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Compute the reliability and the Domination Polynomial of Computer Server

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Abstract

We shall clarify two concepts in this study: the domination polynomial and the reliability of network, because of their impact on how much the network's strength, interconnectivity, and quality. This paper will tackle computer servers as a case study. Un computer program was used Mat lab

Keywords: Reliability, minimal cut, minimal path, Dominating polynomial.

1. Introduction

Reliability and dominating polynomials are two important subjects in network science. The dominating polynomial is useful in computing a system's long-term reliability, contrasting the reliability of various system configurations. Moreover, it can be applied to decision-making procedures pertaining to system upkeep, design, and Reliability enhancement[1]. A Reliability network is a probability that the system will work for aperiodic time. Reliability engineering frequently uses it to examine the availability, complexity and reliability of systems. The elements of the system are represented as nodes, and the connections between them an edges or links[2]. Different system configurations, including series, parallel, and series-parallel, in addition, there are complex systems that cannot be linked in series or parallel. So, the Engineers can show how component failures affect system performance by examining the reliability network[3,4]. The dominant term in a system's reliability function is represented mathematically by the dominating polynomial. The probability of the system working is described during a given time period by the reliability function, also known as R(t) [5]. The reliability function's most notable behavior is captured by the dominating polynomial, especially when t grows big. It offers perceptions of the system reliability's asymptotic behavior and also shows the interconnectedness and strength of the system and its components [6]. This paper will compute the reliability and the dominating polynomial of the computer server. In this study, we will show an appropriate method for determining the reliability and dominating polynomials of computer servers based on network features.

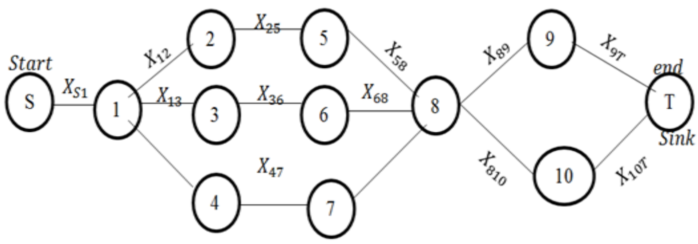

Figure 1: computer server

2. Basic Concept

Lorem ipsum dolor sit amet, consectetur adipiscing elit, sed do eiusmod tempor incididunt ut 1. Parallel system reliability: When a system requires the success of at least one component, it is referred to as a parallel system in order to determine the reliability

$$R_{sys} = 1 - \prod_{i=1}^{n} (1 - R_i) \qquad (1)$$

2. A minimal path is a collection of elements that make up a path; however, if any element is removed, the remaining set ceases to be a path.

3. A cut set is a collection of parts such that the system will fail if every part in the cut fails while every other part succeeds.

4. A minimal cut is a collection of elements that make up a cut; however, if any one element is removed, the set that remains is no longer a cut.

5. Dominating polynomial: A graph polynomial that encodes information on a graph G's domination number is called the domination polynomial, or D(G;x). The smallest number of vertices in a The dominance number of a graph is defined as the dominating set, which is a collection of vertices such that every vertex in the graph is either in the set or next to a vertex in the set.

6. Reliability system is a probability that the system will work for aperiodic time.[7,8].

3. Methods

Using the collection of all minimal paths, this method compiled all of the minimum pathways into an incidence matrix in the event that n minimal pathways exist and are denoted by the notations P1, P2,..., Pn by algorithm (1). We may then determine how to calculate the incidence matrix of all minimal paths IM.

$$\boldsymbol{IM} = \begin{array}{c} \\ \boldsymbol{p1} \\ \boldsymbol{p2} \\ \vdots \\ \boldsymbol{pm} \end{array} \begin{array}{c} \boldsymbol{Edges} \\ \begin{array}{cccc} \boldsymbol{x1} & \boldsymbol{x2} & \cdots & \boldsymbol{xn} \end{array} \\ \begin{bmatrix} \boldsymbol{a11} & \boldsymbol{a_{12}} & \cdots & \boldsymbol{a_{1n}} \\ \boldsymbol{a_{21}} & \boldsymbol{a22} & \cdots & \boldsymbol{a_{2n}} \\ \vdots & \vdots & \ddots & \vdots \\ \boldsymbol{a_{m1}} & \boldsymbol{a_{m2}} & \cdots & \boldsymbol{amn} \end{bmatrix} \end{array}$$

Such that i = 1, 2,..., m, j = 1, 2,..., n and aij ∈ {0, 1}.A state is an n-dimensional raw consisting of 1s and 0s. Aij = 1 if and only if xj ∈ Pi; else, aij = 0. Three steps need to be followed in order to produce the minimal cut sets for the generation of minimal cut sets.

Step 1: xj produces a first order diagonal cut if ∀ aij 0 of any column xj of IM.

Step 2: Instantaneously merge two IM columns. Assuming that xj xj+k is a second order cut, ∀ aij + aij+k = 0 for all k = 1,..., n. and continue until you have cut as many set order cuts as possible. Step 3: Repeat step (2) with three columns at a time to produce the third-order cuts. This time, omit any cuts that involve first or second-order cuts and continue cutting until the set order cut maximum is reached [9, 10]. Thus, the minimum path method is exhibited and used on a digital computer program called MATLAB, which gets the minimal path that a computer server receives

$(P1) = X1_{S1}\, X_{12}\, X_{25}\, X_{58}\, X_{89}\, X_{9T}$

(P2)=$XS1\ X12\ X25\ X58\ X810\ X10T$

(P3)=$XS1\ X13\ X36\ X68\ X89\ X9T$

(P4)=$Xs1\ X13\ X36\ X68\ X810\ X10T$

(P5)=$Xs1\ X14\ X47\ X78\ X810\ X10T$

(P6)=$Xs1\ X14\ X47\ X78\ X89\ X9T$

After applying the minimal cut method on the identity matrix of the computer server, IM=

$$
\begin{array}{c}
\\ P_1 \\ P_2 \\ P_3 \\ P_4 \\ P_5 \\ P_6
\end{array}
\begin{array}{c}
\begin{array}{cccccccccccccc}
x_{s1} & x_{12} & x_{25} & x_{58} & x_{13} & x_{36} & x_{68} & x_{14} & x_{47} & x_{78} & x_{89} & x_{81} & x_{9} & x_{10}
\end{array} \\
\begin{bmatrix}
1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \\
1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \\
1 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \\
1 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 0 & 1 \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 & 1 & 0
\end{bmatrix}
\end{array}
$$

Gets all minimal cut of a computer server,

$\{[x_{s1}], [x_{12}, x_{13}, x_{14}], [x_{25}, x_{36}, x_{47}], [x_{58}, x_{68}, x_{78}], [x_{89}\ , x_{810}], [x_{9T}\ , x_{10T}], [x_{12}, x_{36}, x_{47}], [x_{12}, x_{36}, x_{78}], [x_{12}, x_{36}, x_{810}], [x_{12}, x_{36}, x_{10T}], [x_{14}, x_{36}, x_{58}], [x_{14}, x_{36}, x_{89}], [x_{14}, x_{36}, x_{9T}], [x_{14}, x_{13}, x_{25}], [x_{14}, x_{68}, x_{58}]$ $[x_{14}, x_{68}, x_{89}], [x_{13}, x_{25}, x_{810}], [x_{13}, x_{25}, x_{10T}], [x_{13}, x_{78}, x_{89}], [x_{13}, x_{78}, x_{9T}], [x_{25}, x_{68}, x_{78}], [x_{25}\ , x_{810}, x_{89}], [x_{25}, x_{36}, x_{78}], [x_{89}, x_{10T}], [x_{810}\ , x_{9T}], [x_{47}, x_{68}, x_{89}], [x_{4768}\ , x_{9T}], [x_{58}, x_{68}, x_{810}], [x_{58}, x_{68}, x_{10T}], [x_{68}\ , x_{78}, x_{89}], [x_{68}\ , x_{78}, x_{9T}], [x_{78}, x_{89}, x_{810}]\}$

4. Results

4.1 Evaluation of The Reliability of Compute Server

From generation the least amount of cutting Using the minimal cut, the system is connected in parallel and series. [11], [12], the reliability Computer is determined in Eq.2

$$R_{sys} = 1 - \prod_{i=1}^{n} (1 - R_i) \tag{2}$$

By using Eq.(1), obtain the reliability of the computer server

$$
\begin{aligned}
Rsys = {} & R_{12}R_{25}R_{58}R_{10T}R_{810}R_{s1} + R_{12}R_{36}R_{68}R_{10T}R_{810}R_{s1} + R_{14}\,R_{47}\,R_{78}\,R_{10T}\,R_{810}R_{s1} + R_{12}\,R_{25} \\
& R_{58}\,R_{84}\,R_{9T}\,R_{s1} + R_{12}R_{36}R_{68}\,R_{84}\,R_{9T}\,R_{s1} + R_{14}R_{47}R_{78}R_{84}R_{9T}R_{s1} - R_{12}R_{25}R_{36}\,R_{58}R_{68}R_{10T}R_{810} \\
& R_{s1} - R_{12}R_{25}R_{36}R_{58}R_{68}R_{84}R_{9T}R_{s1} - R_{12}R_{25}R_{58}R_{84}R_{10T}R_{810}R_{9T}R_{s1} - R_{12}R_{36}R_{68}\,R_{84}\,R_{10T}\,R_{810} \\
& R_{9T}R_{s1} - R_{14}R_{47}R_{78}R_{84}R_{10T}R_{810}R_{9T}R_{s1} - R_{12}\,R_{14}R_{25}R_{47}R_{58}\,R_{78}\,R_{10T}\,R_{810}\,R_{s1} - R_{12}\,R_{14}\,R_{36} \\
& R_{47}R_{68}R_{78}R_{10T}R_{810}R_{s1} - R_{12}R_{14}R_{25}R_{47}R_{58}R_{78}R_{84}\,R_{9T}\,R_{s1} - R_{12}\,R_{14}\,R_{36}\,R_{47}R_{68}R_{78}R_{84}\,R_{9T}\,R_{s1} \\
& + R_{12}R_{25}R_{36}R_{58}R_{68}R_{84}R_{10T}R_{810}R_{9T}R_{s1} + R_{12}R_{14}R_{25}R_{36}R_{47}R_{58}R_{68}\,R_{78}R_{10T}R_{810}\,R_{s1} + R_{12}R_{14}R_{25} \\
& R_{36}R_{47}R_{58}R_{68}R_{78}R_{84}R_{9T}R_{s1} + R_{12}R_{14}R_{25}R_{47}R_{58}R_{78}R_{84}\,R_{10T}\,R_{810}R_{9T}R_{s1} + R_{12}R_{14}R_{36}R_{47}R_{68}R_{78}
\end{aligned}
$$

$$R_{84}R_{10T}R_{810}R_{9T}R_{s1} - R_{12}R_{14}R_{25}\,R_{36}\,R_{47}\,R_{58}R_{68}R_{78}R_{84}R_{10T}R_{810}R_{9T}R_{s1}$$

4.2 Evaluate the Dominating Polynomial of Computer Server

The graph polynomial that represents the domination number of a graph G is called the domination polynomial, or D(G;x). Any vertex in the graph that is either in the set or adjacent to a vertex in the set is said to be a dominating set. The domination number of a graph is the smallest number of vertices in a dominating set.

We define the domination polynomial as follows:

$$D(G,x)=\sum_{k=0}^{v} d(G,k)x^{k} \tag{3}$$

where d(G,k) is the number of domination sets of size k in the graph G, [k=0,1,2,…] [13,14].

first compute D(Pi, x) of each path

D(P_1,x)=x^6+6x+5x^4+4x^3+3x^2+x

D(P_2,x)=x^6+6x+5x^4+4x^3+3x^2+x

D(P_3,x)=x^6+6x+5x^4+4x^3+3x^2+x

D(P_4,x)=x^6+6x+5x^4+4x^3+3x^2+x

D(P_5,x)=x^6+6x+5x^4+4x^3+3x^2+x

D(P_6,x)=x^6+6x+5x^4+4x^3+3x^2+x

So the Domination Polynomial of Computer Server

$$D(P_i,x)=\sum_{i=1}^{6} D(P_i,x)$$

$$= 6[x^6+6x^5+5x^4+4x^3+3x^2+x]$$

$$= 6x^6+36x^5+30x^4+24x^3+18x^2+6x$$

5. Conclusions

The dominant polynomial is helpful for evaluating a system's long-term reliability, contrasting the reliability of various system configurations, and figuring out the crucial elements that affect the system's overall reliability. Additionally, it can be applied to decision-making and optimization processes pertaining. The relationships between these polynomials can shed light on the interplay between different structural properties of the graph, such as independence, domination, and matching. This paper computes the reliability of the computer server by the minimum cut method and determines the domination polynomial by depending on the minimal path and shows that all domination polynomials of each path have the same look. That means each part of the computer has the same power of connection and strong reliability.

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Algorthem(1): Generating the minimal path of computer srver

```
Input: clear all
click
n=12;
A = sym('X%d%d', [n n]);
 for i=1:n
    for j=i:n
      A(i,j)=A(j,i);
   end
 end
A(2,3)=0;
A(2,4)=0;
A(3,4)=0;
A(5,6)=0;
A(5,7)=0;
A(5,4)=0;
A(6,7)=0;
A(9,10)=0;
A(8,T)=0;
A(3,7)=0;
A(5,T)=0;
A(2,T)=0;
A(1,T)=0;
A(S,T)=0;
A(7,T)=0;
A(4,T)=0;
A(1,T)=0;
A(3,T)=0;
A(6,T)=0;
 A(2,6)=0;
 A(2,7)=0;
A(2,8)=0;
 A(2,10)=0;
  A(2,9)=0;
  A(3,8)=0;
  A(3,7)=0;
  A(3,9)=0;
  A(3,10)=0;
  A(4,8)=0;
  A(4,10)=0;
  A(5,10)=0;
  A(5,9)=0;
  A(6,10)=0;
  A(6,9)=0;
  A(7,9)=0;
  A(7,10)=0;
  A(1,5)=0;
  A(1,6)=0;
```

```
  A(1,7)=0;
  A(1,8)=0;
  A(1,9)=0;
  A(S,2)=0;
  A(S,5)=0;
  A(S,3)=0;
  A(S,6)=0;
  A(S,4)=0;
  A(S,7)=0;
  A(S,9)=0;
  A(S,10)=0;
w=genconn(n);
W(2,3)=0;
W(2,4)=0;
W(3,4)=0;
W(5,6)=0;
W(5,7)=0;
W(5,4)=0;
W(6,7)=0;
W(9,10)=0;
W(8,T)=0;
W(3,7)=0;
W(5,T)=0;
W(2,T)=0;
W(1,T)=0;
W(S,T)=0;
W(7,T)=0;
W(4,T)=0;
W(1,T)=0;
W(3,T)=0;
W(6,T)=0;
 W(2,6)=0;
 W(2,7)=0;
W(2,8)=0;
 W(2,10)=0;
  W(2,9)=0;
  W(3,8)=0;
  W(3,7)=0;
  W(3,9)=0;
  W(3,10)=0;
  W(4,8)=0;
  W(4,10)=0;
  W(5,10)=0;
  W(5,9)=0;
  W(6,10)=0;
  W(6,9)=0;
  W(7,9)=0;
  W(7,10)=0;
  W(1,5)=0;
  W(1,6)=0;
```

```
  W(1,7)=0;
  W(1,8)=0;
  W(1,9)=0;
  W(S,2)=0;
  W(S,5)=0;
  W(S,3)=0;
  W(S,6)=0;
  W(S,4)=0;
  W(S,7)=0;
  W(S,9)=0;
  W(S,10)=0;
CM=(w+eye(n)).*A;
       for i=1:n
            CM(i,i)=1;
        end
       CM(1:size(CM,1)+1:end)=1;
w
CM
m=2;
CCM=sweepnode(CM,m);
CM1=[0;CCM(2:n,1)];
CM2=[0 CCM(1,2:n) 1];
CM3=[CCM CM1;CM2]
a=minimal(CM3,n);
paths=eval(['{''' strrep(char(a),'+',''';''') '''}']);
path=sym(paths);
k=0;
for i=1:numel(path)-1
    str=char(path(i));
    aa=findstr(str,'X');
    path(i)=sym(str(aa:end));
    if isnotcycle(path(i),n)
        k=k+1;
        allpaths(k,1)=path(i);
    end
end
allpaths
allvar=symvar(allpaths);
Output :minimal  path
Beg
```

Algorithm (2) : Generating the minimal cut of computer server graph

```
Input:all the minimal path
for i=1:numel(allpaths)
      IM(i,:)=findvar(allvar,symvar(allpaths(i)));
  end
  kk=0;
   for k=1:numel(allvar)
        c = combnk(1:size(IM,2),k);
```

```
        for i=1:size(c,1)
        IMMM=sum(allor(IM(:,c(i,:))));
        if IMMM==numel(allpaths)
           if kk==0
              kk=kk+1;
             CUT{kk}=allvar(c(i,:));
             display(CUT{kk})
           else
              if isnewcut(CUT,allvar(c(i,:)))
             kk=kk+1;
              CUT{kk}=allvar(c(i,:));
             display(CUT{kk})
              end
          end
        end
    end
     if kk>37
        break
    end
 end
Output: minimal cut
```

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