cdots ,4$
$\chi(m^2 + s) = 11 + s, m = 3, s = 1,2, \cdots ,6$
$\chi(m^2 + s) = 17 + s, m = 4, s = 1,2, \cdots ,8$
$\chi(m^2 + s) = 25 + s, m = 5, s = 1$

5. **Coding a letter:**

As shown in the image above, numbers referring to the alphabet are based on a chart of three stars written in triplets representing a number.

6. **Coding:**(Letter wise)

| | |
|---|---|
| Between | (1,2,0)(1,3,0)(3,3,0)(2,4,0)(1,3,0)(1,3,0)(1,5,0) |
| first | (1,0,5)(1,1,0)(2,0,3)(2,2,2)(1,3,0) |
| monday | (2,0,1)(2,0,2)(1,5,0)(1,0,1)(1,1,1)(1,0,3) |
| and | (1,1,1)(1,5,0)(1,0,1) |
| third | (1,3,0)(3,1,0)(1,1,0)(2,0,3)(1,0,1) |
| thursday | (1,3,0)(3,1,0)(2,0,4)(2,0,3)(2,2,2)(1,0,1)(1,1,1)(1,0,3) |
| contact | (1,0,4)(2,0,2)(1,5,0)(1,3,0)(1,1,1)(1,0,4)(1,3,0) |
| soldiers | (2,2,2)(2,0,2)(3,2,0)(1,0,1)(1,1,0)(1,3,0)(2,0,3)(2,2,2) |
| conduct | (1,0,4)(2,0,2)(1,5,0)(1,0,1)(2,0,4)(1,0,4)(1,3,0) |
| coaching | (1,0,4)(2,0,2)(1,1,1)(1,0,4)(3,1,0)(1,1,0)(1,5,0) (3,1,0) |
| and | (1,1,1)(1,5,0)(1,0,1) |
| assemble | (1,1,1)(2,2,2)(2,2,2)(1,3,0)(2,0,1)(1,2,0)(3,2,0) (1,3,0) |
| behind | (1,2,0)(1,3,0)(3,1,0)(1,1,0)(1,5,0)(1,0,1) |
| the | (1,3,0)(3,1,0)(1,3,0) |
| headquarters | (3,1,0)(1,3,0)(1,1,1)(1,0,1)(3,2,0)(3,0,4)(1,1,1) (3,0,3)(3,3,0)(1,3,0)(3,0,3)(2,2,2) |

7. **Horizontal string:**

344644662564346234362442524465623464643546452245654626545435426436236643354344262
444443257366466

8. **Matrix coding :**

$$\begin{pmatrix} 3 & 4 & 4 \\ 6 & 4 & 4 \\ 6 & 6 & 2 \\ 5 & 6 & 4 \end{pmatrix} \begin{pmatrix} 3 & 4 & 6 \\ 2 & 3 & 4 \\ 3 & 6 & 2 \\ 4 & 4 & 2 \end{pmatrix} \begin{pmatrix} 5 & 2 & 4 \\ 4 & 6 & 5 \\ 6 & 2 & 3 \\ 4 & 6 & 4 \end{pmatrix} \begin{pmatrix} 6 & 4 & 3 \\ 5 & 4 & 6 \\ 4 & 5 & 2 \\ 2 & 4 & 5 \end{pmatrix}$$

$$\begin{pmatrix}6&5&4\\6&2&6\\5&4&5\\4&3&5\end{pmatrix}\begin{pmatrix}4&2&6\\4&3&6\\2&3&6\\6&4&3\end{pmatrix}\begin{pmatrix}3&5&4\\3&4&4\\2&6&2\\4&4&4\end{pmatrix}\begin{pmatrix}4&4&3\\2&5&7\\3&6&6\\4&6&6\end{pmatrix}$$

## 7. Algorithm

• **step 1:** Begin by selecting a three-star graph and applying the Super Mean Labeling (SML) technique to it.

• **step 2:** Use the EFOF and SPFN methods to split the alphabet into smaller groups.

• **step 3:** Develop a coding method for each individual letter based on the chosen grouping technique.

• **step 4:** Write out the message that needs to be encoded.

• **step 5:** Code the message word by word.

• **step 6:** Convert the coded message into a horizontal string.

• **step 7:** Represent the horizontal string as a matrix or picture.

• **step 8:** Use the resulting matrix or picture as the encoded message.

The transformer needs to pass on the following information to the communicator:

- Clues for identifying the appropriate three star graph.
- A hint on how to split the alphabets using EFOF and SPFN methods.
- Details on the matrix coding technique.

## 9. Methodology

Generating a new ND-Hyperchaotic System: A new ND- chaotic system is introduced in this proposal, its chaotic behaviour and complexity evaluation are investigated by experimental evidence. The ND- chaotic system is defined by the following equations: For $i = 1, \ldots, n \geq 4$; $\frac{dX_i}{dt}$=$(CX_{i+1} - X_{i-2})X_{i-1}$ ; (1)

where it is expected that $X_0 = X_N, X_{-1} = X_{N-1} and X_1 = X_{N+1}$. At this point $X_i$ is the formal of the model and C is an imposing constant. From (1) system generate state with n*n, where n is integer. The dynamic behaviour of the proposed chaotic system In a dynamical system, the term "attraction" refers to a group of points or a series of values in state space that represent the movement route of the system's output. We can guarantee the structure of the hyperchaotic system and safeguard the perfect property of the ergodic dynamical system. Figure 1 (a)-(d) shows the attractor diagrams of the 7D-hyperchaotic system based on different values of the parameters $a = 40; b = 60; c = 8; d = 20; e = 0.1; f = 77; g = 10$. The attractor diagram with different values of the initial states $(x_1, x_2, x_3, x_4, x_5, x_6, x_7) = (1,1,2,2,1,1,1), (1,1,2,2,1,1,0.1), and (1,1,2,2,1,0.1,0.1)$ as blue, red, and green, respectively. A new quantum image representation based on bit-plane (QIRBP) will introduc. QIRBP model utilizes $b + 2n + 6$ qubits to store a color digital image with size $2^n \times 2^n$. Designing a Quantum Circuit for the ND-Hyperchaotic System: Once the new ND-Hyperchaotic system is defined, we aim to design a quantum circuit that can simulate or represent this system. The quantum circuit should capture the behavior and dynamics propose the use of quantum algorithms for implementing S-box substitutions. S-boxes are essential components in many cryptographic operations, and by utilizing quantum algorithms, we aim to enhance the efficiency and security of these operations. This likely involves developing new algorithms or adapting existing classical S-box substitution methods for quantum systems. Investigating Synchronized Chaos and Quantum Key Distribution (QKD): In this part of our proposal, we explore the use of synchronized chaos and quantum key distribution to hide a sent message securely. Specifically, we propose a scheme that enables three distinct observers to exchange cryptographic keys securely using 3-particle GHZ states. The violation of Svetlichny's inequality is utilized to test for potential eavesdropping in this scheme, ensuring the security of the distributed multipartite keys. Building a Complete Encryption System: Finally, we seek to build a complete encryption system that can be used to transfer information securely. This likely involves integrating the various components and schemes proposed in your proposals, such as the ND-Hyperchaotic system, quantum walks, S-box substitutions, and the secure key distribution scheme based on GHZ states. The result is an encryption system capable of protecting information during transmission.

## 10. Quantum Cryptography Protocol

Quantum cryptography, a branch of physics grounded in quantum mechanics, explores fundamental phenomena related to the polarization states of single photons [42]. The principles underpinning quantum cryptography include entanglement, photon polarization, and the Heisenberg uncertainty principle. In 1984, Gilles Brassard from the University of Montreal and Charles Bennett from IBM Research developed the pioneering quantum cryptographic protocol [1]. Price et al. [2] introduced a quantum key distribution protocol aimed at detecting and exposing fake users attempting to monopolize communication links and deny services to legitimate users. This proposed protocol leverages a quantum channel to transport encrypted data using a quantum hyperchaotic system, which then drives a corresponding system to reproduce the chaotic mask and decode the message. A notable aspect of the quantum key distribution scheme is its ability to enable three separate observers to securely exchange cryptographic keys through a sequence of 3-particle GHZ states. These entangled states offer heightened security due to their delicate quantum correlations. To test for eavesdropping, the violation of Svetlichny's inequality is utilized. Remarkably, even when the eavesdropper has full control over the outcomes of two participants' measurements, our scheme ensures the security of the key distribution process, safeguarding communication against potential adversaries.

## 11. Conclusion and future work

In conclusion, this paper has presented a new graph labeling technique called super mean labeling, which has been demonstrated on two graphs: $K_{1,4} \cup K_{1,4} \cup K_{1,4}$ and $K_{1,5} \cup K_{1,6} \cup K_{1,7}$. The technique uses both mathematical and non-mathematical cues and has been shown to be effective at labeling graphs from various graph families. We explain their work in detail with detailed examples and diagrams, making it easy for readers to understand the work and apply it to other images. The use of matrix and graph encoding techniques adds an additional layer of security to tags, making them useful tools in applications such as cryptography. The results of this research have implications for many fields, including mathematics, computer science, and cryptography. Super-average tagging techniques are very effective in many applications, including network analysis, data mining, and cryptography. The use of matrix methods and graphics along with key elements makes tags difficult to decipher, making them an essential tool in security-sensitive applications. Future research may explore the application of the supermean recording technique to other family images and further explore the use of these images. Overall, this work presents a new and useful recording method that has the potential to make a huge impact in many ways.

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